A PARAMETRIC PROGRAMMING APPROACH TO AN INTUITIONISTIC FUZZY QUEUING MODEL

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Abstract
Fuzziness is a new type of unavoidable circumstance. Ambiguity is depicted by the fuzzy set theory. Intuitionistic Fuzzy queuing model with flexible service policy is being investigated in this study. A parametric programming technique is designed to find the membership and non-membership functions of queue length and sojourn time in steady state, in which the arrival rate and service rate are being Intuitionistic pentagon fuzzy numbers. Based on α, β -cuts and Zadeh's extension principle, the intuitionistic fuzzy queues are evolved into family of crisp queues. The model's potency is calculated for various possibilities of α -cuts and β - cuts.

Keywords: Fuzzy Sets, Intuitionistic fuzzy sets, Membership Functions, Non-membership functions, Fuzzy Queuing Model, Intuitionistic Pentagon fuzzy number, Flexible Service Policy, Parametric Programming, α -cuts , β - cuts, Queue length, Sojourn time,

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1. Introduction

Queuing Theory is an important facet of operational research. Applications of waiting line is very essential in day to day life but also in sequence of computer programming, networks, medical field, banking sectors, call centres, telecommunications, manufacturing and production systems. Many researchers have investigated the optimization problems of queuing systems, as well as their fine structure and techniques in several Markovian lines. Various service policies bring different functionality to multi-server queue systems. Burnetas and Economou [1] was to propose an equilibrium strategies of customers in several Markovian queues. Many scholars have designed and contributed Markovian queues in diverse contexts, including Economou and Kanta [2], Guo and Hassin [9], Li and Li [8], Lan and Tang. Multi server queuing systems with different service policies has been analysed by the researchers like T. V. Do, Baumann and Sandmann, Liu and Yu [11].

Fuzzy queuing model have been described by the authors like R.J. Li and E.S. Lee [10], J.J. Buckley [7], R.S. Negi and E.S. Lee [4]. Chen has discussed the queuing system in a fuzzy environment using Zadeh's extension principle. Kao et al derived the membership functions of the system measures for fuzzy queues using parametric programming technique. Atanassov created intuitionistic fuzzy sets (IFS), a generalization of fuzzy sets that incorporates the degree of non-membership function. Intriguing and helpful in a wide range of application domains is the idea of defining intuitionistic fuzzy sets. The intuitionistic fuzzy set is used to describe practical issues in fields including marketing, psychology, financial services, medical diagnostics, sales analysis and so forth.

This paper is structured as follows: second segment provides some basic definitions of this research work. Third segment details a mathematical model which figures queue length and sojourn time. Fourth segment describes the parametric programming technique of a multi-server fuzzy queue with flexible service policy. Fifth segment illustrates a numerical example. Sixth segment concludes the paper.

Preliminaries:

Definition: 2.1

A is a fuzzy set defined on E and can be written as a collection of ordered pairs. If E is a universe of discourse and x is a particular element of E, then $\tilde{A}=\{(x,\phi_{\tilde{A}}(x))/x\in E\}$.

Definition: 2.2

The α - cut of a fuzzy number $\tilde{A}$ is defined as $\tilde{A}=\{x:\phi_{\tilde{A}}(x)\geq\alpha,\ \alpha\in[0,1]\}$

Definition: 2.3

A Trapezoidal fuzzy number is defined as $\tilde{A}=(a,b,c,d)$ where a < b < c < d with its membership function is given by:

$$\phi_{\tilde{A}}(x)=\begin{cases} L(x)=\dfrac{x-a}{b-a}, & a\leq x\leq b \\ 1 & , b\leq x\leq c \\ R(x)=\dfrac{d-x}{d-c}, & c\leq x\leq d \\ 0 & , otherwise \end{cases}$$

Definition: 2.4

A Pentagon fuzzy number is defined as $\tilde{A}=(a_1,a_2,a_3,a_4,a_5)$ where $a_1\leq a_2\leq a_3\leq a_4\leq a_5$ with its membership function is given by:

$$\phi_{\tilde{A}}(x)=1\begin{cases}0 & , \quad x<a_1\\ L_1(x)=\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ L_2(x)=\dfrac{x-a_2}{a_3-a_2}, & a_2\le x\le a_3\\ 1 & , \quad x=a_3\\ R_1(x)=\dfrac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\\ R_2(x)=\dfrac{a_5-x}{a_5-a_4}, & a_4\le x\le a_5\\ 0 & , \quad x>a_5\end{cases}$$

Definition: 2.5

Let $\tilde{C}^I$ be an intuitionistic fuzzy set in universe of discourse U and defined by a set of ordered triple $\tilde{C}^I=\left\{\left(x,\phi_{\tilde{C}^I}(x),\psi_{\tilde{C}^I}(x)\right);x\in U\right\}$ where the functions $\phi_{\tilde{C}^I}(x),\psi_{\tilde{C}^I}(x)$ is a mapping from $U\to[0,1]$, such that $0\le\phi_{\tilde{C}^I}(x)+\psi_{\tilde{C}^I}(x)\le 1,\forall x\in U$. The membership degree of the element $x\in U$ in $\tilde{C}^I$ represents as $\phi_{\tilde{C}^I}(x)$. The non-membership degree of the element $x\in U$ in $\tilde{C}^I$ represents as $\psi_{\tilde{C}^I}(x)$.. And finally, the hesitation degree of $\tilde{C}^I$ is $h(x)$, defined by

$$h(x)=1-\phi_{\tilde{C}^I}(x)-\psi_{\tilde{C}^I}(x),\forall x\in U.$$

Definition: 2.6

A pentagonal intuitionistic fuzzy number of an intuitionistic fuzzy set $\tilde{C}^I$ is denoted as $\tilde{C}^{PI}=\left(c_a,c_b,c_c,c_d,c_e;c_a',c_b',c_c',c_d',c_e'\right)$ where $c_i,c_i'\in R, i=a,b,c,d,e$ and its membership and non-membership function are defined by

$$\phi_{\tilde{C}^{PI}}(x)=\begin{cases}0, & x<c_a\\ \dfrac{1}{2}\left(\dfrac{x-c_a}{c_b-c_a}\right), & c_a\le x\le c_b\\ \dfrac{1}{2}+\dfrac{1}{2}\left(\dfrac{x-c_b}{c_c-c_b}\right), & c_b\le x\le c_c\\ 1, & x=c_c\\ \dfrac{1}{2}+\dfrac{1}{2}\left(\dfrac{c_d-x}{c_d-c_c}\right), & c_c\le x\le c_d\\ \dfrac{1}{2}\left(\dfrac{c_e-x}{c_e-c_d}\right), & c_d\le x\le c_e\\ 0, & x\ge c_e\end{cases}$$

$$\psi_{\tilde{C}^{PI}}(x)=\begin{cases}1, & x<c_a^{'}\\ \frac{1}{2}+\frac{1}{2}\left(\frac{c_b^{'}-x}{c_b^{'}-c_a^{'}}\right), & c_a^{'}\le x\le c_b^{'}\\ \frac{1}{2}\left(\frac{c_c^{'}-x}{c_c^{'}-c_b^{'}}\right), & c_b^{'}\le x\le c_c^{'}\\ 0, & x=c_c^{'}\\ \frac{1}{2}\left(\frac{x-c_c^{'}}{c_d^{'}-c_c^{'}}\right), & c_c^{'}\le x\le c_d^{'}\\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-c_d^{'}}{c_e^{'}-c_d^{'}}\right), & c_d^{'}\le x\le c_e^{'}\\ 1, & x>c_e^{'}\end{cases}$$

Definition: 2.7
Zadeh's Extension Principle:

Let the inter arrival rate $\tilde{\lambda}$ and service rate $\tilde{\mu}$ are all fuzzy numbers. The system performance measure $P(x, y)$ is defined as

$$\phi_{p(\tilde{\lambda},\tilde{\mu})}(Z)=\underset{\substack{x\in X\\ y\in Y}}{Sup}\left\{\min(\phi_{\tilde{\lambda}}(x),\phi_{\tilde{\mu}}(y))\,/\,Z=p(x,y)\right\}$$

Model Formulation
The Queuing system with an infinite waiting room with flexible service policy is considered in an intuitionistic fuzzy environment. The customers arrive according to a Poisson process with arrival rate λ .When the system has at least two customers, each server provides service to one of them separately. When the system only has one customer, however, the two servers serve that customer collectively at the same time. It is called the TTO service policy. If the system has two identical servers, but one server can serve two customers at the same time. If the system has at least two customers, each server services a customer individually, or if the system only has one customer, both servers serve the customer simultaneously. If another customer arrives before servicing a customer, one of the two servers turns to service the new customer immediately. When a customer is serviced by one server, the service time is an exponential distribution with the parameter μ . On the other hand, when a customer is serviced by two servers together, we introduce the interaction parameter q. The traffic intensity $\rho=\frac{\lambda}{2\mu}$

The steady-state equations governing the classical queuing model are derived as follows:

$$\lambda P_0-2\mu qP_1=0,$$
$$\lambda P_0-(\lambda+2\mu q)P_1+2\mu P_2=0,$$
$$\lambda P_i-(\lambda+2\mu)P_{i+1}+2\mu P_{i+2}=0, \qquad i\ge 1$$

Solving the above equations, we obtain:

$P_0=$ Pr{system is empty}

$$=\frac{2\mu q-\lambda q}{2\mu q-\lambda q+\lambda}$$

$$P_i=\left(\frac{\lambda}{2\mu}\right)^i\frac{2\mu-\lambda}{2\mu q-\lambda q+\lambda}, \qquad i\ge 1$$

$P_{NW}=$ Pr{the customer need not to wait}

$$= P_0 + P_1$$
$$= \frac{4\mu^2 q - 2\lambda\mu q + 2\mu\lambda - \lambda^2}{4\mu^2 q - 2\mu\lambda q + 2\mu\lambda},$$

3.1. Performance Measures

(i) Steady-state queue Length $\left(N_q\right)$

$$N_q = \frac{2\mu\lambda}{[(2\mu-\lambda)q+\lambda](2\mu-\lambda)},$$

(ii) Steady-state Sojourn time (W_S)

$$W_s = \frac{2\mu}{[(2\mu-\lambda)q+\lambda](2\mu-\lambda)},$$

1. Formulation of an Intuitionistic Fuzzy Queuing Model

Consider a two server fuzzy queue system with flexible service policy. The inter arrival time $\tilde{A}$ and service time $\tilde{S}$ are denoted by the following intuitionistic fuzzy sets

$$\tilde{A} = \left\{(a, \phi_{\tilde{A}(\alpha,\beta)}(a), \psi_{\tilde{A}(\alpha,\beta)}(a)) / a \in X\right\}$$
$$\tilde{S} = \left\{(s, \phi_{\tilde{S}(\alpha,\beta)}(s), \psi_{\tilde{S}(\alpha,\beta)}(s)) / s \in Y\right\}$$

Where X and Y are crisp sets of an intuitionistic inter arrival time and intuitionistic service time. The (α, β)-cuts of the arrival rate and service rate can be represented by

$$\tilde{A}(\alpha,\beta) = \left\{(a \in X / \phi_{\tilde{A}}(a) \geq \alpha, \psi_{\tilde{A}}(a) \leq \beta)\right\}$$
$$\tilde{S}(\alpha,\beta) = \left\{(s \in Y / \phi_{\tilde{S}}(s) \geq \alpha, \psi_{\tilde{S}}(s) \leq \beta)\right\}$$

Where $\tilde{A}(\alpha,\beta)$, $\tilde{S}(\alpha,\beta)$ are the crisp subsets of X and Y respectively. Let the confidence intervals of intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{S}$ be $\left[l_{\tilde{A}(\alpha,\beta)}, u_{\tilde{A}(\alpha,\beta)}\right]$ and $\left[l_{\tilde{S}(\alpha,\beta)}, u_{\tilde{S}(\alpha,\beta)}\right]$. Thus the crisp queue can be got from the intuitionistic fuzzy queue for different (α,β)-cuts. The membership function and non-membership function of the performance measures $p(\tilde{A}, \widehat{S})$ are defined from Zadeh's extension principle,

$$\phi_{p(\tilde{A},\tilde{S})}(Z) = \underset{\substack{a\in X \\ s\in Y}}{Sup}\left\{\min(\phi_{\tilde{A}}(a), \psi_{\tilde{S}}(s)) / Z = p(a,s)\right\}$$
$$\psi_{p(\tilde{A},\tilde{S})}(Z) = \underset{\substack{a\in X \\ s\in Y}}{Inf}\left\{\min(\phi_{\tilde{A}}(a), \psi_{\tilde{S}}(s)) / Z = p(a,s)\right\}$$

The parametric programming technique is used for finding upper and lower bounds of (α,β)-cuts of $\phi_{p(\tilde{A},\tilde{S})}(Z)$ and $\psi_{p(\tilde{A},\tilde{S})}(Z)$ which are

$$l_{p(\alpha,\beta)} = \min p(a,s) \text{ ; Such that } l_{A(\alpha,\beta)} \leq a \leq u_{A(\alpha,\beta)} \text{ and } l_{S(\alpha,\beta)} \leq s \leq u_{S(\alpha,\beta)}$$
$$u_{p(\alpha,\beta)} = \max p(a,s) \text{ ; Such that } l_{A(\alpha,\beta)} \leq a \leq u_{A(\alpha,\beta)} \text{ and } l_{S(\alpha,\beta)} \leq s \leq u_{S(\alpha,\beta)}$$

Both the upper and lower bounds of $p(\alpha,\beta)$ is invertible with respect to α, β .

The left shape function $L_{\phi}(z)$ and right shape function $R_{\phi}(z)$ of membership functions are obtained from $l^{-1}_{p(\alpha,\beta)}$ and $u^{-1}_{p(\alpha,\beta)}$.

$$\phi_{P(\tilde{A},\tilde{S})}(z)=\begin{cases} L_1(z), & z_1^M \le z \le z_2^M \\ L_2(z), & z_2^M \le z \le z_3^M \\ 1, & z = z_3^M \\ R_1(z), & z_3^M \le z \le z_4^M \\ R_2(z), & z_4^M \le z \le z_5^M \end{cases}$$

Similarly for Non-Membership function $\psi_{p(\tilde{A},\tilde{S})}(Z)$ the left shape function $L_{\psi}(z)$ and right shape function $R_{\psi}(z)$ can be obtained as

$$\psi_{P(\tilde{A},\tilde{S})}(z)=\begin{cases} L_1(z), & z_1^N \le z \le z_2^N \\ L_2(z), & z_2^N \le z \le z_3^N \\ 1, & z = z_3^N \\ R_1(z), & z_3^N \le z \le z_4^N \\ R_2(z), & z_4^N \le z \le z_5^N \end{cases}$$

Numerical Illustration:

Consider a two server fuzzy queuing system that runs on a two-to-one (TTO) service policy basis. The service rate is assumed to follow exponential distribution, whereas the arrival rate follows the Poisson distribution. When a customer is serviced by two servers together, we introduce the interaction parameter q. In this system, we take q=1. A parametric programming technique can be used to evaluate the efficiency of the intuitionistic fuzzy queuing model.

Let the inter arrival rate and service rate are pentagon intuitionistic fuzzy numbers. Let $\tilde{\lambda}=\langle(1,2,3,4,5),(4,5,6,7,8)\rangle$ and $\tilde{\mu}=\langle(11,12,13,14,15),(13,14,15,16,17)\rangle$

The (α,β)-cut of $\tilde{\lambda}$ are $\tilde{\lambda}=\langle(1+2\alpha,5-2\alpha),(4+2\beta,8-2\beta)\rangle$

The (α,β)-cut of $\tilde{\mu}$ are $\tilde{\mu}=\langle(11+2\alpha,15-2\alpha),(13+2\beta,17-2\beta)\rangle$

Where $\alpha,\beta\in[0,1]$.

(i) Steady-State queue Length (N_q)

$$N_q=\frac{2\tilde{\mu}\tilde{\lambda}}{[(2\tilde{\mu}-\tilde{\lambda})q+\tilde{\lambda}](2\tilde{\mu}-\tilde{\lambda})},$$

The parametric programming for the steady state queue length is

$$l_{N_q(\alpha)}=Min\left\{\frac{2xy}{[(2y-x)+x](2y-x)}\right\},\ \text{Such that } 1+2\alpha<x<5-2\alpha \text{ and } 11+2\alpha<y<15-2\alpha$$

$$u_{N_q(\alpha)}=Max\left\{\frac{2xy}{[(2y-x)+x](2y-x)}\right\},\ \text{Such that } 1+2\alpha<x<5-2\alpha \text{ and } 11+2\alpha<y<15-2\alpha.$$

For $l_{N_q(\alpha)}, x\to 1+2\alpha$ *and* $y\to 15-2\alpha$

$$l_{N_q(\alpha)}=\left\{\frac{-8\alpha^2+56\alpha+30}{24\alpha^2-296\alpha+870}\right\}$$

$For\ \ u_{N_q(\alpha)}, x \to 5-2\alpha\ and\ y \to 11+2\alpha$

$$u_{N_q(\alpha)} = \left\{ \frac{-8\alpha^2 - 24\alpha + 110}{24\alpha^2 + 200\alpha + 374} \right\}$$

The membership function is

$$\phi_{N_q}(z) = \begin{cases} L(z), & \left[l_{N_q(\alpha)}\right]_{\alpha=0} \le z \le \left[l_{N_q(\alpha)}\right]_{\alpha=1} \\ R(z), & \left[u_{N_q(\alpha)}\right]_{\alpha=1} \le z \le \left[u_{N_q(\alpha)}\right]_{\alpha=0} \\ 0 & ,\ otherwise \end{cases}$$, Which is defined as

$$\phi_{N_q}(z) = \begin{cases} \dfrac{(296z+56) \pm 64\sqrt{(z^2+2z+1)}}{48z+16} & , 0.0345 \le z \le 0.1304 \\ \dfrac{-(200z+24) \pm 64\sqrt{(z^2+2z+1)}}{48z+16} & , \ 0.1304 \le z \le 0.2941 \\ 0 & , \ otherwise \end{cases}$$

$$l_{N_q(\beta)} = Min\left\{ \frac{2xy}{[(2y-x)+x](2y-x)} \right\}$$, Such that $4+2\beta < x < 8-2\beta\ \ and\ \ 13+2\beta\ < x < 17-2\beta$

$$u_{N_q(\beta)} = Max\left\{ \frac{2xy}{[(2y-x)+x](2y-x)} \right\}$$, Such that $4+2\beta < x < 8-2\beta\ \ and\ \ 13+2\beta\ < x < 17-2\beta$

$For\ \ l_{N_q(\beta)}, x \to 4+2\beta\ and\ y \to 17-2\beta$

$$l_{N_q(\beta)} = \left\{ \frac{-8\beta^2 + 52\beta + 136}{24\beta^2 - 324\beta + 1020} \right\}$$

$For\ \ u_{N_q(\beta)}, x \to 8-2\beta\ and\ y \to 13+2\beta$

$$u_{N_q(\beta)} = \left\{ \frac{-8\beta^2 - 20\beta + 208}{24\beta^2 + 228\beta + 468} \right\}$$

The Non- membership function is

$$\psi_{N_q}(z) = \begin{cases} L(z), & \left[l_{N_q(\beta)}\right]_{\beta=0} \le z \le \left[l_{N_q(\alpha)}\right]_{\beta=1} \\ R(z), & \left[u_{N_q(\beta)}\right]_{\beta=1} \le z \le \left[u_{N_q(\alpha)}\right]_{\beta=0} \\ 0 & ,\ otherwise \end{cases}$$, Which is defined as

$$\psi_{N_q}(z)=\begin{cases}\dfrac{(-324z+52)\pm 84\sqrt{(z^2+2z+1)}}{48z+16} & ,0.1333\le z\le 0.2500\\ \dfrac{-(228z+20)\pm 84\sqrt{(z^2+2z+1)}}{48z+16}, & 0.2500\le z\le 0.4444\\ 0 & ,\ otherwise\end{cases}$$

(ii) Steady-State Sojourn time (W_S)

$$W_s = \frac{2\tilde{\mu}}{[(2\tilde{\mu}-\tilde{\lambda})q+\tilde{\lambda}](2\tilde{\mu}-\tilde{\lambda})},$$

The parametric programming for the steady-state Sojourn time is

$$l_{W_s(\alpha)} = Min\left\{\frac{2y}{[(2y-x)+x](2y-x)}\right\}, \text{ Such that } 1+2\alpha<x<5-2\alpha \ and \ 11+2\alpha<y<15-2\alpha .$$

$$u_{W_s(\alpha)} = Max\left\{\frac{2y}{[(2y-x)+x](2y-x)}\right\}, \text{ Such that } 1+2\alpha<x<5-2\alpha \ and \ 11+2\alpha<y<15-2\alpha .$$

$For \ l_{W_s(\alpha)}, x\to 1+2\alpha \ and \ y\to 15-2\alpha$

$$l_{W_s(\alpha)} = \left\{\frac{30-4\alpha}{24\alpha^2-296\alpha+870}\right\}$$

$For \ u_{W_s(\alpha)}, x\to 5-2\alpha \ and \ y\to 11+2\alpha$

$$u_{W_s(\alpha)} = \left\{\frac{22+4\alpha}{24\alpha^2+200\alpha+374}\right\}$$

The membership function

$$\phi_{W_s}(z)=\begin{cases} L(z), & \left[l_{W_s(\alpha)}\right]_{\alpha=0}\le z\le \left[l_{W_s(\alpha)}\right]_{\alpha=1}\\ R(z), & \left[u_{W_s(\alpha)}\right]_{\alpha=1}\le z\le \left[u_{W_s(\alpha)}\right]_{\alpha=0}\\ 0 & ,\ otherwise\end{cases}$$

Which is defined as

$$\phi_{W_s}(z)=\begin{cases}\dfrac{(296z-4)\pm 4\sqrt{(256z^2+32z+1)}}{48z} & ,0.0345\le z\le 0.0435\\ \dfrac{-(200z-4)\pm 4\sqrt{(256z^2+32z+1)}}{48z}, & 0.0435\le z\le 0.0588\\ 0 & ,\ otherwise\end{cases}$$

$$l_{W_s(\beta)} = Min\left\{\frac{2xy}{[(2y-x)+x](2y-x)}\right\}, \text{ Such that } 4+2\beta<x<8-2\beta \ and \ 13+2\beta<y<17-2\beta$$

$$u_{W_s(\beta)} = Max\left\{\frac{2xy}{[(2y-x)+x](2y-x)}\right\}$$, Such that $4+2\beta < x < 8-2\beta \ and \ 13+2\beta < x < 17-2\beta$

$For \ l_{W_s(\beta)}, x \to 4+2\beta \, and \ y \to 17-2\beta$

$$l_{W_s(\beta)} = \left\{\frac{34-4\beta}{24\beta^2 - 324\beta + 1020}\right\}$$

$For \ u_{W_s(\beta)}, x \to 8-2\beta \, and \ y \to 13+2\beta$

$$u_{W_s(\beta)} = \left\{\frac{26+4\beta}{24\beta^2 + 228\beta + 468}\right\}$$

The Non- membership function is

$$\psi_{W_s}(z) = \begin{cases} L(z), & \left[l_{W_s(\beta)}\right]_{\beta=0} \le z \le \left[l_{W_s(\beta)}\right]_{\beta=1} \\ R(z), & \left[u_{W_s(\beta)}\right]_{\beta=1} \le z \le \left[u_{W_s(\beta)}\right]_{\beta=0} \\ 0 & , \ otherwise \end{cases}$$

, Which is defined as

$$\psi_{W_s}(z) = \begin{cases} \dfrac{(-4+324z) \pm 4\sqrt{(441z^2+42z+1)}}{48z} & ,0..0333 \le z \le 0.0417 \\ \dfrac{(-228z+4) \pm 4\sqrt{(441z^2+42z+1)}}{48z} & , \ 0.0417 \le z \le 0.0556 \\ 0 & , \ otherwise \end{cases}$$

Table1. (α, β) -cuts of queue length, sojourn time

S. No.	α	$l_{N_{q\alpha}}$	$u_{N_{q\alpha}}$	$l_{W_{s\alpha}}$	$u_{W_{s\alpha}}$	β	$l_{N_{q\beta}}$	$u_{N_{q\beta}}$	$l_{W_{s\beta}}$	$u_{W_{s\beta}}$
1	0	0.0345	0.2941	0.0345	0.0588	1	0.2500	0.2500	0.0417	0.0417
2	0.1	0.0423	0.2727	0.0352	0.0568	0.9	0.2358	0.2650	0.0407	0.0427
3	0.2	0.0504	0.2527	0.0360	0.0549	0.8	0.2222	0.2807	0.0397	0.0439
4	0.3	0.0588	0.2340	0.0368	0.0532	0.7	0.2093	0.2973	0.0388	0.0450
5	0.4	0.0677	0.2165	0.0376	0.0515	0.6	0.1970	0.3148	0.0379	0.0463
6	0.5	0.0769	0.2000	0.0385	0.0500	0.5	0.1852	0.3333	0.0370	0.0476
7	0.6	0.0866	0.1845	0.0394	0.0485	0.4	0.1739	03529	0.0362	0.0490

8	0.7	0.0968	0.1698	0.0403	0.0472	0.3	0.1631	0.3737	0.0355	0.0505
9	0.8	0.1074	0.1560	0.0413	0.0459	0.2	0.1528	0.3958	0.0347	0.0521
10	0.9	0.1186	0.1429	0.0424	0.0446	0.1	0.1429	0.4194	0.0340	0.0538
11	1	0.1304	0.1304	0.0435	0.0435	0	0.1333	0.4444	0.0333	0.0556

2. Conclusion

This work investigates the qualitative behaviour of two server fuzzy queuing systems with flexible service policies using the intuitionistic pentagon fuzzy number. Using the (α, β)-cuts and Zadeh's extension principle, we determine the queue length and sojourn time in steady-state condition. The alpha and beta cut strategy breaks down the fuzzy queue into a family of crisp queues to get around the difficulties. Thus, the example has successfully demonstrated the effectiveness and precision of parametric programming technique.

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