Coregular Connected Domination in Line Graph

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Abstract :

A line graph L(G) is the graph whose vertices corresponding to the graph G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent.

A dominating set D$\subseteq V[L(G)]$ is known as coregularConnected dominating set , if the induced sub graph $\langle D\rangle$ is connected such that the induced subgraph < $V[L(G)] - D$> is regular. The minimum cardinality of vertices in such a set is called coregular Connected domination number in L(G) , and is denoted by $\gamma_{coc}[L(G)]$.

In this article, we examine the graph theoretic propertiesof $\gamma_{coc}[L(G)]$, and we find numerous limitations in terms of elements G and its connections to other dominating parameters. Our investigation on this work is to establish the application oriented standard results in the field of domination theory for several kinds of new concepts which are playing an important role of application.

Keywords: Line graph , Co- regular restrained dominating set , Co-regular restrained domination number.

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Introduction

In this paper the graphs considered here are finite and simple. In general we follow the notations of Harary [4].

We begin by recalling some standard definitions from domination theory.

A set S$\subseteq$ V(G) is said to be a dominating set of G , if every vertex in (V-S) is adjacent to some vertex in S. The domination number γ(G) is the least cardinality among all dominating sets in G. [6]

A set X$\subseteq$ E(G) is said to be an edge dominating set if every edge in E(G)$-$X is adjacent to some edge in X. The edge domination number of a graph G is the cardinality of smallest edge dominating set of G and is denoted by $\gamma^{|}$(G).

A dominating set S of a graph G is a total dominating set if the induced subgraph $\langle S\rangle$ has no isolated vertices. The total domination number γ_t(G) of G is the minimum cardinality among all the total dominating sets in G. [1]

A dominating set S⊆ V(G) is a split dominating set, if the induced subgraph < V$[(G)] - S$> is disconnected . The minimum cardinality of vertices in such a set is called a split domination number of a graph G, and is denoted by γ_s(G) . [6]

A dominating set S of a graph G is a connected dominating set if the induced subgraph $\langle S\rangle$ connected. The connected domination number γ_c(G) of G is the minimum cardinality among all the connected dominating sets in G. See [7]

A Roman dominating function on a graph G = (V,E) is a function f: V{0,1,2} satisfying the condition that every vertex v for which f(v) = 0 is adjacent to atleast one vertex of v for which f(v) = 2. The weight of a Roman dominating function is the value f(v) = $\sum_{v\in V} f(v)$. The Roman domination number of a graph G is denoted by γ_R(G) , equals the minimum weight of a Roman dominating function on G.

A dominating set S⊆ V(G) is a restrained dominating set of a graph G if every vertex in V-S is adjacent to a vertex in S and another vertex in V-S. The restrained domination number γ_r(G) of G is the minimum cardinality of a restrained dominating set of G . [3]

A dominating set S is called a perfect dominating set of a graph G if every vertex of V(G) - S is adjacent to exactly one vertex of S. The minimum cardinality of a perfect dominating set of G is a perfect domination number and is denoted by γ_p(G) .See [2]

A connected dominating set S of a graph G is a coregular connected dominating set if the induced subgraph $\langle V - S\rangle$ is regular. The coregular connected domination number γ_{coc}(G) of G is the minimum cardinality of a coregular connected dominating set.

Analogously, connected dominating set D of a linegraph L(G) is a coregular connected dominating set , if the induced subgraph $\langle V[L(G)] - D\rangle$ is regular.The coregular connected domination number $\gamma_{coc}L$(G) is the minimum cardinality of a coregular connected dominating set.

Results:

we list out coregular connected domination number oflinegraph L(G) for some standard graphs,which are straight forward in the following theorem.

Theorem 1:

1. For any PathP_p , with $p \geq 3$ vertices,

$\gamma_{coc}[L(P_p)]$ = p – 3

2. For any Cycle C_p , with $p \geq 3$ vertices,

$\gamma_{coc}[L(C_p)] =$ p – 2

3.For any Wheel W_p , with $p \geq 3$ vertices,

$\gamma_{coc}[L(W_p)] =$ p − 1 .

A dominating set $D_1 \subseteq$ V(G) is called a coregular restrained dominating set , if every vertex of $\mathrm{V} - \mathrm{D}_1$ is adjacent to a vertex in D_1 and another vertex in $\mathrm{V} - \mathrm{D}_1$ such that the induced subgraph $\langle V(G) - D_1 \rangle$ is regular . The minimum cardinality of vertices in such a set is called coregular restrained dominating set in G and is denoted by $\gamma_{coRe}(G)$. See[8].

In the following theorem we relate this definition to our concept.

Theorem 2 : For any connected (p, q) graph G with $p \geq 3$ vertices ,

$\gamma_{coc}L(G) + \gamma_{split}(G) \leq \gamma_{core}(G) + \gamma_R(G) - 1$ and G $\neq K_{1,n}$

Proof : By the definition , the coregular restrained dominating set does not exists for $K_{1,n}$

Suppose S= $\{v_1, v_2, \ldots\ldots, v_n\} \subseteq$ V(G) be the set of all nonend vertices in G. Then $\forall v_i$, $v_i \in$ S, $1 \leq i \leq n$ is adjacent to atleast one vertex of V(G) − S and the induced subgraph $\langle V(G) - S \rangle$ has more than one component with the property N[S] = V(G). Hence S is a γ_{split} set of G.

Suppose H = V(G) − S and $S_1 \subset S$, such that $\mathrm{N}[\mathrm{S}_1 \cup \mathrm{H}] = \mathrm{V(G)}$. Then $\{\mathrm{S}_1 \cup \mathrm{H}\}$ is a minimal dominating set of G. If $\forall v_i \in \mathrm{V(G)} - \{\mathrm{S}_1 \cup \mathrm{H}\}$ is adjacent to atleast one vertex of $\{\mathrm{S}_1 \cup \mathrm{H}\}$ and atleast one vertex of $\mathrm{V(G)} - \{\mathrm{S}_1 \cup \mathrm{H}\}$. Then $\{\mathrm{S}_1 \cup \mathrm{H}\}$ is a restrained dominating set G. Suppose the induced subgraph $\langle V(G) - \{S_1 \cup H\} \rangle$ is regular then $\{\mathrm{S}_1 \cup \mathrm{H}\}$ is a coregular restrained dominating set of G.

Further let the function f: V(G)→{0,1,2} and partition the vertex set V(G) in to (v_0, v_1, v_2) induced by f with $|v_i| = n_i$ for n = 0,1,2 . Suppose the set V_2 dominates V_0 .Then M = $V_1 \cup V_2$ forms a minimal Roman dominating set of G.

Now , since V[L(G)] = E(G) , let D = $\{u_1, u_2, \ldots\ldots, u_n\} \subseteq V_1[L(G)] = E_1$(G), where E_1(G) is the set of edges which are incident with the vertices of S , such that N[D] = V[L(G)]. Then D forms a minimal dominating set of L(G). Further if the induced subgraph $\langle D \rangle$ has exactly one component then D itself is a minimal connected dominating set of L(G). If the induced subgraph $\langle V[L(G)] - D \rangle$ is regular then D is a coregular connecting dominating set of L(G) . If not , then add the minimum set of vertices $\{u_k\} \in \{V[L(G)] - D\}$ to make $\langle V[L(G)] - D \cup \{u_k\} \rangle$ is a coregular connected dominating set of L(G). Hence

$|D \cup \{u_k\}| + |S| \leq |H \cup S| + |M| - 1$ gives

$\gamma_{coc}L(G) + \gamma_{split}(G) \leq \gamma_{core}(G) + \gamma_R(G) - 1$.

Lemma: For any cycle C_p, with $p \geq 3$ vertices , then $\gamma_{coc}L(C_p) = \gamma_c(C_p)$.

Proof: For any Cycle $C_p : v_1, v_2, \ldots\ldots, v_n, v_1$ = V(C_p) and $e_1, e_2, \ldots\ldots, e_n$ = E(C_p). Since $|V(C_p)| = |E(C_p)|$ and V(C_p) = E(C_p), then $v_2, v_{3,} v_{4,}, \ldots\ldots, v_{n-1}$ is a minimal connected dominating set of C_p .Similarly $\{e_1, e_2, \ldots.., e_n\} = V[L(G)] = V[L(C_p)$]and $e_2, e_3, \ldots\ldots, e_{n-1}$ is a minimal connected dominating set of $L(C_p)$. Further $e_1, e_n \in V[L(C_p)] - \{e_{1,} e_n\}$ and $\langle e_{1,} e_n \rangle$ is regular . Since $|\{e_2, e_3, \ldots\ldots, e_{n-1}\}| = |\{v_2, v_{3,} v_{4,}, \ldots\ldots, v_{n-1}\}|$, then $\gamma_{coc} L(C_p) = \gamma_c(C_p)$.

A subset S⊆ V(G) is double dominating set of G if every vertex v ∈ V(G) , |N[v]⋂S| ≥ 2 that is v is in S and has at least one neighbour in S or v is in V(G)-S has atleast two neighbours in S and is denoted by γ_{dd}(G). The double domination number is the smallest cardinality of a double dominating set of G.See [5]

Theorem 3: For any nontrivial (p, q) graph G, with$p \geq 3$ vertices

$\gamma_{coc} L(G) + \gamma_{cot}(G) + 1 \leq 2\gamma_{dd}$(G).

Proof: Suppose $F = \{v_1, v_2, \ldots.., v_p\}$ ⊆ V(G) such that N[F] = V(G). Then F is a dominating set of G . If the induced subgraph $\langle V - F \rangle$ has no isolates , then F is a γ_{cot} set of G. Now consider $V_1 = V(G) - F$and $V_2 = \{v_1, v_2, \ldots.., v_i\} \subseteq V_1$ then D^d = F ∪ V_2 forms a double dominating set of G.

Now in L(G) , let D = $\{u_1, u_2, \ldots.., u_i\}$ ⊆ V[L(G)] be the set $\{u_j\} = \{e_j\}$ ∈E(G), 1≤ j ≤ n where $\{e_j\}$ are incident with the vertices of $\{F \cup V_2\}$. Suppose D be the minimal set of vertices with N[D] = V[L(G)]. Then D is a γ-set of L(G) . Suppose the induced subgraph $\langle D \rangle$ has only one component. Then D itself is a connected dominating set of L(G). If the induced subgraph $\langle D \rangle$has more than one component , then attach the minimum number of vertices $\{w_i\}$ ∈V[L(G)] - D where deg (w_i)≥ 2.So that D_1= D ∪ $\{w_i\}$ forms exactly one component in the induced subgraph $\langle D_1 \rangle$. Clearly D_1forms a minimal γ_c- set of L(G). Suppose $\forall u_i \in \{V[L(G)] - D_1\}$has same degree. Then D_1 is a γ_{coc} set of L(G). Hence $|D_1|$+ $|F|$+ 1≤ 2 $|V_1 \cup V_2|$ gives

$\gamma_{coc} L(G) + \gamma_{cot}(G) + 1 \leq 2\gamma_{dd}$(G).

The following theorem gives a relation of$\gamma_{coc} L(G)$ with total domination and connected domination number of G.

Theorem 4: If graph G is a nontrivial connected (p, q) graph, with$p \geq 3$ vertices

$\gamma_{coc} L(G) + \gamma_c(G) \geq \gamma_t$(G) + diam(G) -1 and G ≠ P_p with p< 8, G≠ $K_{1,n}$.

Proof: SupposeG = P_p with p< 8 and G≠ $K_{1,n}$.

Then $\gamma_{coc}L(G) + \gamma_c(G) \ngeq$ diam(G) + γ_t(G) -1

Hence G $\neq P_p$ with p< 8 and G$\neq K_{1,n}$.

Let F =$\{e_1, e_2, \ldots\ldots, e_k\} \dot{\subseteq} E(G)$ be the minimal set of edges in G, which constitute the diametral path in G . Clearly $|F|$ = diam(G) .

Let $S_1 = \{v_1, v_2, \ldots\ldots, v_n\} \subseteq$ V(G) be the set of all nonend vertices in G.Suppose $S_2 \subseteq S_1$ be the minimum set of vertices , which coveres all vertices in G. If deg(v_i) $\geq 1 \forall v_i \in S_2$, 1≤ i ≤ n, in the induced subgraph $\langle S_2 \rangle$ then S_2 forms a total dominating set of G. Otherwise , if deg(v_i)< 1 , then attach the vertices $w_i \in N(v_i)$ to make deg(v_i)≥ 1,such that the induced subgraph $\langle S_2 \cup \{w_i\} \rangle$ does not contain any isolated vertex. Clearly $S_2 \cup \{w_i\}$ forms a minimal total dominating set of G.

Let $D_1 = \{v_1, v_2, \ldots\ldots, v_p\}$ be the set of all endvertices in G .Suppose $D_2 = \{V(G) - D_1\}$ then there exists a minimal set of vertices such that N[v_i] = V(G) $\forall\, v_i \in D_2$, then D_2 forms a minimal dominating set of G. Further if D_2 has exactly one component then D_2 itself is a connected dominating set of G.

Let $E_1 = \{e_1, e_2, \ldots\ldots, e_k\} \subseteq E(G)$, $E_2 = \{e_1, e_2, \ldots\ldots, e_l\} \subseteq E(G)$ .Then $\forall e_i \in E_1$ are incident with $\forall\, v_i \in \gamma_c$ set of G and $\forall e_j \in E_2$ are incident with $\forall\, v_i \in \gamma_t$ set of. Let E_3 =$\{e_1, e_2, \ldots\ldots, e_l\} = E(G)$. Then $\{u_1, u_2, \ldots\ldots, u_n\} = V[L((G)]$ corresponding to the elements of E_3 . Also $H_1 = \{u_1, u_2, \ldots\ldots, u_k\} \subset V[L((G)]$ corresponding to the elements of E_1 and H_2 = $\{u_1, u_2, \ldots\ldots, u_l\} \subset V[L((G)]$ corresponding to the elements of E_2. Suppose K⊂ V[L(G)] be the set of vertices with deg(u_j) ≥ 1 such that N[K] = V[L(G)] . Clearly K forms a dominating set of L(G). Suppose the induced subgraph$\langle K \rangle$has exactly one component,then K forms a connected dominating set of L(G). But it is easily verify that $|H_1| > |H_2|$ then $|K| + |H_1| \geq |H_2|$ + diam(G) −1 and the induced subgraph $\langle V[L(G)] - K \rangle$ is regular , then K is a γ_{coc} set of L(G). Which gives

$\gamma_{coc}L(G) + \gamma_c(G) \geq \gamma_t$(G) + diam(G) −1 .

Next theorem relates $\gamma_{coc}L(G)$ in terms of dominating set of L(G) and edges of G.

Theorem 5: For any connected (p, q)graph G, with$p \geq 3$ vertices

$$\gamma_{coc}L(G) + \gamma[L(G)] \leq q.$$

Proof: Let E =$\{e_1, e_2, \ldots\ldots, e_n\}$be the edge set of G with $|E|$ = q.

Let J be the set of vertices with deg(u_i) ≥2$\forall u_i \in$ J, 1≤ i ≤ n in L(G). Further let $J_1 = \{u_1, u_2, \ldots\ldots, u_k\} \subseteq J$ such that dist(u, v) ≥ 2. Then there exists a minimal set of vertices J_2 in L(G) such that $\forall u_i \in$ V[L(G)]−J_2 is adjacent to atleast one vertex of J_2. Hence J_2 is minimal γ-set of L(G).Suppose the induced subgraph $\langle J_2 \rangle$ has exactly one component.Then J_2 itself is a minimal connected dominating set of L(G). If in the induced subgraph $\langle V[L(G)] - J_2 \rangle$ every vertex has same degree , then J_2 is a γ_{coc}- set of L(G). If not add the set of vertices $\{w_i\} \in$ V[L(G)] $-$ J_2 such that the induced subgraph $\langle$V[L(G)] $-$ $J_2 \cup \{w_i\}\rangle$is regular. Hence $|J_2 \cup \{w_i\}|$+ $|J_2| \leq |E|$ which gives $\gamma_{coc}L(G)$+ γ[L(G)] $\leq$ q .

Now connecting relation of coregular connected domination number of linegraph with independent domination number and maximal independent vertices of G.

Theorem 6: If graph G is a nontrivial connected (p, q) graph, with$p \geq 3$ vertices

$\gamma_{coc}L(G) - i(G) \leq \beta_0$(G).

Proof: SupposeD = $\{v_1, v_2, \ldots\ldots, v_k\} \subseteq$ V(G) be a minimal dominating set of G. If $\forall v_i \in$ D, deg(v_i) = 0, then D is a minimal independent dominating set of G.

Let A = $\{v_1, v_2, \ldots\ldots, v_m\} \subseteq$ V(G) be a maximal independent set of vertices such that N(u)∩ N(v) ≠x , ∀ u,v ∈ A ,and x ∈ V(G) $-$ A . So that $|A|$ = β_0(G).

Now in L(G) , let B_1= $\{u_1, u_2, \ldots\ldots, u_n\} \subset$ V[L(G)] be the set of vertices corresponding to the edges which are incident to the vertices of A in G. Also the set B_2= $\{u_1, u_2, \ldots\ldots, u_m\} \subset$ V[L(G)] be the set of vertices corresponding to the edges which are incident to the vertices of D in G.

Suppose K⊂ B_1 such that $\forall u_j \in$ K is adjacent to a vertex set of B_1 such that N[K] = V[L(G)] and u,v ∈ K there exist a path between u,v in L(G) .Clearly K forms a connecting dominating set of L(G). If for every vertex of the induced subgraph $\langle$V[L(G)] $-$ K$\rangle$ has same degree , then K is a γ_{coc} set of L(G) . It follows that $|K|$- |D| $\leq |A|$ Hence

$\gamma_{coc}L(G) - i(G) \leq \beta_0$(G).

The following theorem relates $\gamma_{coc}L(G)$ with edge domination and connected edge domination number of G.

Theorem 7: For any (p, q) connected graph G, with$p \geq 3$ vertices

$\gamma_{coc}L(G) \leq \gamma^{|}$(G) + $\gamma_c^{|}$(G).

Proof: Let E_1 =$\{e_1, e_2, \ldots\ldots, e_k\} \subseteq E(G)$ which covers all the edges of G such that N[E_1] = E(G) . Then E_1 is a minimal edge dominating set of G. Let K = $\{v_1, v_2, \ldots\ldots, v_n\}$ be the set of all endvertices of G. Suppose E_2 =$\{e_1, e_2, \ldots\ldots, e_m\}$ be the set of edges which are not incident to the vertices of K and $\forall e_i \in E_2$ is adjacent to atleast one edge of E(G)$-E_2$. If an edge induced subgraph E_2 has exactly one component , then E_2 forms a connected edge dominating set of G. Now suppose there exists a set $E_1^{|} \subseteq E_1$ and $E_2^{|} \subseteq E_2$ such that the set A= $\{u_1, u_2, \ldots\ldots, u_n\}$ = $\{E_1^{|} \cup E_2^{|}\}$ where $\forall u_i \in$ A , 1≤ i ≤ n corresponds to the elements of $\{E_1^{|} \cup E_2^{|}\}$. Further if every vertex of A is adjacent to atleast one vertex of V[L(G)]−A and every vertex of the induced subgraph ⟨V[L(G)] − A ⟩ has same degree . Then A forms a γ_{coc} set of L(G) .

It gives $|A| \leq |E_1| + |E_2|$. Hence $\gamma_{coc}L(G) \leq \gamma^{|}$(G) + $\gamma_c^{|}$(G).

An edge dominating set S ⊆ E(G) of a graph G is an end edge dominating set if S contains all end edges of E(G) . The end edge domination number $\gamma_e^{|}$(G) of G is the minimum cardinality of an end edge dominating set of G.See [9].

In the below theorem , we relates $\gamma_e^{|}$(G) to our concept.

Theorem 8: If graph G is a nontrivial connected (p, q) graph, with$p \geq 3$ vertices

$\gamma_{coc}L(G) \geq \gamma_e^{|}$(G) + δ(G) , G≠ $K_{1,n}$

Proof: Let E_1={$e_1, e_2, \ldots\ldots, e_m$}be the set of all endedges in Gand E_2 = E(G)−E_1. Suppose $E_2^{|} \subseteq E_2$ such that $\forall e_i \in \{E_1 \cup E_2^{|}\}$ is adjacent to atleast one edge of E(G)−$\{E_1 \cup E_2^{|}\}$. Then $\{E_1 \cup E_2^{|}\}$ is a $\gamma_e^{|}$-set of G.

Suppose M ⊂ $\{E_1 \cup E_2^{|}\}$ and H ⊂ E(G) − $\{E_1 \cup E_2^{|}\}$. Then in L(G) , {M}⋃{H}⊂ V[L(G)] be the minimal set of vertices such that N[u_i] = V[L(G)] $\forall u_i$ ∈{M}⋃{H}. Then{M}⋃{H} forms a minimal dominating set of L(G). Further if {M}⋃{H} has exactly one component , then {M}⋃{H} itself is a connecting dominating set of L(G) . If not attach the minimum number of vertices { u_k } which are in every u-v path ∀ u,v ∈ [V[L(G)] − {M} ⋃ {H}]. Hence J = {M}⋃{H}⋃{ u_k } is a minimal connecting dominating set of L(G). If the induced subgraph ⟨ $V[L(G)] - J$ ⟩ is regular, then J itself is a γ_{coc}- set of L(G) . Hence $|\mathbf{J}| \geq |\mathbf{E_1} \cup \mathbf{E_2^{|}}|$ + δ(G) Which gives

$\gamma_{coc}L(G) \geq \gamma_e^{|}$(G) + δ(G).

Theorem 9: For any connected (p, q)graph G, with$p \geq 3$ vertices

$\gamma_{coc}L(G) + \left\lceil \frac{\text{diam(G)}}{2} \right\rceil + 1 \geq p - \gamma_p(G)$, G≠ $K_{1,n}$ when n ≥ 4.

Proof : Suppose G= $K_{1,n}$. Then $\gamma_{coc}L(G) + \left\lceil \frac{\text{diam(G)}}{2} \right\rceil + 1 < p - \gamma_p(G)$.

Hence G≠ $K_{1,n}$ Hence n ≥ 4.

Let V = $\{v_1, v_2, \ldots\ldots, v_n\}$ = V(G) with $|V|$ = p .

Let E_1=$\{e_1, e_2, \ldots\ldots, e_n\}$ ⊆ E(G) be the minimal set of edges which constitute the largest path between any two distinct vertices u,v ∈ V(G) such that dist(u,v) = diam(G).

Now let D = $\{v_1, v_2, \ldots\ldots, v_m\}$ ⊆ V(G) such that $\forall v_i$ ∈ V−D is adjacent to exactly one vertex of D and N[D] = V(G). Then D itself is a γ_p set of G .

Now in L(G) , let B ={$v_1^{|}$,$v_2^{|}$,$v_3^{|}$,..........,$v_n^{|}$}⊂ V[L(G)] be the set of vertices corresponding to the edges which are incident to the vertices of D in G. Suppose K ⊂ B such that $\forall v_j^{|}$ ∈ K is adjacent to a vertex set of K such that N[K] = V[L(G)] and ∀ u,v ∈ K there exist a path between u,v in L(G) . Clearly K forms a connecting dominating set of L(G). If every vertex in the induced subgraph $\langle V[L(G)] - K \rangle$ has same degree, then K is a γ_{coc} set of L(G). It follows that

$|K| + \left\lceil \frac{|E_1|}{2} \right\rceil + 1 \geq |V| - |D|$ Hence the result

$\gamma_{coc}L(G) + \left\lceil \frac{\text{diam(G)}}{2} \right\rceil + 1 < p - \gamma_p(G)$.

Theorem 10: For any connected (p, q)graph G, with $p \geq 3$ vertices

$$\gamma_{coc}L(G) \leq q - \Delta(G), \; G \neq K_{1,n}.$$

Proof: Let E = $\{e_1, e_2, \ldots\ldots, e_n\}$be the edge set of G with $|E|$ = q .

Suppose $V_1 = \{v_1, v_2, \ldots\ldots, v_m\} \subset$ V(G) be the set of all nonend vertices in G, then there exists atleast one vertex of maximum degree $\Delta(G)$ in V(G).

Further let $H = \{u_1, u_2, \ldots\ldots, u_n\}$ be the vertices of L(G) corresponding to the elements of E of G. Let $H_1 = \{u_1, u_2, \ldots\ldots, u_m\} \subseteq$H be the set $\{u_i\} = \{e_i\} \in$ E(G), 1≤ i ≤ n . Suppose H_1 be the minimal set of vertices with N[H_1] = V[L(G)]. Then H_1 is a γ-set of L(G). If the induced subgraph $\langle H_1 \rangle$ has only one component . Then H_1 it self is a connected dominating set of L(G).

If the induced subgraph $\langle H_1 \rangle$ has more than one component , then add the minimum number of vertices $\{k_i\} \in$ H−H_1 where deg(k_i) ≥ 2 such that $H_2 = H_1 \cup \{k_i\}$ forms exactly one component in the induced subgraph $\langle H_2 \rangle$.Clearly H_2forms a minimal $\gamma_c$set of L(G). Suppose $\forall u_i \in \{V[L(G)] - H_2\}$, have the same degree. Then H_2 is a γ_{coc} set of L(G). Hence $|H_2| \leq |E| - |V_1|$ which gives $\gamma_{coc}L(G) \leq q - \Delta(G)$.

Theorem 11: For any connected (p, q) graph G with $p \geq 3$ vertices , then $\gamma_{coc}L(G) \geq \beta_1(G)$.

Proof: Let $E_1 = \{e_1, e_2, \ldots\ldots, e_k\} \subseteq$ E(G) be the maximal set of edges such that for any $e_i, e_j \in E_1$, N(e_i)∩ N(e_j) = e and e∈ E(G) − E_1 . Clearly E_1 forms maximal edge independent set of G with $|E_1| = \beta_1(G)$.

Since in L(G), E(G) = V[L(G)].Suppose M ⊂ E_1 and H ⊂ E(G) − E_1. Then in L(G) , $\{M\} \cup \{E_1\} \subset$ V[L(G)].

Now assume $\forall v_i \in V[L(G)] - \{M \cup E_1\}$, is adjacent to atleast one vertex of M∪ E_1 and N[M∪ E_1] = V[L(G)] . Then {M∪ E_1} is a γ- set of L(G) .

Suppose the induced subgraph $\langle M \cup E_1 \rangle$ has exactly one component . Then{M∪ E_1} be the connected dominating set of L(G). Further the induced subgraph $\langle V[L(G)] - \{M \cup E_1\} \rangle$ is regular. Then {M ∪ E_1} is a γ_{coc} set of L(G) . If not select a set K = $\{u_1, u_2, \ldots\ldots, u_m\}$ in V[L(G)] − {M ∪ E_1} such that the induced subgraph $\langle V[L(G)] - \{M \cup E_1\} \cup K \rangle$ is regular . Hence

$|\{M \cup E_1\} \cup K| \geq |E_1|$ which gives

$\gamma_{coc}L(G) \geq \beta_1(G)$.

Conclusion:

In this work, we looked line graphs with Co regular Connected Domination Number.

These results show a significant correlation between the Co regular Connected domination number in a line graph and many factors, such as the split domination number, the edge domination number , and entire domination number ,of a straightforward, undirected graphs .

The idea behind Coregular ,the regularity of the vertices of V[L(G)] − D. D is the connected domination number of line graph L(G). Here, we have some broad conclusions on the idea of Coregular connected Domination number. Additionally, its relation with additional dominating parameters were discovered.

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