Section A -Research paper

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Interpolation of Inverse Quintic Spline on Applying Polynomial Iteration Method

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Abstract

Quintic Spline interpolation is the process of interpolation using splines of degree five. This paper deals with the derivation of inverse quintic spline using Polynomial Iteration technique and its evaluation using a set of data points, then corresponding error is also calculated.

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Keywords: Interpolation, Spline, Natural Quintic Spline, Polynomial Iteration Method.

1. Introduction

Spline interpolation is a form of interpolation using piecewise polynomial called splines. Spline interpolation is often preferred over polynomial interpolation because the interpolation error can be reduced considerably with different degree splines.

Based on the first derivative spline conditions, splines can be classified into different types such as Natural Spline, Parabolic Runout Spline, Cubic Runout Spline, Periodic Spline, Extrapolated

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Spline. This paper deals with Natural Quintic spline interpolation and the inverse quintic spline are derived using Polynomial Iteration method

2. NATURAL QUINTIC SPLINE

Consider a set of data points (x_i, y_i) where $i = 0,\ 1,\ 2 \ldots \ldots .., n$ of the function $y = f(x)$. Let $I = [x_i, x_{i+1}]$ be a subinterval of $[x_0, x_n]$. Let $S_i(x)$ denote the spline function of n functions and every spline work is characterized in the interval $[x_i, x_{i+1}]$. On account of quintic spline, the functions $S_i(x)$ are the quintic spline polynomials coefficients has to be resolved.

Definition:

A spline polynomial $S(x)$ of degree n which is piecewise, having continuous derivative up to order $n-1$ considering each subinterval $S_i(x_i)$ as polynomials of degree n.

$$S(x) = \begin{cases} S_0(x), & x \in [x_0, x_1] \\ S_1(x), & x \in [x_1, x_2] \\ \quad . & \\ \quad . & \\ \quad . & \\ S_n(x), & x \in [x_{n-1}, x_n] \end{cases}$$

A Formula for Natural Quintic spline has been derived, details of which can be had from [2] based on the following boundary conditions:

Boundary conditions

The limit conditions are,

a) $S_i(x_i) = y_i,\ i = 0,1,2 \ldots .. n$
b) $S_i(x_{i+1}) = y_{i+1},\ i = 0,1,2 \ldots .. n-2$
c) $S_i(x_i),\ S_i{}'(x_i),\ S_i{}''(x_i), S_i{}'''(x_i)\ and\ S_i^{1v}(x)$ are continuous.
d) $S_i{}^{1v}(x_0) = S_{i+1}{}^{1v}\ (x_n) = 0$
e) $S_{n-1}{}''(x_n) = 0$
f) $S_{n-1}{}'''(x_n) = 0$

On applying these conditions, we get a set of equation with coefficients. Solving the coefficients of the functions which on substituting gives the derivation of the natural quintic spline functions $S_i(x)$.

3. Derivation of Quintic Spline

Since $S_i(x)$ is a quintic spline, $S_i{}'^{v}(x)$ is linear.

$$S_i{}'^{v}(x) = \frac{1}{h_i}[(x_{i+1} - x)M_i + (x - x_i)M_{i+1}],\ i = 0,1,2 \ldots .. n-1 \qquad (1)$$

where $h_i = x_{i+1} - x_i$

Integrating (1) four times,

$$S_i(x) = \frac{1}{h_i}\left[\frac{(x_{i+1}-x)^5}{120}M_i + \frac{(x-x_i)^5}{120}M_{i+1}\right] + C_i(x_{i+1}-x)(x-x_i)^2 + D_i(x_{i+1}-x)(x-x_i) + E_i(x_{i+1}-x) + F_i(x-x_i) \quad (2)$$

Using the boundary conditions (a) and (b) in (1),

$$E_i = \frac{y_i}{h_i} - \frac{{h_i}^3}{120}M_i \quad (3)$$

$$F_i = \frac{y_{i+1}}{h_i} - \frac{{h_i}^3}{120}M_{i+1}, \text{for } i = 0,1,2 \ldots\ldots n-1 \quad (4)$$

Differentiating (2), the first derivative of $S_i(x)$ can be obtained:

$$S_i{}'(x) = \frac{1}{h_i}\left[-\frac{(x_{i+1}-x)^4}{24}M_i + \frac{(x-x_i)^4}{24}M_{i+1}\right] + C_i(2xx_{i+1} - 2x_ix_{i+1} - 3x^2 + 4xx_i - {x_i}^2) + D_i(x_{i+1} + x_i - 2x) + E_i(-1) + F_i \quad (5)$$

Then the following recurrence relation is obtained:

$$\left[\frac{{h_i}^3}{24} - \frac{{h_{i+1}}^3}{24}\right]M_{i+1} + \frac{{h_{i+1}}^3}{120}[M_{i+2} - M_{i+1}] - \frac{{h_i}^3}{120}[M_{i+1} - M_i] + [C_{i+1}{h_{i+1}}^2 - C_i{h_i}^2] + [D_{i+1}h_{i+1} - D_ih_i] = \Delta_{i+1} - \Delta_i \quad (6)$$

Where $\Delta_i = \frac{y_{i+1} - y_i}{h_i}$

Differentiating (5), we get $S_i{}''(x)$:

$$S_i{}''(x) = \frac{1}{h_i}\left[\frac{(x_{i+1}-x)^3}{6}M_i + \frac{(x-x_i)^3}{6}M_{i+1}\right] + C_i[2x_{i+1} - 6x + 4x_i] + D_i[-2] \quad (7)$$

On applying the continuity condition defined in (c) (ii), we get the next expression follows:

$$D_i = D_{i+1} - \left[\frac{Z_i}{12}\right]M_{i+1} + 2[C_{i+1}h_{i+1} - C_ih_i] \text{ for } i = 0,1,2 \ldots\ldots n-1 \quad (8)$$

Once again differentiating (7), the third derivative is obtained:

$$S_i{}'''(x) = \frac{1}{h_i}\left[-\frac{(x_{i+1}-x)^2}{2}M_i + \frac{(x-x_i)^2}{2}M_{i+1}\right] - 6C_i \quad (9)$$

Again applying the condition defined in (c), (iii) the new expression is:

$$C_i = C_{i+1} + \left[\frac{h_i - h_{i+1}}{12}\right]M_{i+1} \text{ for } i = 0,1,2 \ldots\ldots n-1 \quad (10)$$

Replacing equation (8) in (6) then the following relation is obtained:

$$h_i^3[M_i - 6M_{i+1}] + h_{i+1}^3[M_{i+2} - 6M_{i+1}] + 120C_{i+1}Z_i + 10h_i^2h_{i+1}M_{i+1} + 120[D_{i+1}h_{i+1} - D_ih_i] = 120[\Delta_{i+1} - \Delta_i] \quad \text{where } Z_i = h_{i+1}^2 - h_i^2 \text{ for } i = 0,1,2 \ldots\ldots n-1 \quad (11)$$

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Using the Boundary Conditions (d), (e) and (f) in (8), (10) and (11) gives
$C_{n-1} = 0 \; and \; D_{n-1} = 0$ (12)

Finally on solving (6), (8) and (9) and replacing it into (2) along with (3), (4) and (12), the spline function $S_i(x)$ is obtained.

4. Inverse By Polynomial Iteration Method

Consider a fifth-degree polynomial

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$$

$$y - a_0 = a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$$

$$a_1x = y - a_0 - a_2x^2 - a_3x^3 - a_4x^4 - a_5x^5$$

$$x = \frac{y-a_0}{a_1} - \frac{a_2}{a_1}x^2 - \frac{a_3}{a_1}x^3 - \frac{a_4}{a_1}x^4 - \frac{a_5}{a_1}x^5 \quad (13)$$

On neglecting the higher powers of x and using the first approximation for x is

$$\boldsymbol{x}^{(1)} = \frac{y-a_0}{a_1} \quad (14)$$

Substituting (2) in (1) the second approximation is

$$x^{(2)} = \frac{y-a_0}{a_1} - \frac{a_2}{a_1}\left[\frac{y-a_0}{a_1}\right]^2 - \frac{a_3}{a_1}\left[\frac{y-a_0}{a_1}\right]^3 - \frac{a_4}{a_1}\left[\frac{y-a_0}{a_1}\right]^4 - \frac{a_5}{a_1}\left[\frac{y-a_0}{a_1}\right]^5$$

$$\begin{aligned} x^{(2)} = \frac{y-a_0}{a_1} &- \frac{a_2}{a_1}\left[\frac{y^2 - 2a_0y + a_0^2}{a_1^2}\right] - \frac{a_3}{a_1}\left[\frac{y^3 - 3a_0y^2 + 3a_0^2y + a_0^3}{a_1^3}\right] \\ &- \frac{a_4}{a_1}\left[\frac{y^4 - 4a_0y^3 + 6a_0^2y^2 - 4a_0^3y + a_0^4}{a_1^4}\right] \\ &- \frac{a_5}{a_1}\left[\frac{y^5 - 5a_0y^4 + 10a_0^2y^3 - 10a_0^3y^2 + 5a_0^4y - a_0^5}{a_1^5}\right] \end{aligned}$$

$$\begin{aligned} x = \left[\frac{-a_5}{a_1^6}\right]y^5 + \left[\frac{5a_5a_0}{a_1^6} - \frac{a_4}{a_1^5}\right]y^4 + \left[\frac{-10a_5a_0^2}{a_1^6} + \frac{4a_4a_0}{a_1^5} - \frac{a_3}{a_1^4}\right]y^3 \\ + \left[\frac{10a_5a_0^3}{a_1^6} - \frac{6a_4a_0^2}{a_1^5} + \frac{3a_3a_0}{a_1^4} - \frac{a_2}{a_1^3}\right]y^2 \\ + \left[\frac{-5a_5{a_0}^4}{a_1^6} + \frac{4a_4a_0^3}{a_1^5} - \frac{3a_3{a_0}^2}{a_1^4} + \frac{2a_2a_0}{a_1^3} + \frac{1}{a_1}\right]y \\ + \left[\frac{a_5{a_0}^5}{a_1^6} - \frac{a_4{a_0}^4}{a_1^5} + \frac{a_3{a_0}^3}{a_1^4} - \frac{a_2{a_0}^2}{a_1^3} - \frac{a_0}{a_1}\right] \end{aligned}$$

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5. Derivation Of Inverse Quintic Spline Using Polynomial Iteration Method

Consider the quintic spline interpolation formula

$$S_i(x) = \left[\frac{h_i^4 M_{i+1}}{120} + F_i h_i\right] + \left[\frac{-5h_i^3 M_{i+1}}{120} + C_i h_i^2 + D_i h_i + E_i - F_i\right] X_i + \left[\frac{h_i^2 M_{i+1}}{12} - 2C_i h_i - D_i\right] {X_i}^2 + \left[C_i - \frac{h_i M_{i+1}}{12}\right] {X_i}^3 + \left[\frac{M_{i+1}}{24}\right] {X_i}^4 + \left[\frac{M_i - M_{i+1}}{120 h_i}\right] {X_i}^5$$

$y_i = S_i(x) = P_i + Q_i X_i + R_i {X_i}^2 + S_i {X_i}^3 + T_i {X_i}^4 + W_i {X_i}^5$ be a fifth-degree polynomial

Applying Polynomial Iteration Method in equation (5), so that we get an inverse equation for X_i in terms of y where $h_i = x_{i+1} - x_i \; i = 0,1,2, \dots \dots . n$

Where $P_i = \frac{h_i^4 M_{i+1}}{120} + F_i h_i$

$Q_i = \frac{-5h_i^3 M_{i+1}}{120} + C_i h_i^2 + D_i h_i + E_i - F_i$

$R_i = \frac{h_i^2 M_{i+1}}{12} - 2C_i h_i - D_i$

$S_i = C_i - \frac{h_i M_{i+1}}{12}$

$T_i = \frac{M_{i+1}}{24}$

$W_i = \frac{M_i - M_{i+1}}{120 h_i}$

$y_i - P_i = Q_i X_i + R_i {X_i}^2 + S_i {X_i}^3 + T_i {X_i}^4 + W_i {X_i}^5$

$Q_i X_i = y_i - P_i - R_i {X_i}^2 - S_i {X_i}^3 - T_i {X_i}^4 - W_i {X_i}^5$

$$X_i = \frac{y_i - P_i}{Q_i} - \frac{R_i}{Q_i} {X_i}^2 - \frac{S_i}{Q_i} {X_i}^3 - \frac{T_i}{Q_i} {X_i}^4 - \frac{W_i}{Q_i} {X_i}^5 \quad (15)$$

The first approximation for X_i is obtained by neglecting the higher powers of X_i, Hence

$$X_i^{(1)} = \frac{y_i - P_i}{Q_i} \quad (16)$$

Now the second approximation is obtained by substituting (4) in (3)

$$X_i^{(2)} = \frac{y_i - P_i}{Q_i} - \frac{R_i}{Q_i}\left[\frac{y_i - P_i}{Q_i}\right]^2 - \frac{S_i}{Q_i}\left[\frac{y_i - P_i}{Q_i}\right]^3 - \frac{T_i}{Q_i}\left[\frac{y_i - P_i}{Q_i}\right]^4 - \frac{W_i}{Q_i}\left[\frac{y_i - P_i}{Q_i}\right]^5$$

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$$X_i^{(2)} = \frac{y_i - P_i}{Q_i} - \frac{R_i}{Q_i}\left[\frac{{y_i}^2 - 2y_iP_i + P_i^2}{{Q_i}^2}\right] - \frac{S_i}{Q_i}\left[\frac{y_i^3 - 3{y_i}^2P_i + 3y_i{P_i}^2 - P_i^3}{{Q_i}^3}\right] - \frac{T_i}{Q_i}\left[\frac{y_i^4 - 4{y_i}^3P_i + 6{y_i}^2{P_i}^2 - 4y_i{P_i}^3 + P_i^4}{{Q_i}^4}\right] - \frac{W_i}{Q_i}\left[\frac{y_i^5 - 5y_i^4P_i + 10{y_i}^3{P_i}^2 - 10{y_i}^2{P_i}^3 + 5y_i{P_i}^4 - P_i^5}{{Q_i}^5}\right]$$

$$X_i^{(2)} = \left[-\frac{W_i}{{Q_i}^6}\right]y_i^5 + \left[\frac{5W_iP_i}{{Q_i}^6} - \frac{T_i}{{Q_i}^5}\right]y_i^4 + \left[\frac{-10W_i{P_i}^2}{{Q_i}^6} + \frac{4T_iP_i}{{Q_i}^5} - \frac{S_i}{{Q_i}^4}\right]{y_i}^3 + \left[\frac{10W_i{P_i}^3}{{Q_i}^6} - \frac{6T_i{P_i}^2}{{Q_i}^5} + \frac{3S_iP_i}{{Q_i}^4} - \frac{R_i}{{Q_i}^3}\right]{y_i}^2 + \left[\frac{-5W_i{P_i}^4}{{Q_i}^6} + \frac{4T_i{P_i}^3}{{Q_i}^5} - \frac{3S_i{P_i}^2}{{Q_i}^4} + \frac{2R_iP_i}{{Q_i}^3} + \frac{1}{Q_i}\right]y_i + \left[\frac{W_i{P_i}^5}{{Q_i}^6} - \frac{T_i{P_i}^4}{{Q_i}^5} + \frac{S_i{P_i}^3}{{Q_i}^4} - \frac{R_i{P_i}^2}{{Q_i}^3} - \frac{P_i}{Q_i}\right]$$

Let $S_i(x) = y$, and $X_i = x_{i+1} - x$, $x = x_{i+1} - X_i$

Then the Inverse quintic spline is ${S_i}^{-1}(y_i) = x_{i+1} - X_i$, Hence

$${S_i}^{-1}(y_i) = x_{i+1} + \left[\frac{W_i}{{Q_i}^6}\right]y_i^5 + \left[\frac{T_i}{{Q_i}^5} - \frac{5W_iP_i}{{Q_i}^6}\right]y_i^4 + \left[\frac{10W_i{P_i}^2}{{Q_i}^6} - \frac{4T_iP_i}{{Q_i}^5} + \frac{S_i}{{Q_i}^4}\right]{y_i}^3 + \left[-\frac{10W_i{P_i}^3}{{Q_i}^6} + \frac{6T_i{P_i}^2}{{Q_i}^5} - \frac{3S_iP_i}{{Q_i}^4} + \frac{R_i}{{Q_i}^3}\right]{y_i}^2 + \left[\frac{5W_i{P_i}^4}{{Q_i}^6} - \frac{4T_i{P_i}^3}{{Q_i}^5} + \frac{3S_i{P_i}^2}{{Q_i}^4} - \frac{2R_iP_i}{{Q_i}^3} - \frac{1}{Q_i}\right]y_i + \left[-\frac{W_i{P_i}^5}{{Q_i}^6} + \frac{T_i{P_i}^4}{{Q_i}^5} - \frac{S_i{P_i}^3}{{Q_i}^4} + \frac{R_i{P_i}^2}{{Q_i}^3} + \frac{P_i}{Q_i}\right]$$

$$\boldsymbol{{S_i}^{-1}(y_i) = x_{i+1} + l_iy_i^5 + m_iy_i^4 + n_i{y_i}^3 + o_i{y_i}^2 + p_iy_i + q_i} \quad \textbf{(17)}$$

Hence ${S_i}^{-1}(y_i)$ is the required inverse equation of a fifth degree polynomial

Where $l_i = \frac{W_i}{{Q_i}^6}$

$m_i = \frac{T_i}{{Q_i}^5} - \frac{5W_iP_i}{{Q_i}^6}$

$n_i = \frac{10W_i{P_i}^2}{{Q_i}^6} - \frac{4T_iP_i}{{Q_i}^5} + \frac{S_i}{{Q_i}^4}$

$o_i = -\frac{10W_i{P_i}^3}{{Q_i}^6} + \frac{6T_i{P_i}^2}{{Q_i}^5} - \frac{3S_iP_i}{{Q_i}^4} + \frac{R_i}{{Q_i}^3}$

$p_i = \frac{5W_i{P_i}^4}{{Q_i}^6} - \frac{4T_i{P_i}^3}{{Q_i}^5} + \frac{3S_i{P_i}^2}{{Q_i}^4} - \frac{2R_iP_i}{{Q_i}^3} - \frac{1}{Q_i}$

$$q_i = -\frac{W_i{P_i}^5}{{Q_i}^6} + \frac{T_i{P_i}^4}{{Q_i}^5} - \frac{S_i{P_i}^3}{{Q_i}^4} + \frac{R_i{P_i}^2}{{Q_i}^3} + \frac{P_i}{Q_i}$$

6. Illustration

An example was illustrated on the inverse quintic spline using polynomial iteration method.

x	-1	-0.7	-0.6	-0.4	-0.3	-0.1	0.2
y	1	0.2401	0.1296	0.0256	0.0081	0.0001	0.0016

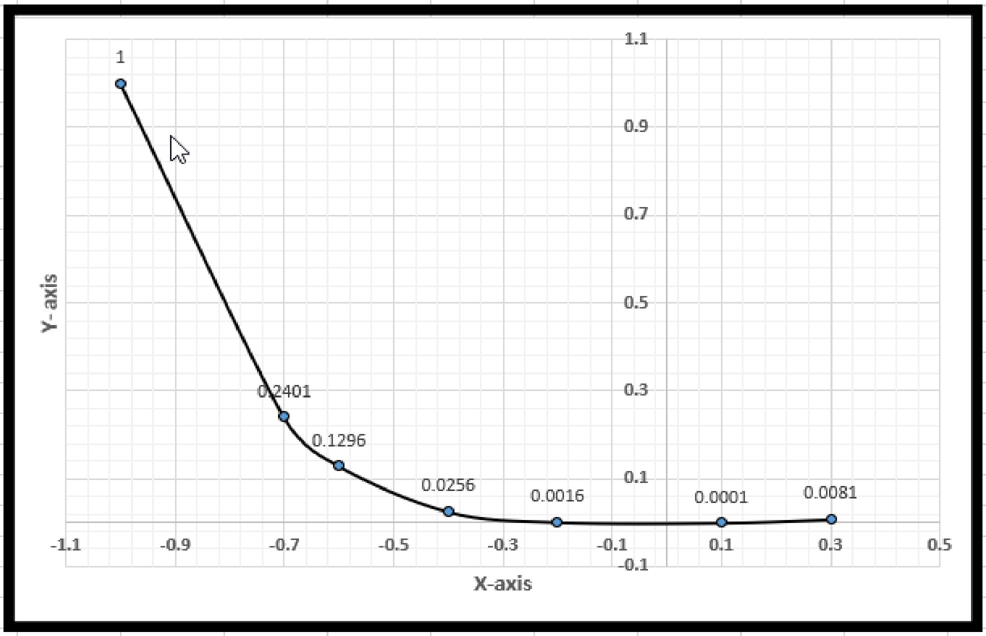

Figure: 1 Quintic Spline

$x_0 = -1$ $\quad y_0 = 1$
$x_1 = -0.7$ $\quad y_1 = 0.2401$
$x_2 = -0.6$ $\quad y_2 = 0.1296$
$x_3 = -0.4$ $\quad y_3 = 0.0256$
$x_4 = -0.3$ $\quad y_4 = 0.0081$
$x_5 = -0.1$ $\quad y_5 = 0.0001$
$x_6 = 0.2$ $\quad y_6 = 0.0016$

$h_i = x_{i+1} - x_i$ $\quad Z_i = h_{i+1}^2 - h_i^2$ $\quad \Delta_i = \frac{y_{i+1} - y_i}{h_i}$

$h_0 = 0.3$ $\quad Z_0 = -0.08$ $\quad \Delta_0 = -2.533$
$h_1 = 0.1$ $\quad Z_1 = 0.03$ $\quad \Delta_1 = -1.105$
$h_2 = 0.2$ $\quad Z_2 = -0.03$ $\quad \Delta_2 = -0.52$
$h_3 = 0.1$ $\quad Z_3 = 0.03$ $\quad \Delta_3 = -0.175$
$h_4 = 0.2$ $\quad Z_4 = -0.05$ $\quad \Delta_4 = -0.04$

$h_5 = 0.3$ $\Delta_5 = 0.005$

In Natural quintic spline, $M_0 = M_6 = 0$,

$M_1 = 12486.60807$
$M_2 = 11396.97861$
$M_3 = -7546.267881$
$M_4 = -899.4587566$
$M_5 = -179.9381436$

$C_i = C_{i+1} + \left[\frac{h_i - h_{i+1}}{12}\right] M_{i+1}$ $D_i = D_{i+1} - \left[\frac{h_i^2 - h_{i+1}^2}{12}\right] M_{i+1}$

$C_0 = 59.244721$ $D_0 = 3.337494$
$C_1 = -148.865413$ $D_1 = -14.586644$
$C_2 = -53.890592$ $D_2 = 5.688956$
$C_3 = 8.994974$ $D_3 = 1.199394$
$C_4 = 1.499485$ $D_4 = 0.149948$
$C_5 = 0$ $D_5 = 0$

$E_0 = 10/3$ $F_0 = -2.009153$
$E_1 = 2.296944933$ $F_1 = 1.201025178$
$E_2 = -0.111798574$ $F_2 = 0.6310845254$
$E_3 = 0.3188855657$ $F_3 = 0.08849548964$
$E_4 = 0.1004639171$ $F_4 = 0.012495876$
$E_5 = 0.04081941564$ $F_5 = 0.0053333$

$P_0 = 0.2401$ $Q_0 = -2.371674182$
$P_1 = 0.1296$ $Q_1 = -2.326272929$
$P_2 = 0.0256$ $Q_2 = 0.7547070786$
$P_3 = 0.0081$ $Q_3 = 0.4777566943$
$P_4 = 0.0001$ $Q_4 = 0.2379165129$
$P_5 = 0.0016$ $Q_5 = 0.035486$

$R_0 = 54.765234$ $S_0 = -252.920481$
$R_1 = 54.76523381$ $S_1 = -243.840235$
$R_2 = 53.85720925$ $S_2 = 71.880539$
$R_3 = -9.286945667$ $S_3 = 16.49046364$
$R_4 = -3.747938308$ $S_4 = 4.49845406$
$R_5 = -1.349536077$ $S_5 = 0$

$T_0 = 520.275336$ $W_0 = -346.850224$
$T_1 = 474.8741088$ $W_1 = 90.80245518$

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$T_2 = -314.4278284$ $W_2 = 789.3019371$
$T_3 = -37.47744819$ $W_3 = -553.9007604$
$T_4 = -7.497423$ $W_4 = -29.98002554$
$T_5 = 0$ $W_5 = -4.998281767$

$l_0 = -1.948999933$ $m_0 - 4.593814813$
$l_1 = 0.5729722523$ $m_1 = -7.341966201$
$l_2 = 4271.423458$ $m_2 = -1830.930948$
$l_3 = -46579.119522$ $m_3 = 380.762465$
$l_4 = -165304.426574$ $m_4 = -9752.681156$
$l_5 = -2503101530.944174$ $m_5 = 20024812.247553$

$n_0 = -2.458536$ $o_0 = -0.475657$
$n_1 = -4.616666176$ $o_1 = -1.827948964$
$n_2 = 381.0573322$ $o_2 = 102.5054244$
$n_3 = 334.747198$ $o_3 = -93.20002299$
$n_4 = 1407.910615$ $o_4 = -278.7252915$
$n_5 = -64079.396205$ $o_5 = -30097.918806$

$p_0 = 1.361965$ $q_0 = 0.027026$
$p_1 = 1.199422576$ $q_1 = -0.1126428647$
$p_2 = -7.208785432$ $q_2 = 0.111713285$
$p_3 = -0.6489704118$ $q_3 = 0.01119035229$
$p_4 = -4.147452221$ $q_4 = 0.0004175310681$
$p_5 = 68.379278$ $q_5 = -0.032199$

Hence the Inverse Quintic Spline using Polynomial Iteration method is

$$S_0^{-1}(y) = -1.948999933y^5 - 4.593814813y^4 + -2.458536y^3 - 0.475657y^2 + 1.361965y - 0.672974, \quad y \epsilon [1, \ 0.2401]$$

$$S_1^{-1}(y) = 0.5729722523y^5 + 6988.150179y^4 - 4.616666176y^3 - 1.827948964y^2 + 1.199422577y - 0.7126428647 \quad y \epsilon [0.2401, \ 0.1296]$$

$$S_2^{-1}(y) = 4271.423458\, y^5 - 1830.930948y^4 + 381.0573322y^3 + 102.5054244y^2 - 7.208785432y - 0.288286715 , \quad y \epsilon [0.1296, \ 0.0256]$$

$$S_3^{-1}(y) = -46579.11952y^5 + 380.762465\, y^4 + 334.747198y^3 - 93.20002299 - 0.6489704118\, y - 0.2888096477, \quad y \epsilon [0.0256, \ 0.0081]$$

$$S_4^{-1}(y) = -165304.426574\, y^5 - 9752.681156y^4 + 1407.910615y^3 - 278.725291y^2 + -4.147452y - 0.099582, \quad y \epsilon\, [0.0081, 0.0001]$$

$$S_5^{-1}(y) = -2503101530.944174y^5 + 20024812.247553y^4 - 64079.396205\, y^3 - -30097.918806y^2 + 68.379278y + 0.167801, \quad y \epsilon\, [0.0001\,, 0.0016\,]$$

7. Error Calculation

Error = Actual value – Computational value

Value of y	Interval	Actual value of x	Polynomial iteration method	
			Calculated value of x	Error
0.522	[1, 0.2401]	-0.85	−0.857948	0.007948
0.1385	[0.2401, 0.1296]	-0.61	-0.596525	-0.013475
0.041	[0.1296, 0.0256]	-0.45	−0.389951	-0.060049
0.021	[0.0256, 0.0081]	-0.38	−0.340555	-0.039445
0.0041	[0.0081, 0.0001]	-0.266	−0.127118	-0.138882
0.0015	[0.0001, 0.0016]	0.197	0.202516	-0.005516

8. Conclusion

In this paper we figure out the Inverse quintic Spline from the derivation of the Qunitic spline using Polynomial Iteration Method. Here an example was also illustrated and the error is calculated successfully.

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