STUDY OF HEAT AND MASS TRANSFER THROUGH A CHANNEL FILLED WITH POROUS MEDIUM WITH MOVING PLATES & VISCOUS DISSIPATION IN THE PRESENCE OF MAGNETIC FIELD

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Abstract

Because of its relevance not only in the realm of academia but also in the business world, the study of fluid movements through porous media has attracted a lot of interest over the course of its history. In particular, heat and the passage of material through a channel or cylinder that is filled with a porous substance. The fluxes in the presence of a magnetic field but with clear fluid, that is, without a porous medium, have been researched by several writers, while other researchers have taken the plates and let them sit still. In this paper, a non-Darcy viscous dissipating MHD flow and heat transfer in a channel filled with porous media and moving plates is considered. All of these factors are taken into consideration. In order to solve the governing equation, a simulation approach that goes by the name Differential Transform Method or Taylor Transform Method is utilised. It is assumed that the effects of the fluid's thermophysical characteristics and the very porous medium's thermophysical qualities remain the same. In order to make accurate predictions about the flow and temperature field, analytical solutions for the velocity and temperature are produced. It is decided to carry out a parametric analysis in order to investigate the impact that a variety of parameters have on the flow and heat transfer performance. Calculations of the skin friction coefficient and the Nusselt Number are carried out, and their respective tables and figures are included.

Keywords: MHD, Porous Media, Heat Transfer, Differential Transform Method (DTM)

INTRODUCTION

Because of its relevance not only in the realm of academia but also in the business world, the study of fluid movements through porous media has attracted a lot of interest over the course of its history. Fluid motions have many applications in many different kinds of industrial and biological processes, such as the food industry, irrigation problems, oil exploitation, motion of blood in the cardiovascular system, chemistry and bio-engineering, soap and cellulose solutions, and in biophysical sciences, where the human lungs are considered to be a porous layer, etc., amongst other things. The investigation of the flow of an electrically conducting fluid has many applications, including the solution of engineering problems such as magneto hydrodynamics (MHD) generators, plasma studies, nuclear reactors, the extraction of geothermal energy, and the control of boundary layers in the field of aerodynamics. Vafai [1] investigated the effects of varying porosity and inertial forces on the convective flow and heat transfer in porous media, as well as the effects of changing porosity and inertial forces on the forced convection in packed beds in the vicinity of an impermeable barrier. Fluid flow and heat transfer at the interface area in deepness were examined by Thiyagaraja et al. [2] for three broad and fundamental types of issues in porous media. There are three types of interface regions: an interface region between a fluid area and a porous medium, an interface region between an impermeable medium and a porous medium, and an interface region between two separate porous media. Ettefagh et al. [3] explored the significance and relevance of non-Darcian effects related with the buoyancy driven convection in open ended cavities filled with fluid-saturated porous media. These cavities were filled to capacity with fluid. Karimi et al.[4] provided a numerical explanation for double-diffusive natural convection in a square cavity that was filled with a porous medium. They also investigated the impact of a non-Darcian fluid on the average heat and mass transfer rates. Marafie et al.[5] conducted an investigation on forced convection flow down a channel that was filled with a porous media. The Darcy-Forchheimer-Brinkman model was utilised in order to simulate the fluid transport that occurred inside the porous medium.

For the purpose of accurately representing fluid and solid energy transfer, a non-thermal equilibrium, two-equation model is utilised. Chakraborty [6] investigated the MHD flow and heat transfer of a dusty viscoelastic stratified fluid travelling down an inclined channel in a porous media while the viscosity was varied. The research conducted by Chamkha [7] investigated unsteady laminar hyderomagnetic flow and heat transmission in porous channels with temperature dependent characteristics. Nield and colleagues [8] investigated the effects of axial conduction and viscous dissipation on the thermal development of a forced convection parallel plate channel filled by a saturated porous medium with a uniform temperature at the wall. Their research focused on the thermal development of the channel. Hooman et al. [9] addressed the effects of viscous dissipation on thermal entranced heat transfer in a parallel plate channel filled with a saturated porous material. Their discussion was built on the foundation of the Darcy model. The authors Mankinde et al. [10] discussed the collaborative effects of a transverse magnetic field and radiative heat transfer on the unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform wall temperature. The researchers focused on the effects of the flow on the walls of the channel.

Figen Kangalgi et al. [11] conducted research to determine how to acquire accurate and approximate solutions for the nonlinear dispersive KdV and mKdV equations using an initial profile. The Nakamura-Sawada rheological model was utilised by Rawat et al. [12] in order to conduct an investigation of the pulsatile hydro magnetic flow and heat transfer of a non-Newtonian biofluid as it travelled through a saturated non-Darcian porous medium channel with viscous heating. The unsteady magnetohydrodynamic (MHD) periodic flow of a non-Newtonian fluid via a porous channel was investigated by Taklifi et al. [13]. Additionally displayed are the impacts of the rheological behaviour of the fluid on the velocity and shear stress profiles along the channel width at a variety of different time periods. The steady magnetohydrodynamic (MHD) flow and heat transfer of an electrically conducting viscous incompressible fluid was investigated by Sharma M. and colleagues [14] using a non-Darcian porous medium that was bounded between two horizontal infinitely impermeable parallel plates with viscous and Joule dissipation effects. The differential transform approach was used to describe how Sepasgozar et al. [15] solved the equations for momentum and heat transfer that arose from the flow of a non-Newtonian fluid through an axisymmetric conduit with porous walls. Using machine learning approaches, Han Wei et al. [16] explored the effective thermal conductivities of composite materials and porous media. These methods are highly helpful tools that can quickly estimate the effective thermal conductivities of composite materials and porous media. H.J. Xu [17] talked about the fully developed forced convection heat transfer in a micro channel that was partially filled with a porous medium core. This was done by assuming that there was a local thermal non equilibrium effect between the solid and liquid phases. The magnetic effect of an impulsively moving semi-infinite vertical cylinder was investigated by P. Ganesan et al. [18] in the presence of a continuous heat flow and a magnetic field that was applied normal to the surface of the cylinder. SN Aristov et al. [19] studied two-dimensional time-dependent viscous fluid flow between moving rigid surfaces in either the transverse or longitudinal direction. The effective thermal conductivity, permeability, and inertial coefficient of metal foam samples with varying porosities were examined by A Bhattacharya et al [20].

In this particular investigation, we will make use of a numerical technique called the Differential Transform Method (DTM), which is predicated on the concept of Taylor series. Zhou JK. [21] is credited with having initially devised the differential transform technique. CL Chen et al. [22] used the differential transformation approach in their research on two-point boundary-value issues. The differential transform method was used by Biazar J. et al. [23] to solve quadratic Riccati differential equations, Ali J [24] solved the fifth and sixth order boundary value problems along with two conditions in a finite domain, and Mirzaee Farshid [25] solved a large number of linear and non-linear ordinary differential equations using DTM (DTM). This approach is a numerical methodology that may be used to discover approximate solutions to initial value issues that are both linear and non-linear, as well as Eigen value problems. However, it provides an exact, an approximate, and a purely numerical solution for the systems of differential equations; it also reduces the size of the computational domain; it is easily applicable to a wide variety of problems; and it gives the exact, approximate, and purely numerical solution for the systems of differential equations.

As we have reviewed the findings of the study performed by a large number of writers, we have found that some of them have addressed the flow via parallel plates while others have considered parallel plates that are moving. Some writers considered the non-Darcy flow of fluid, while others thought about the Darcy flow of fluid. We analyse heat and mass transfer via a channel filled with a porous material with moving plates and a non-Darcy viscous dissipation in the current work. In the presence of a magnetic field, MHD flow will occur. The continuity equation, the Navier-Stokes equations of fluid dynamics, and Maxwell's equations of electromagnetism are all combined to form the MHD flow equations. These equations are used to explain the flow of MHD. Differential approach is utilised to determine velocity profile and temperature profile after non-dimensionalizing the governing equations, and the influence of change of various physical factors is investigated at velocity profile and temperature profile through graphs.

Formulation of Problem

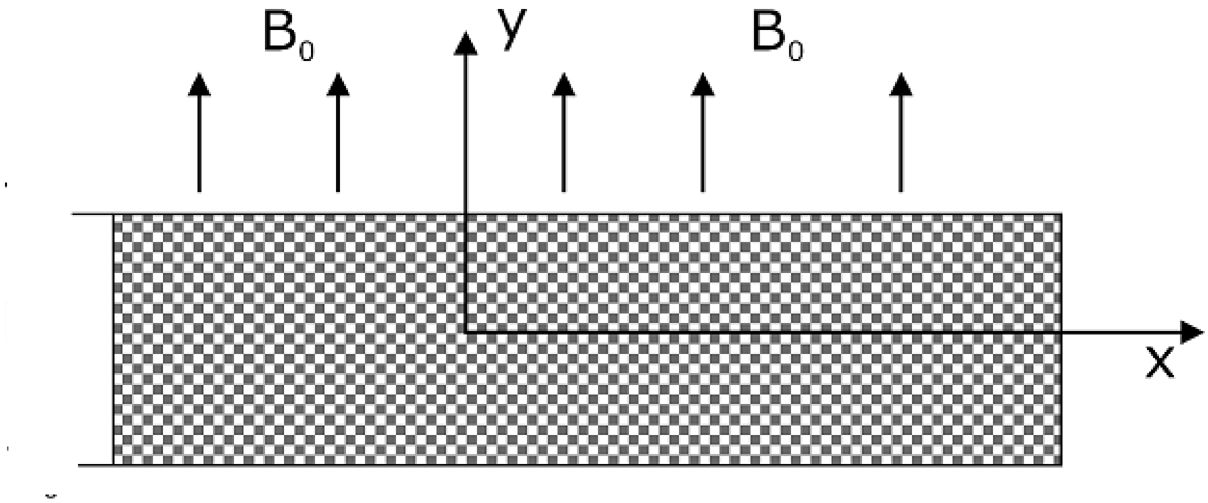

Figure 1.Physical model of the problem

Consider two semi-infinite parallel plates that are kept at a distance of 2d apart and that are parallel to the x-axis. The y-axis is normal to the "channel that is filled with porous" media and moving plates. At the exact centre of the channel, the temperature of both the bottom plate and the top plate is T1 and at its highest point. A static magnetic field with a strength of B0 is applied at an angle that is perpendicular to the direction of flow. It is assumed that the effects of the fluid's thermophysical characteristics and the extremely porous medium's thermophysical qualities remain constant. Due to channelling near to the wall, the porosity of certain porous media may not be consistent throughout the material. On the other hand, for the purpose of this investigation, the porosity and permeability were assumed to be constant along the channel's walls.

The Maxwell equations for MHD flow are, Shercliff [26]

$\mathrm{Div}\vec{B} = 0$ (1)

$\mathrm{Curl}\vec{B} = \mu_m \vec{J}$, (Amper's law) (2)

$\mathrm{curl}\vec{E} = \partial\vec{B}/\partial t$ (Faraday's law) (3)

In this equation, E stands for electric field, B for magnetic field, m for electric permeability, and J for current density. The current density is defined by the generalised version of Ohm's law as

$\vec{J}=\sigma(\vec{E}+\vec{V}\times\vec{B})$ (Ohm's law, without Hall Effect) (4)

Where σ is the electrical conductivity of the fluid

The induced electromagnetic force $F^{(em)}$ is defined as

$\vec{F}^{(em)}=\vec{J}\times\vec{B}=\sigma(\vec{E}+\vec{V}\times B)\times\vec{B}$ (5)

The governing equations can be stated as The equation of continuity for an incompressible fluid is The equation of continuity for a fluid that does not compress is

$\nabla.\vec{q}=0$ (6)

The following is the form that the momentum equation for the flow through a non-Darcy porous media using the Darcy–Brinkman–Forchheimer model, Nield and Bejan (1992) [27], needs to assume in order to account for the existence of a magnetic field:

$\rho D\vec{q}/Dt=-\nabla p+\mu_{eff}\nabla^2\vec{q}-\left\{\frac{\mu}{K}\vec{q}+\frac{\rho C_d}{\sqrt{K}}\vec{q}|\vec{q}|\right\}+\vec{J}\times\vec{B}$ (7)

Where q represents the velocity field for the flow of incompressible viscous electrically conducting fluid, ρ denotes the density of fluid, μ_{eff} denotes the effective viscosity of fluid in the porous medium, μ represents the viscosity of fluid, $_{Cd}$ denotes the drag coefficient, K represents the permeability ,B(0,B_0,0) represents the magnetic field.

Following Cowling (1957) [28] and Gupta (1960) [29], there is no applied or polarization voltage so that $\vec{E}$=0. The resultant electromagnetic force $\vec{F}^{(em)}=\vec{J}\times\vec{B}=-\sigma B_0^2 u\acute{J}$.

When viscous dissipation and joule's dissipation are taken into account in the energy equation, the resulting expression looks like this:

$\rho C_p\frac{DT}{Dt}=k\nabla^2T+\emptyset+\frac{J^2}{\sigma}$ (8)

Where C_p identifies the value of the specific heat, T stands for the temperature of the fluid, and stands for the viscous dissipation term. The variations along the x-direction are insignificant in comparison to the variations along the y-direction since the plate extends to a length that is only about finite. As a result, velocity and temperature are considered to be unrelated to x. Additionally, there is no flow in the z-direction, and the plates are extended to a length that is almost infinite in the z-direction. This results in the following: w=0 and$\frac{\partial}{\partial z}(.)=0.$ After factoring in these assumptions and taking into account the structural make-up of the issue, the solutions to equations (7) and (8) come out to be

$\mu_{eff}\frac{\partial^2u}{\partial y^2}=\frac{\partial p}{\partial x}+\{\frac{\mu}{K}u+\rho\frac{C_d}{\sqrt{K}}u^2\}+\sigma B_0^2u$ (9)

$$k\frac{\partial^2 T}{\partial y^2} + \sigma B_0^2 u^2 + \mu(\frac{\partial u}{\partial y})^2 = 0 \quad (10)$$

The boundary conditions are:

$$Y=0; \quad \frac{\partial u}{\partial y}=0 \quad \frac{\partial T}{\partial y} = 0 \quad (11)$$

$$Y=d; \quad u=v_0 \quad T=T_1$$

METHOD OF SOLUTION

The following dimensionless quantities are introduced, all of which are employed in the equations of motion and energy, as well as boundary conditions.

$$x^*=\frac{x}{d} \quad Y^*=\frac{y}{d} u^*=\frac{u}{v_0} \theta = \frac{T-T1}{T-T0}$$

$$D=\frac{K}{d^2} \quad F=\frac{\rho C_d G d^4}{\mu^2\sqrt{K}} \quad P=\frac{\mu_{eff}}{\mu} \quad H=\sqrt{\frac{\sigma d^2 {B_0}^2}{\mu}} \quad Bm =\frac{G^2 d^4}{K\mu(T_1-T_0)} \quad R=\frac{Gd^2}{\mu v_0}$$

Where $G=-\frac{\partial p}{\partial x}$ is the gradient of the pressure, D is the Darcy number, F is the Forchheimer number, P is the ratio of the effective viscosity to the viscosity of the fluid, and n is the number of particles in the fluid. The letter H stands for the Hartman number, while the letter m stands for the brinkman number.

The dimensionless version of the equation that describes motion and energy is as follows:

$$\frac{d^2u^*}{dy^2} = -\frac{R}{P} + \frac{1}{P}(\frac{1}{D} + H^2)u^* - F/Pu^{*2} \quad (12)$$

$$\frac{d^2\theta}{dy^2} + Bm(\frac{\partial u^*}{\partial y})^2 + H^2 Bmu^{*2} = 0 \quad (13)$$

The corresponding boundary conditions are:

$$y^*=0 \quad \frac{\partial u^*}{\partial y} = 0 \quad \theta = 0$$

$$y^*=1 \quad u^*=1 \quad \theta=1 \quad (14)$$

Taking a variable without a trick out of the dimensionless version of the governing equations does not result in a reduction in the equations' general applicability. Therefore, the equation of motion and energy may be written in a dimensionless form as

$$\frac{d^2u}{dy^2} = -\frac{R}{P} + \frac{1}{p}(\frac{1}{D} + H^2)u - \frac{F}{P}u^2 \quad (15)$$

$$\frac{d^2\theta}{dy^2} + Bm(\frac{\partial u}{\partial y})^2 + H^2 Bmu^2 = 0 \quad (16)$$

The corresponding boundary conditions are:

y=0 $\frac{\partial u}{\partial y} = 0$ $\theta = 0$

y=1 u=1 θ=1 (17)

Solution for Velocity Profile

For the purpose of resolving the (non-linear ordinary differential equation) momentum problem, the Differential Transform Method, abbreviated as DTM, is utilised (15).

The differential transform(s) of the derivative $\frac{d^s u(x)}{dx^s}$ is defined as

U(s) =$\frac{1}{s!}[\frac{d^s u(x)}{du^s}]_{x=x_0}$

Table 1. The fundamental operation in mathematical theory in DTM

Function	Differential Transform
u(y)=f(y)$\pm$g(y)	U(s)=F(s)$\pm$G(s)
u(y)=λg(y)	U(s)=λG(s)
u(y)=$\frac{\partial g(y)}{\partial y}$	U(s)=(s+1)G(s+1)
u(y)=$\frac{\partial^m g(y}{\partial y^m}$	U(s)=(s+1)…….(s+m)G(s+m)
u(y)=y^m	U(s)=$\delta(s-m) = \begin{cases} 1, & s = m \\ 0, & otherwise \end{cases}$
u(y)=f(y)g(y)	U(s)=$\sum_{r=0}^{s} F(r)G(s-r)$
u(y)=$f_1(y)\ f_2(y) \ldots \ldots f_m(y)$	U(s) =$\sum_{s_1}^{s} \ldots . \sum_{s_{m-1}=0}^{s_2} F_1\ (s_1) F_2(s_2 - s_1) \ldots \ldots . F_m(s - s_{m-1})$

And the definition of the inverse differential transform of U(s) is as follows:

U(x) =$\sum_{s=0}^{\infty} U(s)(x-x_0)^s$ (18)

The momentum equation (15) with the transform parameter 'r' provides the following recurrence relation when the differential transform technique is applied.

$\frac{(r+2)!}{r!}U(r+2) = MU(r) + N\sum_{t=0}^{r} U(t)U(r-t) + Z\delta(r)$ (19)

Where M = $\frac{1}{P}(\frac{1}{D} + H^2)$N=$-\frac{F}{P}$Z=-$\frac{R}{P}$

$\delta(r) = \begin{cases} 1 & r = 0 \\ 0 & r \neq 0 \end{cases}$ (20)

DTM of the boundary condition for velocity at lower plate i.e. $\left[\frac{\partial u}{\partial y}\right]_{y=0}$=0 gives

U (1) =0 (21)

The intial value of u(0) is unknown ,therefore its DTM U(0) is also not known ,so taking u(0)=α(constant) (22)

Where is a constant that is not known and must be found using the flow field and the boundary conditions that have been set.

Applying DTM on (23) gives

U (0) =α (23)

Put r=0, 1, 2,3,4,5 the recurrence relation (20) gives

U (2) =$\frac{1}{2}(M\alpha + N\alpha^2 + Z)$

U (3) =0

U (4) =$\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + (\frac{M^2}{24} + \frac{NZ}{12})\alpha + \frac{MZ}{24}$

U (5) =0

The inverse differential transform of U(s) may be defined using equation (19), which states that

U(y) =$\sum_{s=0}^{5} U(s)y^s$ (24)

Put value of U (0) to U (5) the solution of the problem up to 4th order is given by

U(y) =$\alpha + \frac{1}{2}(M\alpha + N\alpha^2 + Z)y^2 + (\frac{N^2\alpha^3}{12} + \frac{MN\alpha^2}{8} + (\frac{M^2}{24} + \frac{NZ}{12})\ \alpha + \frac{MZ}{24})y^4$ (25)

Equation (26) produces a polynomial of the third degree in when applied to the boundary condition u (1) = 1.

$$\frac{N^2}{12}\alpha^3 + \left(\frac{N}{2} + \frac{MN}{8}\right)\alpha^2 + \left(1 + \frac{M}{2} + \frac{M^2}{24} + \frac{NZ}{12}\right)\alpha + \left(\frac{Z}{2} + \frac{MZ}{24} - 1\right) = 0 \quad (26)$$

the profile of the fluid flow that was derived by solving equation (27) and computing the value of. By computing the value of in MATLAB, one may obtain the values of the physical parameters P,D,F, H, and R; the resulting velocity profile can then be studied using graphs.

Solution of Energy Equation

The differential transform method is used to derive the solution to the heat equation (16).

The differential transformφ(s) of the derivative $\frac{d^s u(x)}{dx^s}$ is defined as

φ(s)=$\frac{1}{s!}[\frac{d^s u(x)}{du^s}]_{x=x_0}$ (27)

And inverse differential transform of θ(s) is defined as

$\theta(x)=\sum_{s=0}^{\infty}\varphi(s)(x-x_0)^s$ (28)

When the differential transform technique is used, the energy equation (16), along with the transform parameter 'r,' produces the following recurrence relation:

$$\frac{(r+2)!}{r!}\theta(r+2)=E\sum_{t=0}^{r}u(t)u(r-t)+F\sum_{t=0}^{r}(t+1)(r-t+1)u(t+1)u(r-t+1)$$

(29)

Where $E=-H^2Bm$, $F=-Bm$

DTM of the boundary condition for temperature at lower plate i.e. $\left[\frac{\partial\theta}{\partial y}\right]_{y=0}=0$ gives

$\varphi(1)=0$ (30)

The intial value of $\theta(0)$ is unknown, therefore its DTM $\varphi(0)$ is also not known, so taking

$\varphi(0)=\beta(\text{constant})$ (31)

Whereβ is an undetermined constant that has to be found using the flow field along with the given boundary conditions.

Putting r= 0, 1, 2,3,4,5 the recurrence relation (30) gives

$\varphi(2)=\frac{E\alpha^2}{2}$

$\varphi(3)=0$

$\varphi(4)=\frac{E\alpha(M\alpha+N\alpha^2+Z)}{12}+\frac{F(M\alpha+N\alpha^2+Z)^2}{12}$

$\varphi(5)=0$

$\varphi(6)=\frac{E\alpha}{15}(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+(\frac{M^2}{24}+\frac{NZ}{12})\alpha+\frac{MZ}{24})+\frac{E}{120}(M\alpha+N\alpha^2+Z)^2+\frac{4E}{15}(M\alpha+N\alpha^2+Z)(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+(\frac{M^2}{24}+\frac{NZ}{12})\alpha+\frac{MZ}{24})$ (32)

From equation (29), inverse differential transform of φ(s) can be defined as

$\theta(y)=\sum_{s=0}^{5}\varphi(s)y^s$ (33)

Put value of $\varphi(0)$ to $\varphi(6)$ the solution of the problem upto 6th order is given by

$\theta(y)=\beta+\frac{E\alpha^2}{2}y^2+(\frac{E\alpha(M\alpha+N\alpha^2+Z)}{12}+\frac{F(M\alpha+N\alpha^2+Z)^2}{12})y^4+(\frac{E\alpha}{15}(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+(\frac{M^2}{24}+\frac{NZ}{12})\alpha+\frac{MZ}{24})+\frac{E}{120}(M\alpha+N\alpha^2+Z)^2+\frac{4E}{15}(M\alpha+N\alpha^2+Z)(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+(\frac{M^2}{24}+\frac{NZ}{12})\alpha+\frac{MZ}{24}))y^6$ (34)

At the boundary condition $\theta(1)=1$, the value of β is obtained from the equation (35)

$$\beta = 1-\left(\frac{E\alpha^2}{2}+\frac{E\alpha(M\alpha+N\alpha^2+Z)}{12}+\frac{F(M\alpha+N\alpha^2+Z)^2}{12}+\frac{E\alpha}{15}\left(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+\left(\frac{M^2}{24}+\frac{NZ}{12}\right)\alpha+\frac{MZ}{24}\right)+\frac{E}{120}(M\alpha+N\alpha^2+Z)^2+\frac{4E}{15}(M\alpha+N\alpha^2+Z)\left(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+\left(\frac{M^2}{24}+\frac{NZ}{12}\right)\alpha+\frac{MZ}{24}\right)\right) \tag{35}$$

The Temperature profile obtained from equation (34) by calculating value of β. By computing the value of the physical parameters P, D, Fr, H, Bm, and R, one may determine their respective values. of β through MATLAB.

SKIN FRICTION COEFFICIENT

The non-dimensional shearing stress that is placed on the top plate is calculated based on the local skin-friction coefficient, and the results of this calculation may be found in Table 1.

$S_f=[\partial u/\partial y]_{y=1}$

$$=Z+\frac{MZ}{6}+\left(M+\frac{M^2}{6}+\frac{NZ}{3}\right)\alpha+\left(N+\frac{MN}{2}\right)\alpha^2+\frac{N^2}{3}\alpha^3 \tag{36}$$

NUSSELT NUMBER

In this step, the non-dimensional coefficient of heat transfer at the top plate is determined, and the resulting calculated values are presented in Table 2.

$N_s=-(\partial\theta/\partial y)_{y=1}$

$$=\left(E\alpha^2+4\left(\frac{E\alpha(M\alpha+N\alpha^2+Z)}{12}+\frac{F(M\alpha+N\alpha^2+Z)^2}{12}\right)+6\left(\frac{E\alpha}{15}\left(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+\left(\frac{M^2}{24}+\frac{NZ}{12}\right)\alpha+\frac{MZ}{24}\right)+\frac{E}{120}(M\alpha+N\alpha^2+Z)^2+\frac{4E}{15}(M\alpha+N\alpha^2+Z)\left(\frac{N^2\alpha^3}{12}+\frac{MN\alpha^2}{8}+\left(\frac{M^2}{24}+\frac{NZ}{12}\right)\alpha+\frac{MZ}{24}\right)\right)\right) \tag{37}$$

RESULT AND DISCUSSION

Plotting the Velocity profile and the Temperature profile allows one to determine the influence that different factors have. As demonstrated in figures 2 and 3, the influence of the Hartman number on the velocity profile is depicted with relative viscosity of 0.5 and 2, respectively. As the Hartman number grows, the flow of the fluid accelerates, although the flow also accelerates when the relative viscosity is equal to 2. At P=0.5, an increase in the Darcy number causes a reduction in the velocity of the fluid, but at P=2, an increase in the Darcy number causes an acceleration of the fluid, as seen in figures 4 and 5. In the situation where P is equal to 0.5, the fluid velocity goes up significantly when the Forchheimer number goes from 5 to 10, while in the instance where P is equal to 2, the fluid flow is not much impacted by the rise in Forchheimer number, as shown in figures 6 and 7. In the instance of P=0.5, a rise in the Reynolds number causes the flow to slow down, whereas an increase in the Reynolds number causes the flow to speed up, as illustrated in figures 8 and 9. If the effective viscosity is higher than the fluid viscosity, then the fluid flow will be similar to what is shown in figure 10. This is because the relative viscosity will have a greater impact on the flow of the fluid. In the scenario in which P is equal to 0.5, the temperature of the fluid rises as the Hartman number rises, as shown in figure 11. In the scenario in which P is equal to 2, the temperature of the fluid first rises but then falls as the Hartman number rises, as shown in figure 12. The temperature exhibits a decreasing trend with increasing Darcy number when P=0.5, while it exhibits an increasing trend with increasing Darcy number when P=2 as seen in figures 13 and 14. The patterns for fluid flow may be seen with an increasing Forchheimer number in figures 15 and 16. These figures can be seen below. As seen in figure 17, the temperature drops as the relative viscosity of the substance increases. If the fluid has a pressure of 0.5, then an increase in the Reynolds number and the Brinkman number will cause the temperature of the fluid to rise; however, if the pressure is 2, then the temperature will fall, as seen in figures 18,19,20, and 21 correspondingly. According to Table 1, the rate of heat transfer decreases as the ratio of effective viscosity to the viscosity of the fluid and the Darcy number are raised, while the rate of heat transfer increases when the Hartmann and Forchheimer numbers are increased. When the Reynolds and Brinkman numbers are raised, there is a modest increase in the rate at which heat is transferred. At the upper plate, the magnitude of the skin-friction coefficient is determined, and the results are listed in Table 2. When the viscosity of the fluid is larger than the effective viscosity of the fluid, the skin friction coefficient is at its highest possible value. When the Darcy number is increased, the skin-friction coefficient is also increased. When the value of the Forchheimer number is between 5 and 20, there is a significant drop in the skin friction coefficient. On the other hand, there is only a modest decrease in the skin friction coefficient when the Reynolds number is increased.

Table 1 Values of Nusselt Number at the Upper Plate

P	R	D	H	F	Bm	Ns
0.5	0.01	0.1	5	10	0.01	60.5971
1	0.01	0.1	5	10	0.01	16.3457
2	0.01	0.1	5	10	0.01	0.0133
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.01	0.5	5	10	0.01	22.7306
0.5	0.01	1	5	10	0.01	19.6926
0.5	0.01	0.1	1	10	0.01	0.0129
0.5	0.01	0.1	3	10	0.01	2.6734
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.01	0.1	5	1	0.01	0.0014
0.5	0.01	0.1	5	5	0.01	0.0015
0.5	0.01	0.1	5	10	0.01	60.5971
0.5	0.1	0.1	5	10	0.01	0.0015
0.5	0.5	0.1	5	10	0.01	0.0018
0.5	1	0.1	5	10	0.01	0.0024
0.5	0.01	0.1	5	10	0.5	0.0732
0.5	0.01	0.1	5	10	1	0.1438
0.5	0.01	0.1	5	10	2	0.2773

Table 2 Values of Skin -Friction Coefficient at the Upper Plate

P	R	D	H	F	Sf
0.5	0.01	0.1	5	10	-63.5293
1	0.01	0.1	5	10	-31.8536
2	0.01	0.1	5	10	3.0250
0.5	0.01	0.1	5	10	-63.5293
0.5	0.01	0.5	5	10	-37.1335
0.5	0.01	1	5	10	-34.2804
0.5	0.01	0.1	1	10	-0.5803
0.5	0.01	0.1	3	10	-17.1536
0.5	0.01	0.1	5	10	-63.5293
0.5	0.01	0.1	5	1	3.6467
0.5	0.01	0.1	5	5	3.7227
0.5	0.01	0.1	5	10	-63.5293
0.5	0.1	0.1	5	10	3.6474
0.5	0.5	0.1	5	10	3.6144
0.5	1	0.1	5	10	3.5874

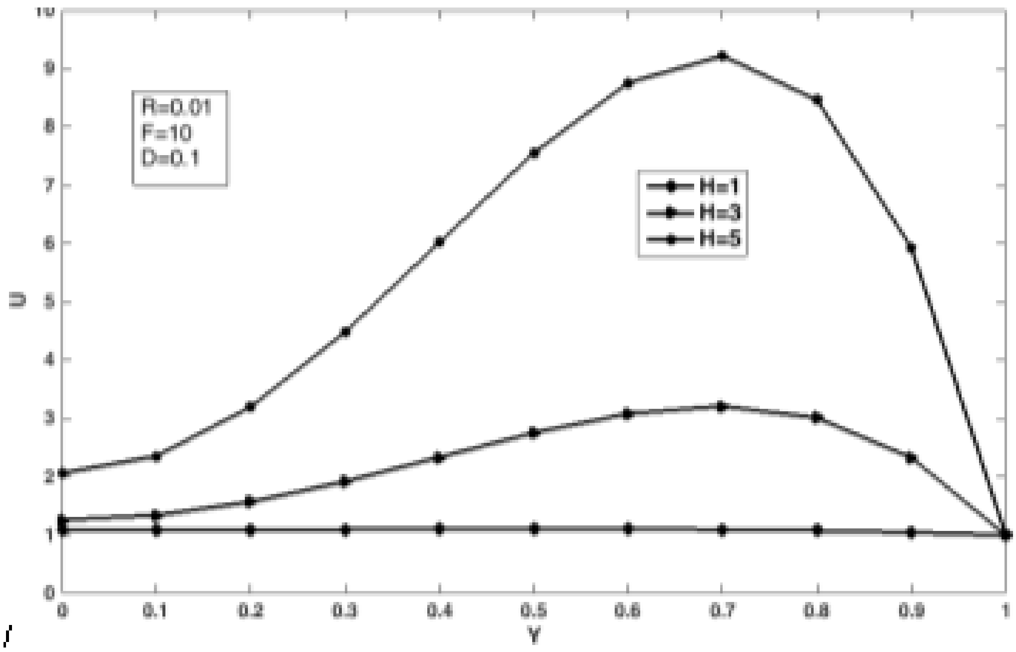

Figure 2. Effect Of Hartman No. On Velocity Profile At P=0.5

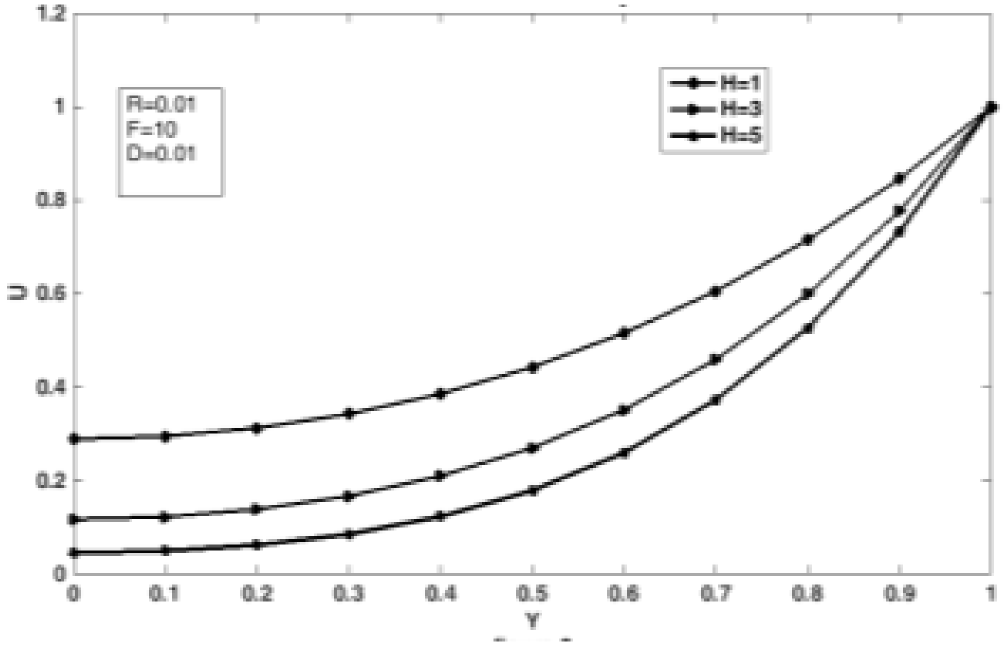

Figure 3. Effect Of Hartman No. On Velocity Profile At P=2

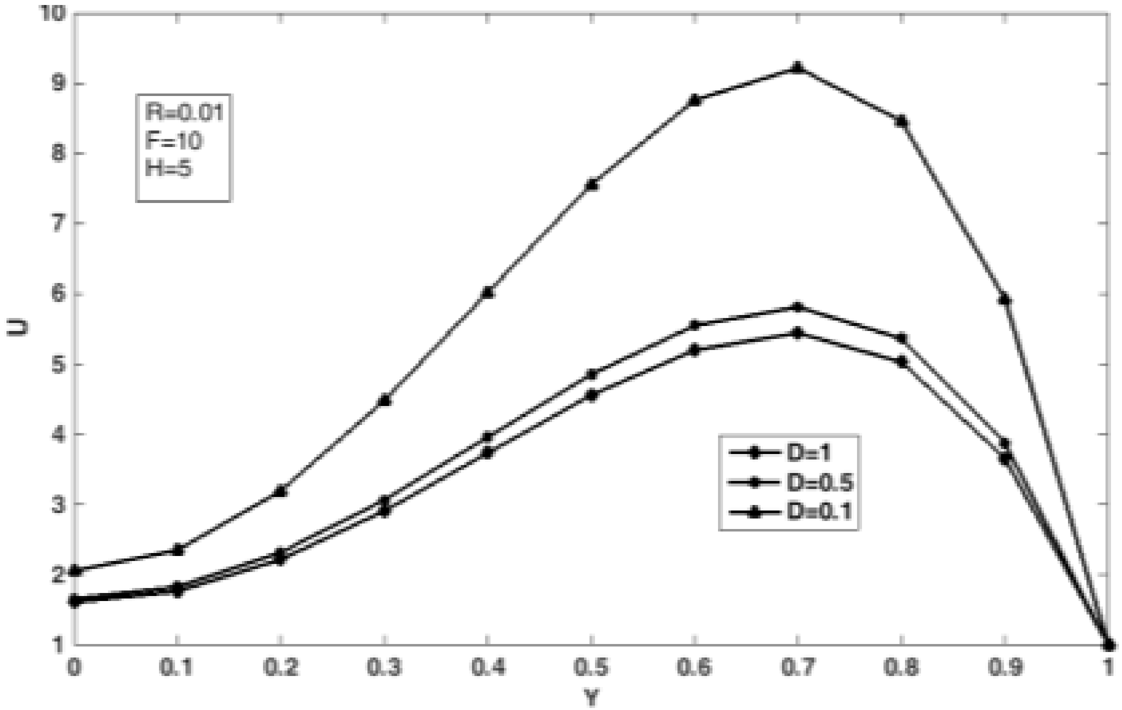

Figure 4. Effect Of Darcy No. On Velocity Profile At P=0.5

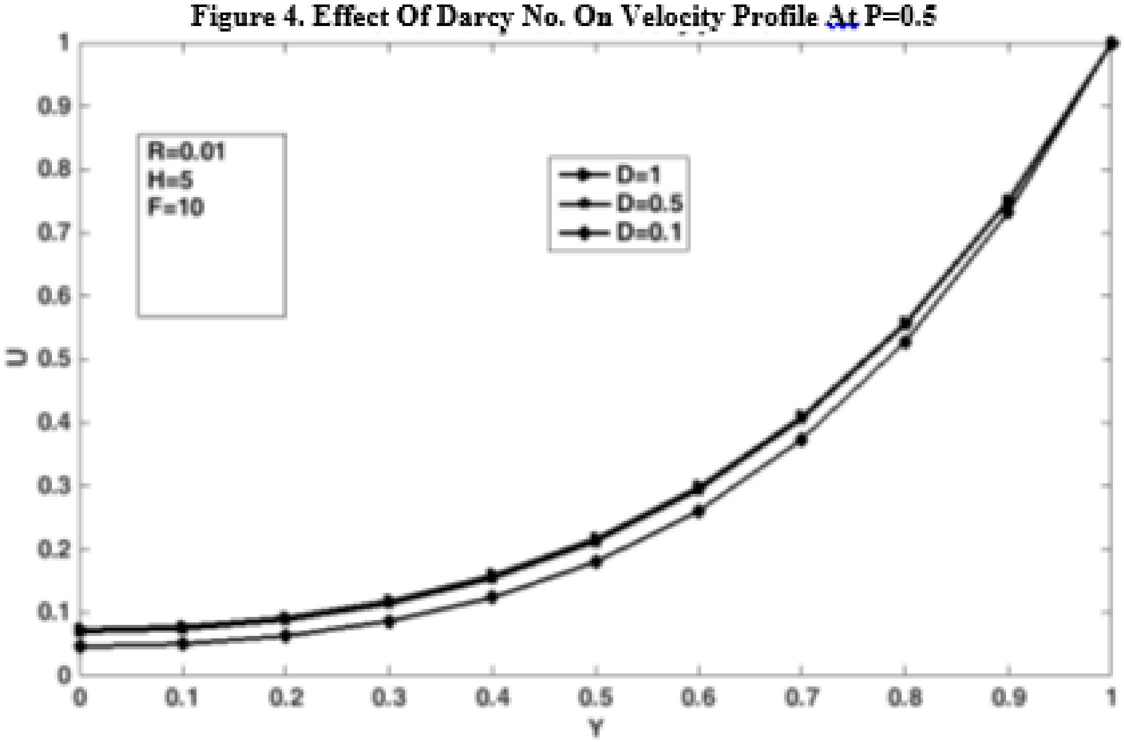

Figure 5. Effect Of Darcy No. On Velocity Profile At P=2

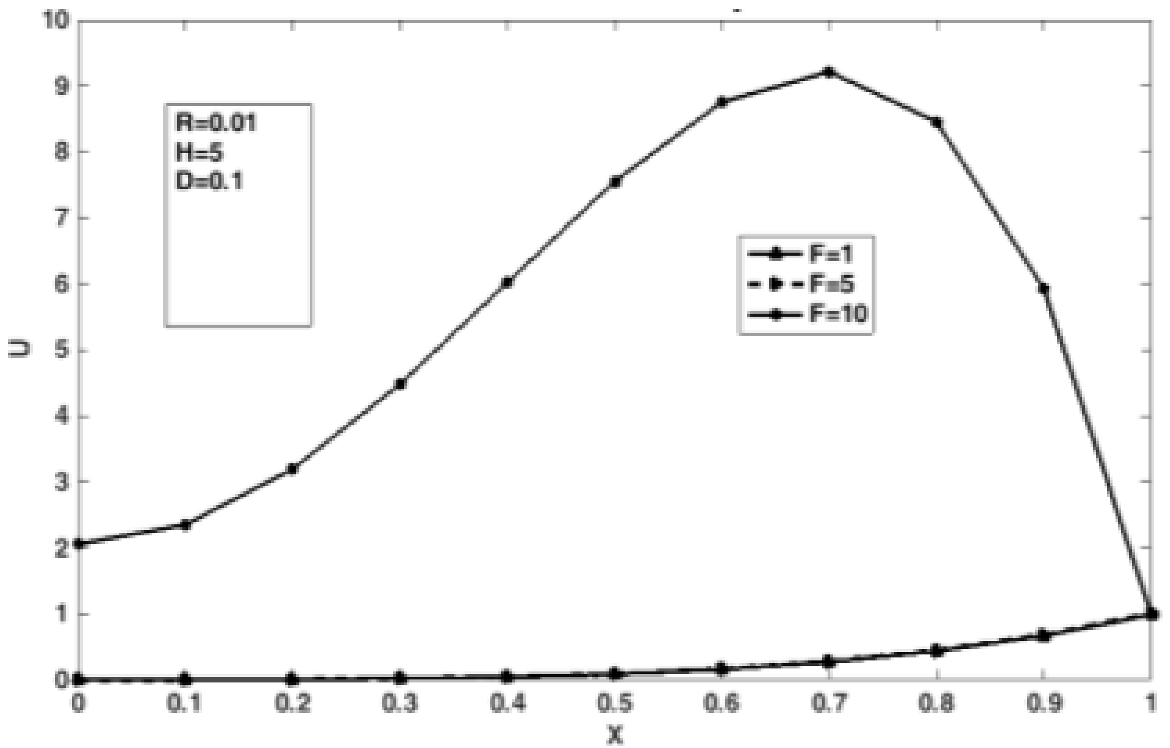

Figure 6. Effect Of Darcy No. On Velocity Profile At P=0.5

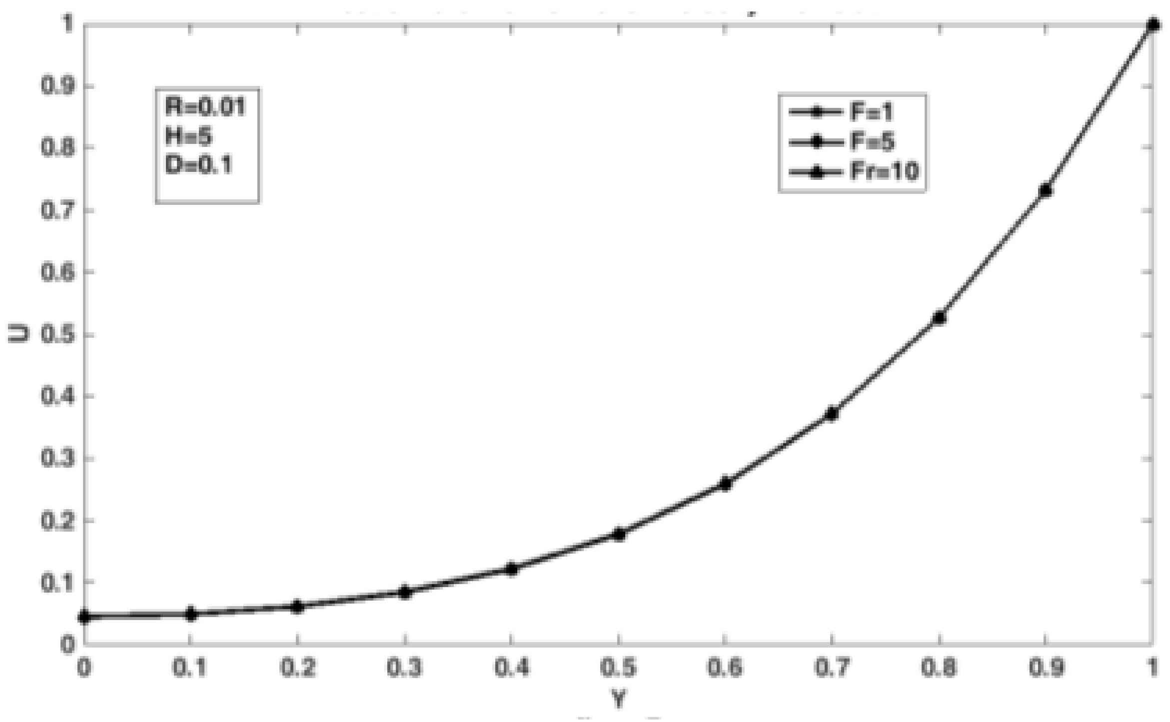

Figure 7. Effect Of Darcy No. On Velocity Profile At P=2

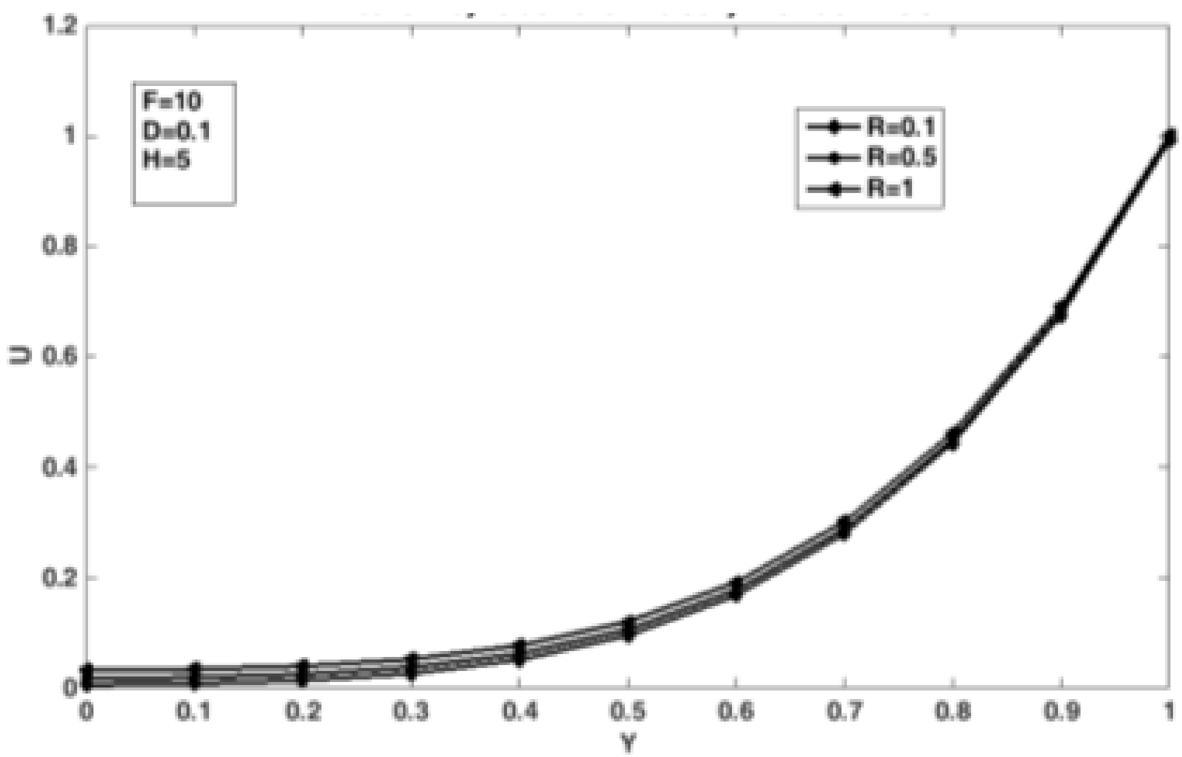

Figure 8. Effect Of Reynolds no. on Velocity Profile At P=0.5

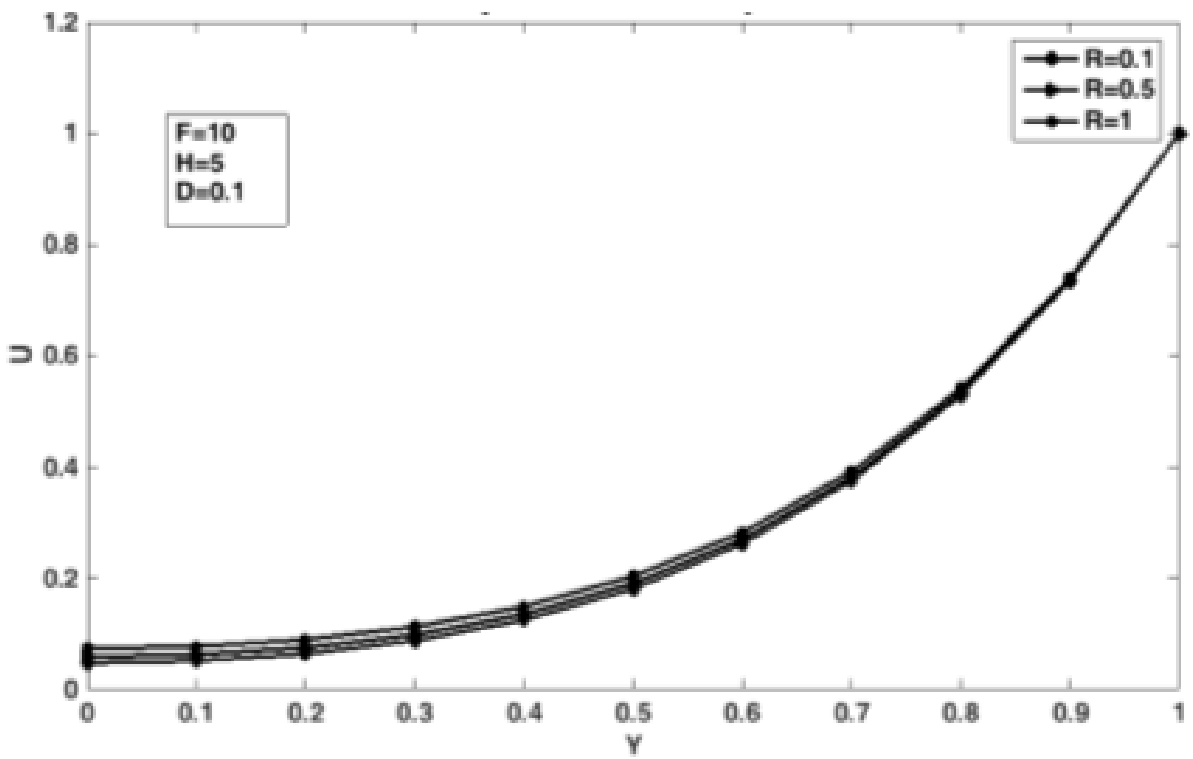

Figure 9. Effect Of Reynolds no. on Velocity Profile At P=2

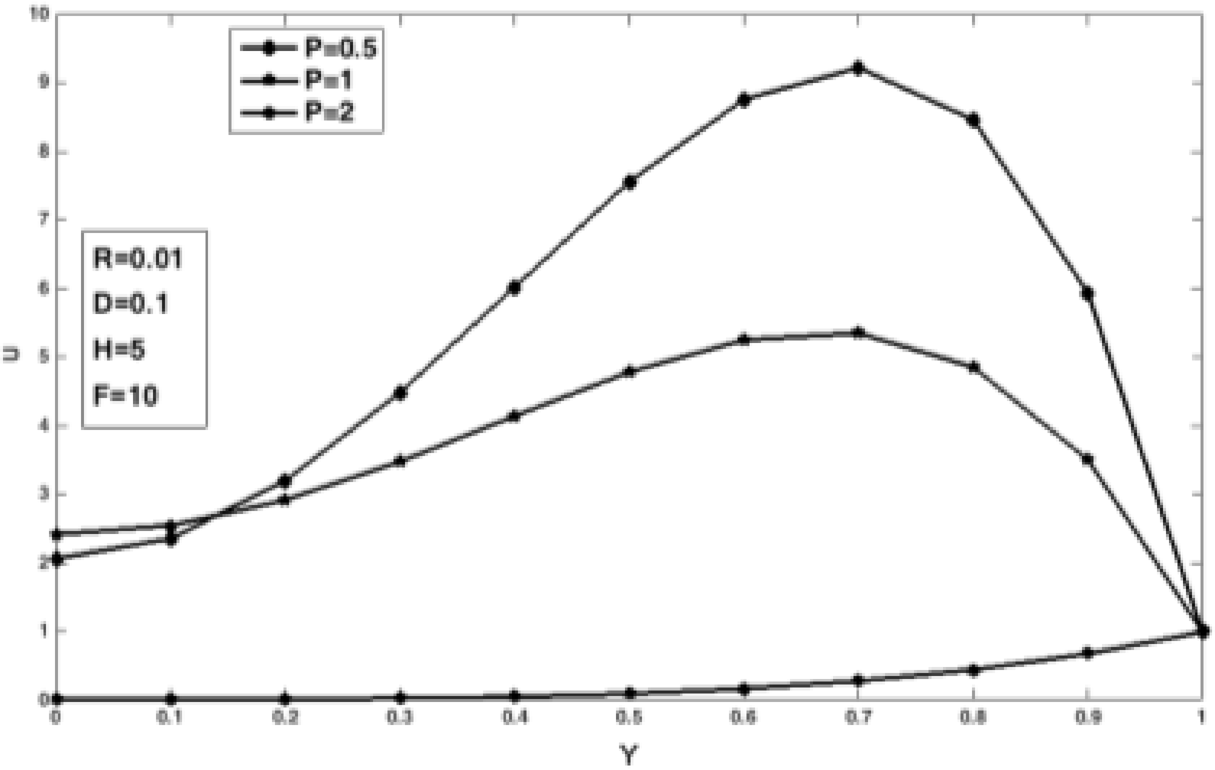

Figure 10. Effect Of P Velocity Profile

CONCLUSION

When the Reynolds and Brinkman numbers are raised, there is a modest increase in the rate at which heat is transferred. When the ratio of effective viscosity to the viscosity of the fluid and the Darcy number are both raised, the rate of heat transfer is reduced, whereas the rate of heat transfer is enhanced when the Hartmann and Forchheimer numbers are both increased. When the viscosity of the fluid is larger than the effective viscosity of the fluid, the skin friction coefficient is at its highest possible value. In direct proportion to the increase in the Hartman number, the temperature of the fluid also rises.

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