Section A-Research paper

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# Restrained Domination in Middle Graph

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**ABSTRACT**

The middle graph of a graph G, denoted by $M(G)$ is a graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is an edge incident with it. A dominating set $D$ of $M(G)$ is called a restrained dominating set of $M(G)$ if every vertex not in $D$ is adjacent to a vertex in $D$ and to a vertex in $V[M(G)] - D$. The minimum cardinality of $D$ is called the restrained domination number of $M(G)$ and is denoted by $\gamma_r\,[M(G)]$.

In this paper, we establish the upper and lower bounds on $\gamma_r\,[M(G)]$ and compare with other dominating parameters of G and elements of G were obtained.

**KEYWORDS:_** Graph, Middle graph, Restrained Domination number.

**SUBJECT CLASSIFICATION NUMBER: AMS 05C69, O5C70.**

## INTRODUCTION:

By a graph $G = (V, E)$ be mean of finite undirected graphs without loops and multiple edges. Terms not here are used in the sense of Harary[2].

As usual the maximum degree of a vertex in $V(G)$ is denoted by $\Delta(G)$ and maximum edge degree of edge in $E(G)$ is denoted by $\Delta'(G)$.

The notation $\alpha_0(G)(\alpha_1(G))$ is the minimum number of vertices(edges) in a vertex(edge) cover of G. The notation $\beta_0(G)(\beta_1(G))$ is the minimum number of vertices(edges) in a maximal independent set of a vertex(edge) of G.

A subset $D$ of $V(G)$ is a dominating set, if every vertex not in $D$ id adjacent to some vertex in $D$. The domination number $\gamma$ of $G$ is the minimum cardinality taken over all the minimal dominating sets of $G$ . The study of domination in graphs was begun by Ore[8] and Berge[1].

A set $D \subseteq E(G)$ is an edge dominating set, if every edge not in $D$ is adjacent to atleast one edge in $D$.The edge domination number of $G$, is denoted by $\gamma'(G)$ and is minimum cardinality of an edge dominating set. Edge domination number was studied by S.L.Mitchell and Hedetniemi in [6].

For any real number $x$, $\lfloor x \rfloor$ denotes the greater integer not greater then $x$.

In [4] defined the strong domination is denoted by $\gamma_{st}(G)$. A dominating set $D$, if for every vertex $u \in V(G) - D$ there exist a vertex $v \in D$ with $\deg(v) \geq \deg(u)$ and $u$ is adjacent to $v$.

Let $D$ is a weak dominating set is denoted by $\gamma_w(G)$, if for every vertex $u \in V(G) - D$ there exist a vertex $v \in D$ with $\deg(v) \leq \deg(u)$ and $u$ is adjacent to $v$ see[4].

The concept Roman domination function (RDF) in a graph $G = (V, E)$ is a function $f: \longrightarrow \{0,1,2\}$ of satisfying the condition that every vertex u for which f(u)=0 is adjacent to atleast one vertex of v for which f(v)=2 in G. The weight of a Roman dominating function is the value f(v)=$\sum_{u \in v} f(u)$. The minimum weight of a Roman domination function of a graph G is called a Roman domination number and is denoted by $\gamma_R(G)$.

A dominating set $D$ is a dominating set whose induces subgraph $< D >$ is connected. The connected domination number of a graph G denoted by $\gamma_c(G)$ , is the minimum cardinality of a dominating set of G. Similarly , the connected edge domination number $\gamma_c'(G)$ is the minimum cardinality of a connected edge dominating set of G.

A total dominating set $D$ of G is a coregular total dominating set if the induced subgraph $< V - D >$ is regular. The coregular total domination number $\gamma_{crt}(G)$ of G is the minimal cardinality of a coregular total dominating set see[7].

**RESULTS:**

The following theorem relates the upper bound for $\gamma_r[M(G)]$ in terms of $\gamma_R(G)$ and strong domination number of G $\gamma_{st}(G)$.

**Theorem 1:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \gamma_R(G) + \gamma_{st}(G) \text{ and } G \neq W_p.$$

**Proof:** Suppose $G = W_n$. Then $\gamma_r[M(G)] \nleq \gamma_R(G) + \gamma_{st}(G)$.Hence $G \neq W_p$. Let $f: V(G) \rightarrow \{0,1,2\}$and partition the vertex set $V(G)$ into $(V_0, V_1, V_2)$ induced by f with $|V_i| = n_i$ for dominates i=0,1,2. Suppose the set $V_2$ dominated the set $V_0$. Then $H = V_1 \cup V_2$ forms a minimal Roman dominating set of G.

Suppose $A = \{v_1, v_2, v_3, \dots \dots v_n\} \subseteq V(G)$ be the minimum set of vertices with maximal degree in $G$ such that $\deg(v_i) \geq \deg(u_i)$ where $\forall v_i \in A$ and $\forall u_i \in V(G) - A$ and $v_i$ is adjacent to $u_i$. Further, if $N[A] = V(G)$, then $A$ is a $\gamma_{st}$ of G.

Further, since $V[M(G)] = V(G) \cup E(G)$, let there exist a set $B = \{v_1, v_2, v_3, \dots \dots v_m\}$ which divides each edge of G with $|B| = q$. Suppose $A_1 \subseteq A$ and $B_1 \subset B$. Then $\{A_1 \cup B_1\}$ cover all vertices of $M(G)$ with $N[A_1 \cup B_1] = V[M(G)]$. Clearly, $\{A_1 \cup B_1\}$ is a minimal dominating set of $M(G)$. If every vertex of $V[M(G)] - \{A_1 \cup B_1\}$ is adjacent to at least one vertex of $\{A_1 \cup B_1\}$ and a vertex of $V[M(G)] - \{A_1 \cup B_1\}$. Then $\{A_1 \cup B_1\}$ is a restrained dominating set of $M(G)$. It is clear that,

$$|\{A_1 \cup B_1\}| \leq |H| + |A| \text{ gives}$$

$$\gamma_r[M(G)] \leq \gamma_R(G) + \gamma_{st}(G).$$

The following theorem relates $\gamma_r[M(G)]$ with $\beta_1(G)$ and weak domination number of G.

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**Theorem 2:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \beta_1(G) + \gamma_w(G).$$

**Proof:** Let $B = \{ e_1, e_2, e_3 \ldots\ldots . e_n\} \subseteq E(G)$be the maximal set of edges with $N(e_i) \cap N(e_j) = e$, for every $e_i$, $e_j \in B, 1 \leq i \leq n$, $1 \leq j \leq n$ and $e \in E(G) - B$. Clearly, B forms a maximal independent edge set in G. Suppose $A = \{v_1, v_2, v_3, \ldots\ldots v_k\} \subseteq V(G)$ be the set of vertices. If every vertex $u_j \in V(G) - A$ is adjacent with $\forall v_i \in A$, $1 \leq i \leq k$ and $\deg(v_i) \leq \deg(u_j)$ ,$u_j$ is adjacent to $v_i$. Such that $N[A] = V(G)$. Then $A$ is a weak dominating set of G.

Further, $V[M(G)] = V(G) \cup E(G)$. Let $H = \{v_1, v_2, v_3, \ldots\ldots v_m\}$ be the set of vertices which are dividing each edge of G such that $|H| = q$. Now we consider $H_1 \subset H$ and a set $N = \{v_1, v_2, v_3, \ldots\ldots v_n\}$ the set of all end vertices of $M(G)$. Suppose $\{H_1 \cup N\}$ be the minimal set of vertices which covers all the vertices of $M(G)$ and every vertex not in $\{H_1 \cup N\}$ is adjacent with atleast one vertex of $\{H_1 \cup N\}$ and atleast one vertex of $V[M(G)] - \{H_1 \cup N\}$. Clearly, $\{H_1 \cup N\}$ is a minimal restrained dominating set of $M(G)$.

Hence, $|\{H_1 \cup N\}| \leq |B| + |A|$ gives

$$\gamma_r[M(G)] \leq \beta_1(G) + \gamma_w(G).$$

A dominating set D of a graph G is a cototal dominating set if the induced subgraph $< V - D >$ has no isolated vertices. The cototal domination number $\gamma_{cot}(G)$ is the minimum cardinality of a cototal dominating set of G.

The following theorem relates the domination number, cototal domination number of G and $\gamma_r[M(G)]$.

**Theorem 3:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \gamma_{cot}(G) + \gamma(G) + 1 \text{ and } \neq W_p \text{ , } G \neq K_{1,p} \text{ ; } p \geq 4.$$

**Proof:** Suppose G is either $W_p$ *or* $K_{1,p}(p \geq 4)$. Then $\gamma_r[M(G)] \nleq \gamma_{cot}(G) + \gamma(G) + 1$. Hence $G \neq W_p$ and $G \neq K_{1,p}$ *with* $p \geq 4$.

Let $D = \{v_1, v_2, v_3, \ldots\ldots v_n\} \subseteq V(G)$ be the minimal set of vertices, such that every vertex of $V(G) - D$ is adjacent to at least one vertex of D. Then D is a $\gamma$ −set of G.

Suppose the induced subgraph $< V(G) - D >$has no isolated vertex, then D forms a cototal dominating set of G. Otherwise, there exists at least one vertex $w \in N[V(G) - D]$ such that induced subgraph $< [V(G) - D] \cup \{w\} >$ has no isolates. Clearly, $[D \cup \{w\}]$ forms a cototal dominating set of G.

Further, let $S = \{u_1, u_2, u_3, \ldots\ldots\ldots\ldots u_n\}$ be the vertex set of $M(G)$. Suppose there exists $S_1 \subseteq S$ such that every vertex not in $S_1$ is adjacent to at least one vertex of $S_1$ and $V[M(G)] - S_1$, so that $N[S_1] = V[M(G)]$. Hence $S_1$is a minimal restrained dominating set of $M(G)$.

Clearly, $|S_1| \leq |[D \cup \{w\}]| + |D| + 1$.

Hence, $\gamma_r[M(G)] \leq \gamma_{cot}(G) + \gamma(G) + 1$.

A dominating set $D \subseteq V(G)$ is a double dominating set of G, if each vertex in V is dominated by at least two vertices of D. A subset $D^d$of G is a double dominating set if for every vertex $v \in V(G)$, $|N(v) \cap D^d| \geq 2$, that is v is in $D^d$and has at least o ne neighbour in $D^d$

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or v is in $V(G)-D^d$ has at least two neighbours in $D^d$. The double domination number $\gamma_{dd}(G)$ of G is the minimum cardinality of a double dominating set of G see[3].

**Theorem 4:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \gamma_{dd}(G)+\Delta(G).$$

**Proof:** Let $V_1=\{v_1, v_2, v_3, \ldots\ldots v_n\} \subseteq V(G)$ be the set of all non-end vertices in G. Then there exists atleast one vertex $v \in V_1$ such that $\deg(v)=\Delta(G)$.

Suppose $D=\{v_1, v_2, v_3, \ldots\ldots v_n\} \subseteq V(G)$ and every vertex of $V(G)-D$ is adjacent to at least one vertex of $D$ such that $N[D]=V(G)$. Then $D$ is a minimal dominating set of G. Now, consider $V_2=V(G)-D$ and $D_2=\{v_1, v_2, v_3, \ldots\ldots v_j\} \subseteq V_2$,then $D^d=V_1 \cup V_2$ forms a double dominating set of G.

Further, let $A_1=\{u_{1,} u_{2,} u_{3,} \ldots\ldots\ldots\ldots u_n\}=V[M(G)]$. Suppose $K \subseteq A_1$ so that if every vertex not in $\{K\}$ is adjacent with at least one vertex of $\{K\}$ and at least one vertex of $V[M(G)]-K$ such that $N[K]=V[M(G)]$. Then $K$ is a minimal restrained dominating set of $M(G)$. Hence,

$$|K| \leq |D^d|+\Delta(G), \text{ gives}$$

$$\gamma_r[M(G)] \leq \gamma_{dd}(G)+\Delta(G).$$

**Theorem 5:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq p-\alpha_0(G)+\gamma_c'(G).$$

**Proof:** Let $M=\{v_1, v_2, v_3, \ldots\ldots v_p\}=V(G)$ such that $|M|$.Suppose $B=\{v_1, v_2, v_3, \ldots\ldots v_n\} \subseteq V(G)$ be the minimum number of vertices which covers all the edges of G. Then $|B|=\alpha_0(G)$. Let, now $S=\{e_1, e_1, e_2, e_3 \ldots\ldots . e_i\}$ be the minimal edge dominating set of G and the subgraph $<S>$ does not contain more then one component. Then $S$ itself is a connected edge dominating set of G. Otherwise, if the induced subgraph $<S>$ has more than one component , then attach the minimum number of edges $\{e_k\} \in E(G)-S$ such that $S_1=\{S \cup e_k\}$ forms exactly one component. Clearly, $S_1$ is a $\gamma_c'$ −set of G.

Suppose $A \subset V[M(G)]$, if every vertex not in {A} is adjacent with at least one vertex of $\{A\}$ and at least one vertex of $V[M(G)-\{A\}]$ and $N[A]=V[M(G)]$.Then $A$ is a minimal $\gamma_r$ −set of $M(G)$. Since $M \subset A, B \subset A$, $S_1 \subset A$ and $V[M(G)]=V(G) \cup E(G)$.

Then it follows that, $|A| \leq |M|-|B|+|S_1|$ which gives

$$\gamma_r[M(G)] \leq p-\alpha_0(G)+\gamma_c'(G).$$

In the following theorem, we develop an upper bound for $\gamma_r[M(G)]$ in terms vertices of G, $\gamma_{ss}(G)$ and $\beta_0(G)$.

A dominating set D of a graph G is a strong split dominating set if the induced subgraph $<V-D>$ is totally disconnected with atleast two vertices. The strong split domination number $\gamma_{ss}(G)$ of a graph G is the minimum cardinality of a strong split dominating set of G. See[5].

**Theorem 6:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq p-\beta_0(G)+\gamma_{ss}(G)+1 \text{ and } G \neq K_{1,p} \text{ ; } p \geq 3.$$

**Proof:** Suppose $G=K_{1,n}$ .Then $\gamma_r[M(G)] \nleq p-\beta_0(G)+\gamma_{ss}(G)+1$. Let $A=\{v_1, v_2, v_3, \ldots\ldots v_n\}$ be the set of all end vertices in G. Consider $K=\{v_1, v_2, v_3, \ldots\ldots v_k\} \subseteq$

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$V(G)$ be the maximum set of vertices with $\deg(u,v) \geq 2$ and $N(u) \cap N(v) = x$, $\forall u, v \in K$ and $x \in V(G) - K$.Clearly,$|K| = \beta_0(G)$. Now, let $S = \{v_1, v_2, v_3, \dots\dots v_k\} \subseteq V(G)$such that every vertex of $V(G) - S$ is adjacent to at least one vertex of $S$ and $N[S] = V(G)$. If the induced subgraph$< S >$ is totally disconnected, then $S$ is a $\gamma_{ss}$ −set of G.

Further, suppose $L = \{u_1, u_2, u_3, \dots\dots\dots\dots u_m\}$ be the set of vertices and each vertex of $L$ is dividing each edge of G in $M(G)$. Assume there exists a set $L_1 \subset L$ and formsS $\{A \cup L_1\}$ is a dominating set of $M(G)$. If every vertex not in $\{A \cup L_1\}$ is adjacent with at least one vertex of $\{A \cup L_1\}$ and a vertex of $V[M(G)] - \{A \cup L_1\}$. Thus, $\{A \cup L_1\}$ is the minimal restrained dominating set of $M(G)$.

Hence, $|A \cup L_1| \leq p - |K| + |S| + 1$ implies that

$$\gamma_r[M(G)] \leq p - \beta_0(G) + \gamma_{ss}(G) + 1.$$

Again in the following theorem we establish an upper bound for $\gamma_r[M(G)]$ in terms of connected domination of G and maximum edge degree of G.

**Theorem 7:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \gamma_c(G) + \Delta^{'}(G).$$

**Proof:** Let $S_1 = \{v_1, v_2, v_3, \dots\dots v_k\}$ be the set of all end vertices in G. Suppose $S_2 \subset V(G) - S_1$ be the minimal set of vertices such that $N[v_i] = V(G), \forall v_i \in S_2$. Then $S_2$ form a minimal dominating set of G.

Further, if $S_2$ has exactly one component then $S_2$ itself is a connected dominating set of G. Suppose $S_2$ has more than one component. Then attach the minimum set of vertices $\{v_k\}$, such that $S_3 = S_2 \cup \{v_k\}$ which are in every $u - v$ path in $V(G) - S_2$. Hence $S_3$ is a minimal connected dominating set of G. Let e be an edge in G with maximum degree $\Delta'$.

Now, let $K = \{v_1, v_2, v_3, \dots\dots v_k\} \subset V(G) - S_1$. Suppose $K_1 \subset K$ and $\{S_1 \cup K_1\}$ is minimal dominating set of $M(G)$. If every vertex not in $\{S_1 \cup K_1\}$ is adjacent with at least one vertex of $\{S_1 \cup K_1\}$ and a vertex of $V[M(G)] - \{S_1 \cup K_1\}$. It follows that $\{S_1 \cup K_1\}$ is the $\gamma_r$ −set of $M(G)$.

Hence, $|S_1 \cup K_1| \leq |S_3| + \Delta^{'}(G)$ gives

$$\gamma_r[M(G)] \leq \gamma_c(G) + \Delta'(G).$$

Following theorem gives an upper bound for our concept.

**Theorem 8:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \leq \alpha_1(G) + \gamma^{'}(G).$$

**Proof:** Let $S = \{e_1, e_2, e_3 \dots\dots e_i\} \subseteq E(G)$ be the set of all end edges in G. Then $S \cup J$ where $J \subseteq E(G) - S$ be the minimal set of edges which covers all the vertices of G, such that $|S \cup J| = \alpha_1(G)$. Suppose $H = \{e_1, e_2, e_3 \dots\dots e_m\} \subseteq E(G)$ such that for each $e_i \in H$, $i = 1,2,3, \dots\dots m$, $N[H] \cap H = \Phi$. Then $|H| = \gamma^{'}(G)$.

Further, let $V = \{v_1, v_2, v_3, \dots\dots v_m\}$be the vertex set of $M(G)$. Suppose there exists $K \subseteq V$ and $S_1 = \{u_1, u_2, u_3, \dots\dots\dots\dots u_i\}$being the set of vertices sub dividing each edge $e_k \in S$, $1 \leq k \leq i$, in $M(G)$. Suppose there exists $S_2 \subseteq S_1$such that $\{S_2 \cup k\} \subset V[M(G)]$. Now assume that every vertex not in $\{S_2 \cup k\}$ is adjacent to at least one vertex $\{S_2 \cup k\}$ and at least one vertex of $V[M(G)] - \{S_2 \cup k\}$. Then $\{S_2 \cup k\}$ is a minimal restrained dominating set of $M(G)$.

Thus, $| S_2 \cup K| \leq |S \cup J| + |H|$ gives

$$\gamma_r[M(G)] \leq \alpha_1(G) + \gamma'(G).$$

Now we established the lower bound for $\gamma_r[M(G)]$ in terms of co-regular total domination.

**Theorem 9:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \geq \gamma_{crt}(G) - 2.$$

Proof: Let $D$ be a minimal dominating set of G. If the induced subgraph $< D >$ has no isolates, then $D$ be a total dominating set of G. Suppose in the induced subgraph$< V(G) - D >$,each vertex has same degree. Then $D$ be a co-regular total dominating set of G.

Suppose $V_1 = \{v_1, v_2, v_3, \ldots\ldots v_m\}$ be the vertex set of $M(G)$. Then there exists a set $K \subset V_1$, if every vertex not in $\{K\}$ is adjacent with at least one vertex of $\{K\}$ and a vertex of $V[M(G)] - K$ so that $N[K] = V[M(G)]$. It shows that $\{K\}$ is a restrained dominating set of $M(G)$. Which gives,

$$|K| \geq |D| - 2.$$

Hence $\gamma_r[M(G)] \geq \gamma_{crt}(G) - 2$.

A dominating set $D \subseteq V(G)$ is split dominating set if the induced subgraph $< V(G) - D >$ has more than one component. The split domination number $\gamma_s$ of G is the minimum cardinality of a minimal split dominating set. For details see[5].

In the following theorem our concept has been related with the vertices and split domination of G.

**Theorem 10:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \geq p - \gamma_s(G) - 2.$$

Proof: Suppose $A = \{v_1, v_2, v_3, \ldots\ldots v_m\} \subseteq V(G)$ be the set of all end vertices in G. Let $A' = V(G) - A$ and consider the set $F \subseteq A'$ such that $< V(G) - F >$ is disconnected and $N[F] = V(G)$. Hence $F$ is a $\gamma_s$ −set of G.

Further, $D_1 \subseteq V[M(G)] = V(G) \cup E(G)$. Now, let $D_2 \subset D_1 - A$ and for the set $\{A \cup D_2\}$, if every vertex not in $\{A \cup D_2\}$ is adjacent with a vertex of $\{A \cup D_2\}$ and a vertex of $V[M(G)] - \{A \cup D_2\}$. then $\{A \cup D_2\}$ is the minimal restrained dominating set of $M(G)$.

Hence, $|A \cup D_2| \geq p - |F| - 2$,

$$\gamma_r[M(G)] \geq p - \gamma_s(G) - 2.$$

**Theorem 11:** For any connected (p,q) graph G,

$$\gamma_r[M(G)] \geq \left\lfloor \frac{2q}{3} \right\rfloor - 1 \text{ and } G \neq W_p;\ p \geq 5.$$

**Proof:**Suppose$G = W_p$. Then $\gamma_r[M(G)] \ngeq \left\lfloor \frac{2q}{3} \right\rfloor - 1$. Hence$G \neq W_p;\ p \geq 5$.
Let $E = \{ e_1, e_2, e_3 \ldots\ldots e_q\}$ be the edge set of G with $|E| = q$. Further, if $H = \{u_1, u_2, u_3, \ldots\ldots\ldots\ldots u_m\}$ be the vertex set of $M(G)$. Suppose there exists $K \subseteq H$, if every vertex

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not in $\{K\}$ is adjacent to at least one vertex of $\{K\}$ and a vertex of $V[M(G)] - K$ such that $N[K] = V[M(G)]$. Hence $K$ is the minimal restrained dominating set of $M(G)$.

$$\text{Clearly, } |K| \geq \left\lfloor \frac{2q}{3} \right\rfloor - 1 \text{ Which gives}$$

$$\gamma_r[M(G)] \geq \left\lfloor \frac{2q}{3} \right\rfloor - 1.$$

## CONCLUSIONS

In this paper we surveyed selected results on restrained domination in middle graph. These results establish optimal upper bounds and lower bounds on the restrained middle domination number in terms of the vertex covering, line covering, the independence domination number, split domination number, double domination number and different domination parameters of a graph G.

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