Analysis of a Supply Chain Model with Weibull Demand and Deterioration under Finite Planning Horizon: Mathematical Inventory Model, Optimality, and Sensitivity Analysis

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Abstract

A significant financial loss can occur in a supply chain system due to the degradation of items in inventory. In this study, we have analyzed a supply chain model that examines deterioration in a finite planning horizon model with Weibull market demand. We have discussed the formulation of a mathematical inventory model with a finite planning horizon and its solution. To illustrate the optimality of replenishment cycles in this inventory model, we have provided a numerical example. We have determined the total optimal cost, which is a convex nonlinear function by establishing the positive definiteness of the Hessian matrix. We have conducted a sensitivity analysis of all parameters using different tables and graphs. In addition, we have dedicated a separate section to discuss the results obtained and provide managerial insights.

Keywords: -Weibull Demand, Finite Planning Horizon, Supply Chain Management, deterioration

Introduction:

In 2014, Jaggi explained that lead time refers to the duration between placing an order and receiving the items, which indicates the time needed for delivery. Lead time can vary for different inventory problems and may require clarification. In the pre-processing, processing, and post-processing stages, companies analyze lead time for manufacturing, inventory management in the supply chain process, and project management. By comparing outcomes to predetermined benchmarks, they can identify inefficiencies. Shortening lead time can simplify procedures, accelerate production, and improve output and profits. Lead time can be influenced by manufacturing processes and inventory management. For instance, keeping finished goods in local warehouses can increase lead time compared to finishing specific parts elsewhere. Late delivery of components due to transportation issues can disrupt or delay manufacturing, affecting productivity.

The Weibull distribution can be characterized as a generalized gamma distribution with both shape and size parameters represented by k. When dealing with items that degrade according to the Weibull distribution, demand can vary depending on the selling price in different payment stages, namely pre-payment, interim-payment, and post-payment scenarios. This can be modeled using two approaches: a crisp inventory model and a fuzzy inventory model. Both approaches consider Weibull degrading inventory models with the demand that is dependent on selling-price settings, which can be achieved by employing pre-, interim-, and post-payment tactics.In 2022, Bhavani and G conducted a study to explore the optimal approach for inventory control of products that degrade according to the Weibull distribution, where demand is dependent on selling price in a crisp environment. They developed a model that incorporates positive lead time and all payment methods. The results of this inventory model are more suitable for addressing real-world inventory control challenges without any external cost optimization effects.

According to Fonseca et al. (2002), perishable goods such as vegetables and fruits experience hydrolysis and respiration even after they are harvested. This process utilizes the available oxygen (O2) in the surrounding atmosphere and generates heat, moisture, carbon dioxide (CO2), and sometimes, ethylene gas. Consequently, the quality of fruits and vegetables deteriorates as they undergo ripening. The rate of respiration is the quantity or concentration of oxygen consumed or carbon dioxide produced by the product per unit of mass over a specific time period. Aerobic respiration, which occurs in the presence of oxygen (O2), involves breaking down complex organic compounds into simpler molecules such as carbon dioxide (CO2) and water, releasing energy in the process. The rate of respiration is considered a physiological indicator of stress.Benjaafar et al. (2012) and Mishra, N.K., &Ranu (2022) highlight that logistics and supply chain operations contribute significantly to carbon emissions, which in turn contribute to climate change. In 2005, the Kyoto Protocol was implemented, prompting governments and environmental organizations to take action in implementing legislation and technology solutions to reduce carbon emissions. This led to the emergence of green laws, green energy, and green supply chains. In 2015, the United Nations Framework Convention on Climate Change was established at the 21st Conference for the States Parties in Paris, France, with the aim of promoting the development of renewable and alternative energy sources, as well as the establishment of resource and coal regulations over time. With a growing emphasis on reducing carbon emissions, there has been an increasing harmonization of carbon reduction programs.

Literature Review:

The aim of this study, as described in Bankole et al. (2022) and Singh et al. (2019), was to develop an Economic Order Quantity (EOQ) approach for perishable products with a Weibull lifespan distribution and a price-proportional exponential demand rate. The authors conducted simulation experiments and found that increasing the Weibull parameter improves the EOQ. Real data from the Afe Babalola University bakery on six loaves of bread were used to demonstrate the model's performance. The usual EOQ range of 60 to 400 loaves with ordering intervals of one or two days was employed. The proposed model aimed to investigate the inventory system for perishable goods under inflationary conditions, where demand is a function of inflation, and degradation is modeled using a two-parameter Weibull distribution. The model calculated the Economic Order Quantity that optimizes the average total cost per unit of time while accounting for the time value of money and inflation. In this case, degradation begins after a predetermined time interval. The study also investigated the effects of the time value of money and inflation on the system's inventory control using numerical examples.

In their study, Mishra et al. (2012) presented an algorithm that utilizes a stochastic dynamic programming framework to optimize revenue for an online retailer within a finite timeframe. The approach incorporates dynamic pricing and sponsored search advertising strategies for perishable products with fixed inventory. It provides a unified solution for advertising and pricing, highlights the essential features of an optimal policy, develops an efficient algorithm to handle large-scale cases, and conducts numerical experiments to compare the proposed algorithm with an ideal approach.Salehi et al. (2023) found that the proposed method is capable of reaching the optimal solution much faster than the ideal strategy. Additionally, they observed that the runtime of the proposed method does not follow a linear relationship with the inventory level.

Agarwal and Badole (2022) proposed a real-time inventory model for degradable items, which involves developing a mathematical approach to determine the total system cost and the order quantity replenished within a finite planning horizon. They examined the effects of money, scarcity, lead time, and information technology on the lead time interval. The demand rate is a log-concave function of time, and they investigated the impact of various factors such as the backlogging parameter, order quantity, degradation rate, and overall supply chain process cost. Data and sensitivity analysis were performed using Mathematica software.

The aim of this study is to develop an economic production quantity approach (EPQ) for deteriorating goods that takes into account Weibull degradation for a two-parameter scenario, demand that depends on price, and a trade-credit scheme. The carrying cost of inventory is modeled as a linear function of time, and three scenarios are considered: build-up, time after decay begins, and partially backlogged time. The approach maximizes overall average profits while achieving an optimal ordering scheme. The model's validity is demonstrated through sensitivity analysis using Mathematica software by Sunita et al. (2022). The study primarily focuses on a finite planning horizon. Several researchers, including P. Singh et al. (2017), V. Singh et al. (2017 and 2019), S. Saxena et al. (2020), and S. Mishra et al. (2020), have examined the finite planning horizon.

Assumptions and Notations

We have considered the following assumptions in this paper and will apply additional assumptions and notations as needed

1. We do not allow shortages in this research study.
2. In this research work, it is assumed that there is a finite planning horizon.
3. $\boldsymbol{\theta}$ refers to thereferstant rate of deterioration.
4. A single retailer and supplier are to be apprised of a single commodity.
5. Both the starting and ending inventory are considered zero
6. Lead time is negligible, therefore lead time is considered zero.
7. Two parameters of Weibull Demand areconsidered.

Notations:

Additionally, the following notations were employed during the construction of the proposed model

1. $h_r$is the holding cos of ($/un. /yr.)
2. S_s is the total cost comprisedof both the setup cost and the transportation cost. ($/order)
3. S_r The cost pertains to the order. ($/order)
4. $\boldsymbol{Q_{nt}}$, the ordered quantity in (i)th cycle at time t where $\boldsymbol{t_i \le t \le t_{i+1}}$
5. $\boldsymbol{I_{i+1}(t)}$, The level of inventory's level during the (i)th cycle when i=1,2,3……. n.
6. n represents thenumber of replenishment cycles.
7. $\boldsymbol{T_i}$, is the replenishment cycle length.
8. $\boldsymbol{T_s^b}$, The overall cost of the supplier during the finite period for the decentralized case.
9. $\boldsymbol{T_r^b}$, The overall cost of the retailer during the finite period for the decentralized case.
10. $\boldsymbol{T_s^c}$, The overall cost of the supplier during the finite period for the centralized case.
11. $\boldsymbol{T_r^c}$, The overall cost of the retailer during the finite period for the centralized case.
12. The planning horizon is represented by H (year)
13. Wrepresents the wholesale price.
14. $\boldsymbol{C_p}$is the cost per unit of purchase. ($/unit).
15. $\boldsymbol{\alpha}$ referred the shape parameter of Weibull Demand
16. $\boldsymbol{\beta}$referred the scale parameter of Weibull Demand.

Mathematicalsolutions:

A mathematical expression for the finite planning horizon is developed in this section of the paper. There are two phases of this study. One is a decentralized phase and the second is for thecentralizedcases.

Decentralized system

In this case, the decision-making authority is distributed among both retailers and suppliers. Both retailer and supplier independently make their own decisions. As a result, there may be a lack of coordination or alignment between the decisions made by each member and the overall goals of the supply chain**.**

$$\frac{\boldsymbol{I_{i+1}(t)}}{dt} = -D_t(t) - \theta * \boldsymbol{I_{i+1}(t)} \quad \text{where } t_i < t < t_{i+1} \quad \textbf{(1)}$$

$$\boldsymbol{I_{i+1}(t)} = e^{-\theta t} * \alpha * \beta \int_t^{t_{i+1}} u^{\beta-1} e^{\theta * u}\, \mathrm{dt} \textbf{(2)}$$

$$\boldsymbol{Q_{i+1}(t)} = \boldsymbol{I_{i+1}}(t_i) = \alpha * \beta \int_{t_i}^{t_{i+1}} t^{\beta-1} e^{\theta(t-t_i)}\, \mathrm{dt} \ \textbf{(3)}$$

The total cost of retailer for decentralized case:

$$T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) = n_1 * S_r + h_r \sum_{i=1}^{n_1} \int_{t_i}^{t_{i+1}} \boldsymbol{I_{i+1}(t)}\, dt + W * \sum_{i=1}^{n} \boldsymbol{Q_{i+1}(t)}$$

$$T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) = (n_1 * S_r) + \sum_{i=0}^{n_1-1} \left(\frac{h_r * \alpha * \beta}{\theta}\right) \int_{t_i}^{t_{i+1}} t^{\beta-1}(e^{\theta(t-t_i)} - 1) dt + W * \alpha * \beta * e^{-\theta t_i} \int_{t_i}^{t_{i+1}} t^{\beta-1} e^{\theta t} dt$$

(4)

The total cost of retailer for decentralized case:

$$T_s^D(n_1, t_0, t_1, \ldots\ldots\ldots, t_{n1}) = (n_1 * S_s) + \sum_{i=0}^{n_1-1} (\alpha * C_p * \beta) \int_{t_i}^{t_{i+1}} t^{\beta-1}(e^{\theta(t-t_i)} - 1)dt$$

(5)

$$\text{Qnt} = \sum_{i=0}^{n_1-1} (\alpha * \beta) \int_{t_i}^{t_{i+1}} t^{\beta-1}(e^{\theta(t-t_i)})dt \quad \textbf{(6)}$$

Partially differentiate Eqn (4) to find the root value of t_i by putting $\frac{\delta T C_r (n_1, t_0, t_1, \ldots\ldots\ldots, t_{n1})}{\delta t_i} = 0$. After that, on the basis of unique and optimal values of t_i then we can find the minimum value of total cot of retailer, supplier and optimal order quantity.

$$\frac{\boldsymbol{\delta T C_r(n_1, t_0, t_1, \ldots\ldots\ldots, t_{n1})}}{\boldsymbol{\delta t_i}} = \frac{\delta}{\delta t_i}\left(n_1 S_r + \sum_{i=0}^{n_{i-1}} h_r \int_{t_i}^{t_{i+1}} I_{i+1}(t)dt + \sum_{i=0}^{n_1-1} W Q_{i+1}\right)$$

$$\frac{\partial}{\partial_{t_i}} T_r(n_1, t_0, t_1, t_2 \ldots\ldots\ldots t_{n_1})$$
$$= -h_r \alpha\beta\left(\int_{t_i}^{t_{i+1}} t^{\beta-1}\left(e^{\theta(t-t_i)}\right)dt - \frac{{t_i}^{\beta-1}}{\theta}\left(e^{\theta(t_i - t_{i-1})} - 1\right)\right)$$
$$- W\alpha\beta\left(e^{-\theta t_i}\left({t_i}^{\beta-1} e^{\theta t_i}\right) + \theta e^{-\theta t_i}\int_{t_i}^{t_{i+1}} t^{\beta-1} e^{\theta t} dt - e^{-\theta t_{i-1}}\left({t_i}^{\beta-1} e^{\theta t_i}\right)\right)$$

$(\mathbf{7})$

Centralizes system:

On the other hand, a centralized supply chain is characterized by a central authority or decision-making body that coordinates the actions of all members of the supply chain. This can result in more favorable outcomes for the entire supply chain, as the decisions made are aligned with the overall goals and objectives. The coordination and alignment of decisions in a centralized supply chain can lead to improved efficiency and effectiveness in achieving supply chain goals.

In the **Centralizes** scenario, where the retailer and supplier collaborate, they share the benefits obtained from the shorter replenishment cycles, as opposed to the unassociated situation. These benefits are reflected in the Improved cost.

$$\text{Oc} = \sum_{i=0}^{n_1-1} (T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) * r\% * (t_{i+1} - 0.1 - t_i)) \quad \textbf{(8)}$$

The total cost of the retailer in centralized case:

$$T_r^C(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) = \left\{T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) - \frac{\text{Oc}}{2}\right\} \quad \textbf{(9)}$$

The total cost of the supplier in the centralized case:

$$T_s^C(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) = \left\{T_s^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) - \frac{\text{Oc}}{2}\right\} \quad \textbf{(10)}$$

Now percentage Improved cost of retailer:

$$= \left\{\frac{T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) - T_r^C(n_1, t_0, t_1, \dots\dots\dots, t_{n1})}{T_r^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1})}\right\} \times 100 \quad \textbf{(11)}$$

Now percentage Improved cost of supplier:

$$= \left\{\frac{T_s^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1}) - T_s^C(n_1, t_0, t_1, \dots\dots\dots, t_{n1})}{T_s^D(n_1, t_0, t_1, \dots\dots\dots, t_{n1})}\right\} \times 100 \quad \textbf{(12)}$$

Numerical example:

Let us consider the parametric values$\alpha = 8.1$, $\beta = 2$, $\theta = 0.6$, $W = 0.9$, h_r =4.5, Sr = 40, Ss = 12, CP = 0.01 and r = 0.2 . Solve this numerical problem with the help of mathematics iterative method by Mathematica software.

Table 1, Table 2, Figure 1, Figure 2, and Figure 3 provide a detailed analysis of the optimal retailer's overall cost for various values of " hr ". For the decentralized case, Optimal total cost values for hr =3.5, 45, and 5.5are $**71.48**, $**88.9**, and $**96.01**, respectively, and are achieved at 3, 5, and 6 optimal replenishment cycles. For all subsequent cycles after reaching the minimum at n_1=2, 2, and 3 for a=0.5, 0.125, and 0.625 respectively, the overall cost gradually increases. In the centralized case, Table 3, Table 4 Figure 4, Figure 5, and Figure 6 show an analysis of optimal total cost values for hr =3.5, 45, and 5.5 are $**55.03**, $**65.02**, and $**64.02**, respectively, and are again achieved at n_1=3, 5, and 6 optimal replenishment cycles.

The solution to the numerical problem is presented in both tabular and graphical formats.

Decentralized system:

Table 1. The overall expense incurred by the retailer in the decentralized scenario.

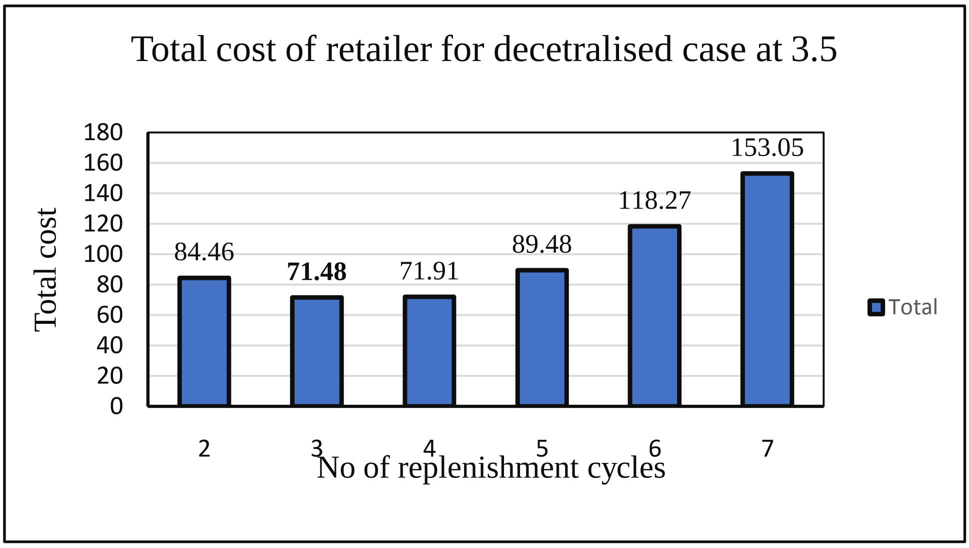
↓ hr →n_1	1	2	3	4	5	6	7
3.5	71.63	84.46	**71.48**	71.91	89.48	118.27	153.05
4.5	105.55	137.32	111.72	90.39	**88.9**	104.42	131.04
5.5	142.77	199.12	164.45	121.39	97.97	**96.01**	110.27

Table 2. In the decentralized scenario, the most favorable retailer cost, supplier cost, the number of replenishment cycles, and replenishment quantity have been determined.

↓ h_r → t_i	t_0	t_1	t_2	t_3	t_4	t_5	t_6	n_1	Q_{nt}	T_r^D	T_s^D
3.5	0	1.695	2.874	**4.**					110.55	71.48	37.10
4.5	**0**	1.424	2.446	3.164	3.616	**4.**			123.04	88.9	61.23
5.5	0	1.298	2.243	2.942	3.424	3.727	**4.**	6	124.43	96.01	72.12

Figure 1. An optimal level of retailer cost in the case of decentralized case 4.5

Figure 2. The optimal level of retailer cost in the decentralized case for 3.5



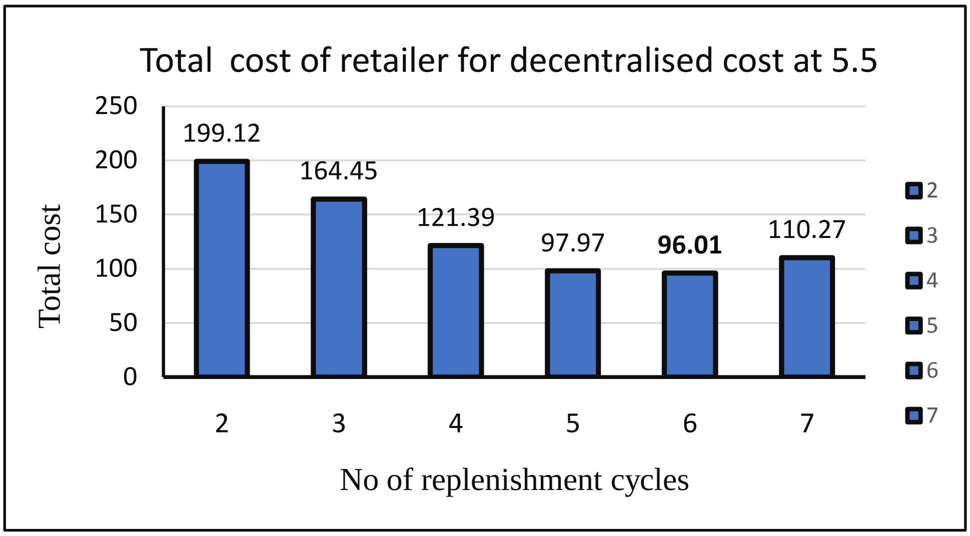
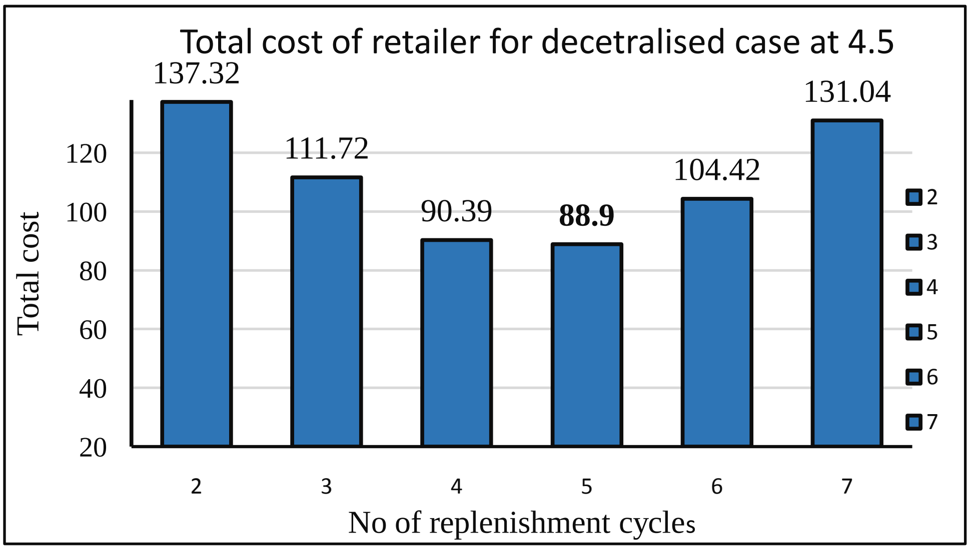


Figure 3. For 5.5, the decentralized case indicates the optimal level of retailer cost



Centralizes system:

Table 3. Retailer's total cost in the centralized case

$\downarrow h_r$ $\rightarrow n_1$	1	2	3	4	5	6	7
3.5	55.18	68.02	**55.03**	55.47	73.03	101.83	136.60
4.5	81.66	113.44	87.84	66.51	**65.02**	80.54	107.15
5.5	110.78	167.13	132.46	89.40	65.99	**64.02**	78.28

Table 4. In the decentralized and centralized scenario, the most favourable retailer cost, supplier cost, the number of replenishment cycles, and replenishment quantity have been determined.

		Decentralized case				Centralized case			
Parameters	n_1	T_D^r	T_D^s	Q_{nt}		T_r^C	T_s^C	$Imp\ cost$ T_r^C	$Imp\ cost$ T_s^C
3.5	3	71.48	37.10	110.55		55.03	35.31	22.95	2.21
4.5	5	88.9	61.23	123.04		65.02	58.18	22.62	3.21
5.5	6	96.01	72.12	124.43		64.02	69.01	22.4	4.31

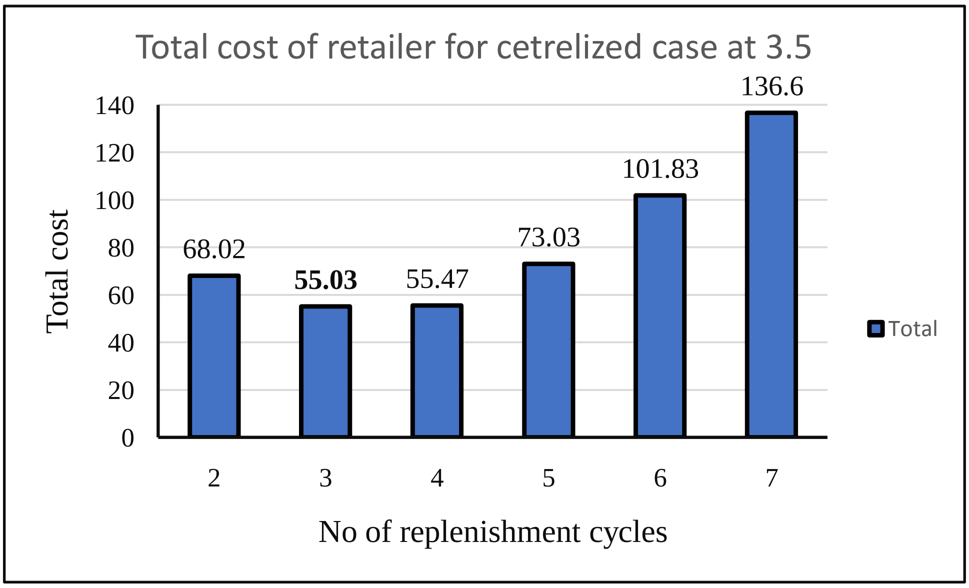


Figure 5. The centralized case indicates that the optimal level of retailer cost for 3.5.

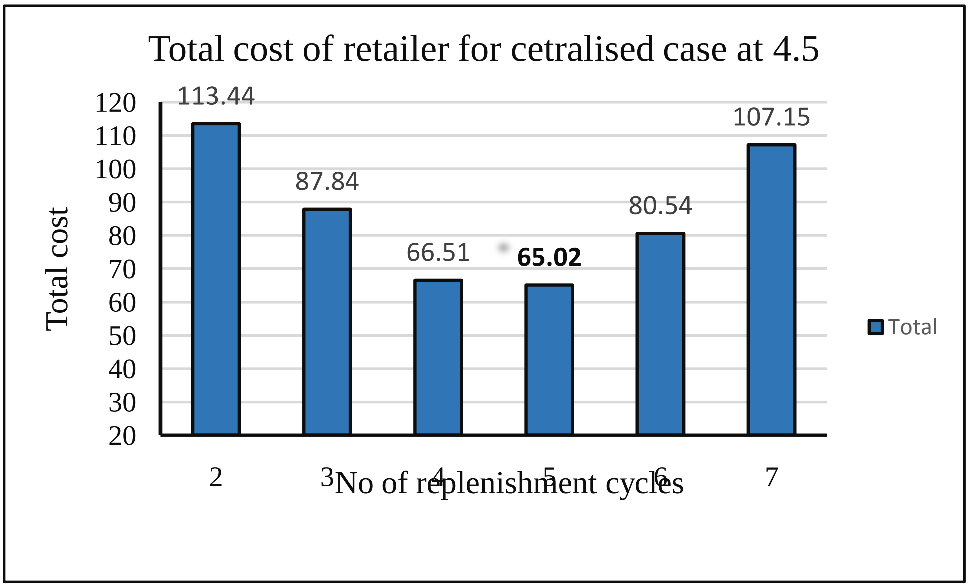


Figure 4. For 4.5, the centralized case shows that the retailer cost has reached its optimal level

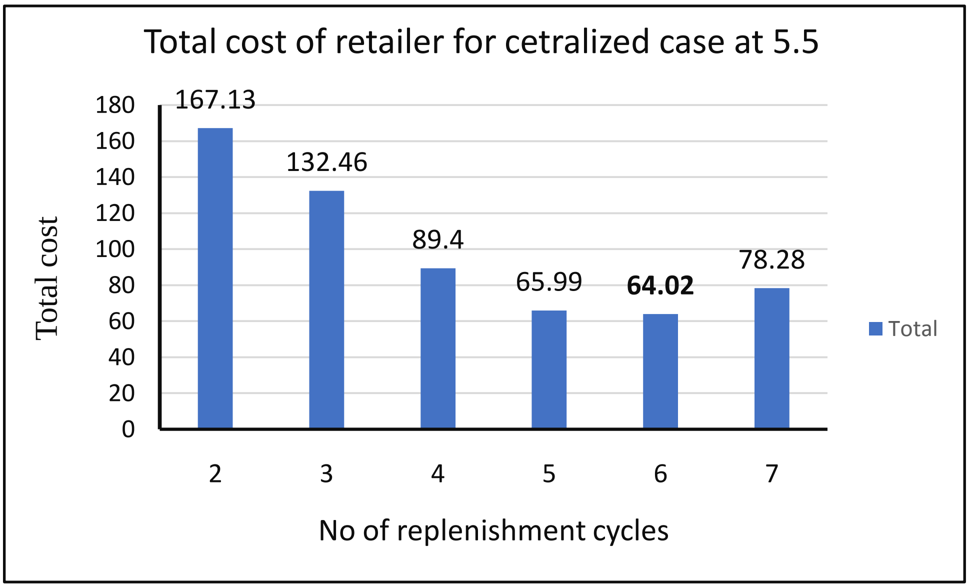


Figure 6. For 5.5, the centralized case shows that the retailer cost has reached its optimal level

Sensitivity Study of this Research Study:

Here, this research work describes the sensitivity demonstration of different parameters that conducted to evaluate the effectiveness of proposed framework when specific parameters are modified while others remain constant. Each parameter is changed one at a particular time while taking the remaining parameters constant. The critical parameters include α, β, θ, W, h_r, Sr, Ss, CP and r are changed one by one. Table 5 evaluates and visually presents the impact of modifying retailer and supplier cost parameters by -2% to +2%, as well as changes in replenishment quantity on the expected total cost of both parties. The central aim of this research analysis is to identify the management applications' significance and the model's solution's importance and relevance for industrial areas that are used to find the optimum value. To confirm the validity of the proposed approach, "MATHEMATICA" version-12 mathematical problem-solving software and an iterative numerical mathematical method were utilized to solve non-linear differential equations. A detailed analysis can be found in Table 5.

1. Table 5 provides a comprehensive analysis of the total cost for the retailer in both centralized and decentralized scenarios**.** T_D^r is very sensitive to the parameters hr, and Sr is moderately sensitive to $\alpha,$W , and practically insensitive to Ss. Similarly for the centralized case.

2. As Table 5 indicates, the supplier's total cost T_D^s is very sensitive to the parametric value of Ss , moderately sensitive to α , w . and practically insensitive to hrand Sr. This observation is also the same for the centralized case T_C^s.

3. Table 5 presents a detailed and lucid analysis, revealing the supplier's replenished order quantity to the retailer Q_{nt} is highly sensitive to the parameter α,moderately sensitive to W, hr and practically insensitive to Ss and Sr.

4. Table 5 Illustrates that 'n'is identifiedthe replenishment cycles's numbersfor both the centralized and decentralized cases concerning parameters such as α, β, θ, W, hr, Sr, Ss, CP

Table 5. Conducting sensitivity analysis on various parameters

			Decentralized case			Centralized case			
Parameters	% change	n_1	T_D^r	T_D^s	Q_{nt}	T_r^C	T_s^C	$Imp\ cost$ T_r^C	$Imp\ cost$ T_s^C
W	-2	5	61.070	60.120	120.133	38.568	58.297	22.494	3.031
	-1	5	80.482	60.122	122.490	41.707	56.981	29.623	5.223
	0	5	82.421	60.122	122.714	48.337	57.361	32.488	4.591
	$+1$	5	84.358	60.122	122.936	55.763	57.806	33.576	3.852
	$+2$	5	86.294	60.123	123.155	57.184	57.765	33.733	3.921

h_r	−2	5	101.537	60.123	123.836	77.25	58.156	22.652	3.271
	−1	5	98.500	60.123	123.609	56.024	56.683	29.668	5.722
	0	5	95.46	60.123	123.384	56.934	57.002	32.488	5.190
	+1	5	92.421	60.123	123.162	60.562	57.542	33.530	4.292
	+2	5	89.379	60.123	122.942	59.294	57.686	33.660	4.053
α	−2	5	91.131	67.543	120.588	67.543	58.248	22.627	3.114
	−1	5	90.02	60.121	121.819	49.478	56.870	29.646	5.407
	0	5	88.909	60.123	123.049	52.61	57.182	32.488	4.890
	+1	5	87.798	60.124	124.280	57.70	57.661	33.553	4.095
	+2	5	86.687	60.125	125.510	57.476	57.712	33.696	4.013
Sr	−2	5	84.909	60.123	23.050	61.206	58.203	22.627	3.193
	−1	5	86.909	60.123	123.049	46.435	56.844	29.646	5.452
	0	5	88.909	60.123	123.049	52.61	57.182	32.488	4.890
	+1	5	90.909	60.123	123.049	60.04	57.622	33.553	4.158
	+2	5	92.909	60.123	123.049	61.601	57.587	33.696	4.217
Ss	−2	5	88.909	58.923	123.049	65.025	56.988	22.627	3.283
	−1	5	88.909	59.523	123.049	48.197	56.225	29.646	5.540
	0	5	88.909	60.123□	123.049	52.610	57.182	32.488	4.890
	+1	5	88.909	60.723	123.049	58.577	58.266	33.553	4.046
	+2	5	88.909	61.324	123.049	58.949	58.896	33.696Ss	3.957

Conclusion:

In the article, a model is introduced for inventory production planning that takes into account Weibull demand with two parameters for both centralized and decentralized supplier-retailer relationships within a finite time frame. This proposed research has advantages for the retail and manufacturing sectors.

The study includes a numerical example to demonstrate sensitivity analysis, where various parameters are modified to observe their impact. The findings are discussed, and potential future work is suggested, such as incorporating additional factors like inflation, fuzzy logic, non-linear mixed integer programming, and carbon emission regulations.

Moreover, the proposed approach can be extended by incorporating Weibull demand with three parameters, as well as including multiple retailers, suppliers, and items.

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