Randić Energy of Face Magic Labeled Graphs

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Abstract

Graph Theory plays a vital role in many areas of science and technology. Let $G = (V, E)$ be a simple graph with vertex set $V = \{v_1, v_2, \dots, v_r\}$ and edge set $E = \{e_1, e_2, \dots, e_m\}$. One of the fast-developing area in graph theory is labeling. A labeling is said to be magic labeling, if for every positive integer s, all s-sided faces have the same weight. In this paper we proposed Randić energy $RE(G)$ for the face magic labelled graphs.

Keywords: Randić energy; labeling; face magic; ladder graph; friendship graph; prism graph

1. Introduction

Graph Theory was introduced by Leonhard Euler in 1736. One of the important sections in graph theory is labeling. The vertices of a graph can be labeled in many different ways. Graph labeling is where the vertices are assigned some values subject to certain conditions [1]. Labeling of vertices and edges play a vital role in graph theory [2]. One of the interesting sections is face magic labeling. Recently the concept of face magic labeling was introduced and many research articles are being published in this topic [3-5]. The face of the graph of order n are labeled by non-negative integers 1,2,3 $\dots$, n.

Let X be the adjacency matrix for the simple graph G, which has n vertices. We'll call the eigenvalues of X as $\rho_1, \rho_2, \dots, \rho_n$. These are referred to as the graph G's eigenvalues and make up its spectrum. In keeping with this line of thinking, we may consider the randić energy to be the total of the eigenvalues of the randić matrix, expressed in absolute terms. Formally: Assign the eigenvalues of the randić matrix R(G) to $\rho_1, \rho_2, \dots, \rho_n$. The Randić energy [6-8] may be described as: Given that these eigenvalues are inescapably real numbers and that their total is zero,

$$RE = RE(G) = \sum_{i=1}^{n} |\rho_r|$$

Let $G = (V, E)$ be a simple binary labeled graph with n vertices $v_1, v_2, \dots, v_n$ [9]. We define,

$$l_{ij} = \begin{cases} a, & \text{if } v_i v_j \in E(G) \text{ and } l(v_i) = l(v_j) = 0 \\ b, & \text{if } v_i v_j \in E(G) \text{ and } l(v_i) = l(v_j) = 1 \\ c, & \text{if } v_i v_j \in E(G) \text{ and } l(v_i) = 0, l(v_j) = 1 \text{ or vice-versa} \\ 0, & \text{otherwise.} \end{cases}$$

where a, b and c are distinct non-zero real numbers.

The adjacency matrix of the labelled graph G, or simply the label matrix of G, is the $n \times n$ matrix $A_l(G) = [l_{ij}]$. The adjacency matrix describes a graph's spectral characteristics. Similar to this, the label matrix $A_l(G)$ describes the characteristics of a binary labeled graph and directs research into its spectral characteristics. The following is the definition of the label matrix $A_l(G)$'s characteristic polynomial:

$$\begin{aligned} \phi(A_l(G), \eta) &= \det(\eta I - A_l(G)) \\ &= c_0\eta^n + c_1\eta^{n-1} + c_2\eta^{n-2} + \cdots + c_n \end{aligned}$$

where I is the order n unit matrix. Label eigenvalues of binary labelled graph G are the roots $\eta_1, \eta_2, \dots, \eta_n$ considered as non-increasing order of $\phi(A_l(G), \eta) = 0$. The formula for a graph's label energy is $E_l(G) = \sum_{i=1}^{n} |\eta_i|$. Since $A_l(G)$ is a real symmetric matrix, the binary labeled graph's eigenvalues are real and have sums of zero. Therefore, $\eta_1 \geq \eta_2 \geq \cdots \geq \eta_n$ and $\sum_{i=1}^{n} \eta_i = 0$. For certain graphs, Alikhani and Ghanbari [10] defined the randić energy and the randi characteristic polynomial.

In this research, we create Randić energy $RE(G)$ for Ladder graph, Friendship graph, Para chain Hexagon, and Prism graph face magic labeling.

2. Preliminaries

Definition 2.1[2]

A labeling or valuation of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$.

Definition 2.2 [2]

A labeling is said to be a face magic labeling if the weights of all face values are equal.

Lemma 2.3 [8]

Let P_n be the path on n vertices. Then the Randić energy of path P_n is represented as

$$RE(P_n) = 2 + \frac{1}{2}E(P_{n-2})$$

Lemma 2.4 [11]

Let C_{2n} be the cycle on $2n$ vertices for $n \geq 2$. Then the Randić energy of even cycles is defined as

$$RE(C_{2n}) = \frac{2\sin\left(\left(\left\lfloor\frac{n}{2}\right\rfloor + \frac{1}{2}\right)\frac{\pi}{n}\right)}{\sin\frac{\pi}{2\pi}}$$

Definition 2.5 [12]

The ladder graph $\mathrm{L_n}$ is a planar undirected graph with 2n vertices and $3n - 2$ edges.

Definition:2.6 [13]

The Friendship graph (or Dutch windmill graph or n-fan) F_n is a planar undirected graph with $2n + 1$ vertices and $3n$ edges. The friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Definition 2.7 [13]

A prism is also called by the name of a circular ladder graph. That is a graph corresponding to the skeleton of an prism. Therefore, Prism graphs are planar and polyhedral.

3. Randić Energy of Face Magic Labeled Graphs

Definition 3.1. Let $G\ be\ a$ simple graph on n vertices and X be its adjacency matrix. Let $\rho_1, \rho_2, \dots, \rho_n (n \in \{1,2, \dots, r\})$ be the eigenvalues of randić matrix of $RE(G)$, which are real numbers and their sum is zero. If the weights W_i of all face values are equal, then the randić energy for the face magic labelled can be defined as Randić energy for face magic labeled graph.

$$RE(G) = \sum_{i=1}^{n} |\rho_r| + W_i$$

where $W_i = L(v_{1,j}) + L(v_{1,j+1}) + L(v_{2,j}) + L(v_{2,j+1}) + \cdots + L(v_{\mathrm{n},j}) + L(v_{\mathrm{n},j+1})$

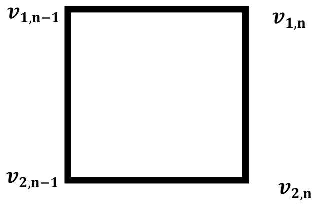

Figure 1. Simple graph

Example 3.2. Let G *be a* simple graph on n vertices and X be its adjacency matrix. When all faces have same weights then it is a face magic labeled graph.

We find eigen values for figure 1.

$V = \{v_{1,n-1}, v_{1,n}, v_{2,n-1}, v_{2,n}\}$

$$X = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

∴ The eigen value of adjacency matrix

$$X \quad = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ is } -2,2,0,0$$

$$\begin{aligned} \mathrm{RE}(G) = & \sum_{i=1}^{r} \mid \rho_r + W_i \\ & = -2 + 2 + 0 + 0 + 10 \\ & = 10 \end{aligned}$$

Theorem 3.3. Let GL_r be a face magic labeled ladder graph, for $r \geq 3$. If the randić energy $RE(GL_r)$ is rational, then $RE(GL_r) \equiv \mathrm{trace}(\mathrm{X}(GL_r))(\mathrm{mod}2)$.

Proof :

Let GL_r be a ladder graph. Let $\rho_1, \rho_2, \dots, \rho_n (n \in \{1,2,\dots,r\})$ be positive, and the remaining eigenvalues be non-positive.

The vertex set of GL_r is represented by

$$V(GL_r) = \{v_{ij} / 1 \leq \mathrm{i} \leq \mathrm{r}, 1 \leq \mathrm{j} \leq 2$$

Then,

$$\begin{aligned}\mathrm{RE}(GL_r) &= \sum_{i=1}^{r} |\rho_i| \\ &= (\rho_1 + \rho_2 + \cdots + \rho_n) - (\rho_{n+1} + \rho_{n+2} + \cdots + \rho_r) \\ &= 2(\rho_1 + \rho_2 + \cdots + \rho_n) - \mathrm{trace}(\mathrm{X}(GL_r))\end{aligned}$$

The labeling of GL_r can be represented as

$$L(\mathrm{v_{ij}}) = \begin{cases} 2r + i - 6, & 1 \leq \mathrm{i} \leq \mathrm{r}, \quad j = 1,2,1,2,\ldots \\ 2r - i + 1, & 1 \leq \mathrm{i} \leq \mathrm{r}, \quad \mathrm{j} = 2,1,2,1 \ldots \end{cases}$$

Let $\mathrm{W_i}$ be the weight of the $\mathrm{j^{th}}$ face of GL_r contains the vertices

$$\mathrm{v}_1, \mathrm{v}_{1,j+1}, v_{2,j}, v_{2,j=1}$$

We have to prove the weight of sum of the labels assigned to the vertices.

Let in each face of GL_r are equal.

It is enough to prove $\mathrm{W_i}$ is true for any three continuous values of $i-1, i, i+1$.

$\mathrm{W_i}$ is the sum of the labels formed by 4 vertices.

$$W_i = L(v_{1,j}) + L(v_{1,j+1}) + L(v_{2,j}) + L(v_{2,j+1})$$

where

$$L(v_{i,j}) = \{2r + i - 6,\ 1 \leq \mathrm{i} \leq \mathrm{r}, \mathrm{j} = 1,2,1,2 \ldots$$
$$\mathrm{L}(\mathrm{v_{i,j+1}}) = \{2r - i = 1,\ 1 \leq \mathrm{i} \leq \mathrm{r}, \mathrm{j} = 2,1,2,1 \ldots$$

Let W_i be the weight of i^{th} face value of GL_r

$$\begin{aligned}\mathrm{W_i} &= 2\mathrm{r} + \mathrm{i} - 6 + 2\mathrm{r} - \mathrm{i} + 1 + 2\mathrm{r} + \mathrm{i} - 6 + 2\mathrm{r} - \mathrm{i} + 1 \\ &= 8\mathrm{r} - 12 + 2 \\ &= 8\mathrm{r} - 10 \qquad (1)\end{aligned}$$

Let $\mathrm{W_{i-1}}$ be the weight of $(\mathrm{i}-1)^{\mathrm{th}}$ face value

$$\begin{aligned}\mathrm{W_{i-1}} &= 2r + (i-1) - 6 + 2r - (i-1) + 1 + 2r + (i-1) - 6 + 2r - (i-1) + 1 \\ &= 8\mathrm{r} - 12 + 2 \\ &= 8\mathrm{r} - 10 \qquad (2)\end{aligned}$$

Let W_{i+1} be the weight of $(i+1)^{\text{th}}$ value of GL_r

$$\begin{aligned} W_{i+1} &= 2r + (i+1) - 6 + 2r - (i+1) + 1 + 2r + (i+1) - 6 + 2r - (i+1) + 1 \\ &= 8r - 12 + 2 \\ &= 8r - 10 \qquad (3) \end{aligned}$$

From (1), (2) and (3) $W_{i-1} = W_i = k$ for all values of $1 \leq i \leq r$.

since $\rho_1, \rho_2, \ldots, \rho_r$ are algebraic integers, so is their sum.

Hence $(\rho_1 + \rho_2 + \cdots + \rho_n)$ must be an integer if $RE(GL_r)$ is rational. Therefore, $RE(GL_r) \equiv \text{trace}(X(GL_r))(\bmod 2)$.

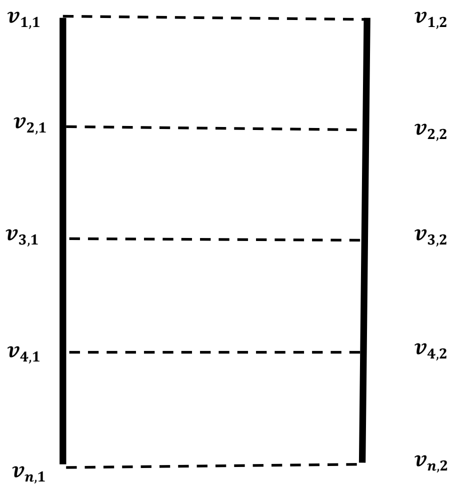

Figure 2. Ladder graph GL_r

Theorem 3.4

Let GH_r be a face magic para chain Hexagon graph for $n \geq 4$. If the randić energy $RE(GH_r)$ is rational, then $RE(GH_r) \equiv \text{trace}(X(GH_r))(\text{mod}2)$.

Proof:

Let GH_r be a para chain Hexagon,

Let $\rho_1, \rho_2, \ldots, \rho_n (n \in \{1,2, \ldots, r\})$ be positive, and the remaining eigenvalues be non-positive.

The vertex set of GH_r is represented as

$$V(GH_r) = \{V_{ij} / \ 1 \leq i \leq 3, \ 1 \leq j \leq r\}$$

Then,

$$\begin{aligned} \mathrm{RE}(GH_r) &= \textstyle\sum_{i=1}^{r} |\rho_i| \\ &= (\rho_1+\rho_2+\cdots+\rho_n)-(\rho_{n+1}+\rho_{n+2}+\cdots+\rho_r) \\ &= 2(\rho_1+\rho_2+\cdots+\rho_n)-\mathrm{trace}(\mathrm{X}(GH_r)) \end{aligned}$$

The labeling of GH_r can be labeled as

$$L(v_{ij}) = \begin{cases} j & & i=1 \quad j \text{ is odd} \\ 2r-1-j & 1\le j\le r & i=1 \quad j \text{ is even} \\ 3r-1-j & 1\le j\le r & i=2 \\ 3r-2+j & 1\le j\le r & i=3 \quad j \text{ is even} \end{cases}$$

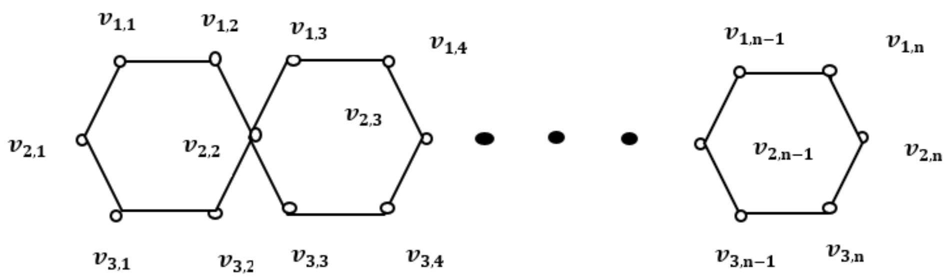

Figure 3. Face magic para chain Hexagon graph

Let W_j be the weight of the j^{th} face of GH_r containing the vertices $v_{1,j}, v_{1,j+1,}, v_{2,j}, v_{3,j} v_{3,j+1}$. we have to prove the weight of the labels assigned to the vertices in each face of GH_r are equal. It is enough to prove W_j is true for any 3 continuous values of $j-1, j, j+1$.

Let W_j be some of the labels formed by thesix vertices.

$$W_j = L(v_{1,j}) + L(v_{1,j+1}) + L(v_{2,j}) + L(v_{2,j+1}) + L(v_{3,j}) + L(v_{3,j+1})$$
$$L(v_{1,j}) = \{j /\ i = 1 \ \text{ and j is odd}$$
$$L(v_{1,j+1}) = \{2r-1-j /\ \mathrm{i} = 1 \ \text{ and j is even}$$
$$\mathrm{L}(\mathrm{v}_{2,\mathrm{j}}) = \{3\mathrm{r}-1-\mathrm{j} /\ \mathrm{i} = 2$$
$$\mathrm{L}(\mathrm{v}_{3,\mathrm{j}}) = \{3r-2+j /\ \mathrm{i} = 3 \ \text{ and j is odd}$$
$$\mathrm{L}(\mathrm{v}_{3,j+1}) = \{3r+1+j /\ \mathrm{i} = 3 \ \text{ and j is even}$$

Let W_j be the weight of $(j-1)^{\text{th}}$ face value of G

$$\mathrm{W_{j-1}} = L(v_{1,j-1}) + \mathrm{L}(\mathrm{v}_{1,j-1}) + \mathrm{L}(\mathrm{v}_{2,j-1}) + \mathrm{L}(\mathrm{v}_{2,j-1}) + \mathrm{L}(\mathrm{v_{3,j-1}}) + L(v_{3,j-1})$$

$$= j + 2r - 1 - j + 3r - 1 - j + 3r - 1 - j + 3r - 2 + j + 3r = 1 + j$$

Replacing j by $(\mathrm{j}-1)$

$$= (j-1) + 2r - 1 - (j-1) + 3r - 1 - (j-1) + 3r - 1 - (j-1) + 3r - 2 + (j-1) + 3r + 1 + (j-1)$$

$$= 2\mathrm{r} - 1 + 3\mathrm{r} - 1 + 3\mathrm{r} - 1 + 3\mathrm{r} - 2 + 3\mathrm{r} + 1$$

$$= 14\mathrm{r} - 4 \quad (4)$$

Let W_j be the weight of j^{th} face value of GH_r

$$\begin{aligned} &= j + 2r - 1 - j + 3r - 1 - j + 3r - 1 - j + 3r - 2 + j + 3r + 1 + j \\ &= 2r - 1 + 3r - 1 + 3r - 1 + 3r - 2 + 3r + 1 \\ &= 14r - 4 \end{aligned} \quad (5)$$

Let W_{j+1} be the weight of $(j+1)^{\text{th}}$ face value of GH_r

$$= (j+1) + 2r - 1 - (j+1) + 3r - 1 - (j+1) + 3r - 1 - (j+1) + 3r - 2 + (j+1) + 3r + 1 + (j+1)$$

$$= 2r - 1 + 3r - 1 + 3r - 1 + 3r - 2 + 3r + 1$$

$$= 14r - 4 \quad (6)$$

From above equations (4), (5) & (6),
$\mathrm{W_{j-1}} = W_j = W_{j+1}$ for all j
That is $W_{\mathrm{j}} = \mathrm{k}$ for all values of $1 \leq \mathrm{j} \leq \mathrm{r}$
since $\rho_1, \rho_2, \ldots, \rho_r$ are algebraic integers, so is their sum.

Hence $(\rho_1 + \rho_2 + \cdots + \rho_n)$ must be an integer if $RE(GH_r)$ is rational. Therefore,
$\mathrm{RE}(GH_r) \equiv \mathrm{trace}(\mathrm{X}(GH_r))(\bmod 2)$.

Theorem 3.5

Let GF_r be a face magic labeled friendship graph, for $r \geq 2$. If the randić energy $R\mathrm{E}(GF_r)$ is rational, then $RE(GF_r) \equiv \mathrm{trace}(\mathrm{X}(GF_r))(\mathrm{mod}2)$.

Proof:
Let GF_r be a friendship graph with $2r + 1$ vertices and $3n$ edges.

Let $\rho_1, \rho_2, \ldots, \rho_n (n \in \{1,2, \ldots, r\})$ be positive, and the remaining eigenvalues be non-positive.

Then the vertex set GF_r is represented by

$$V(GF_r) = \begin{cases} v_0 = 1 \\ v_{ij} = 1 \leq i \leq r, 1 \leq j \leq 2 \end{cases}$$

Then,

$$\begin{aligned} RE(GF_r) &= \sum_{i=1}^{r} |\rho_i| \\ &= (\rho_1 + \rho_2 + \cdots + \rho_n) - (\rho_{n+1} + \rho_{n+2} + \cdots + \rho_r) \\ &= 2(\rho_1 + \rho_2 + \cdots + \rho_n) - \text{trace}(X(GF_r)) \end{aligned}$$

Let W_j be the weight of the j^{th} face of GF_r contain the vertices $v_0, v_{i,1}, v_{i,2}$. We have to prove the weight of sum of the labels assigned to the vertices in each face of GF_r are equal. It is enough to prove W_j is true for any three continuous values of $j-1, j, j+1$. Let W_j be the sum of the labels formed by three vertices

$$W_j = L(v_o) + L(v_{i,1}) + L(v_{i,2})$$

where,

$$\begin{aligned} &L(v_0) = 1 \\ &L(v_{i,1}) = \{r + i - j / j = 1 \quad 1 \leq i \leq r \\ &L(v_{i,2}) = \{2r + j - 1 / j = 2 \quad 1 \leq i \leq r \end{aligned}$$

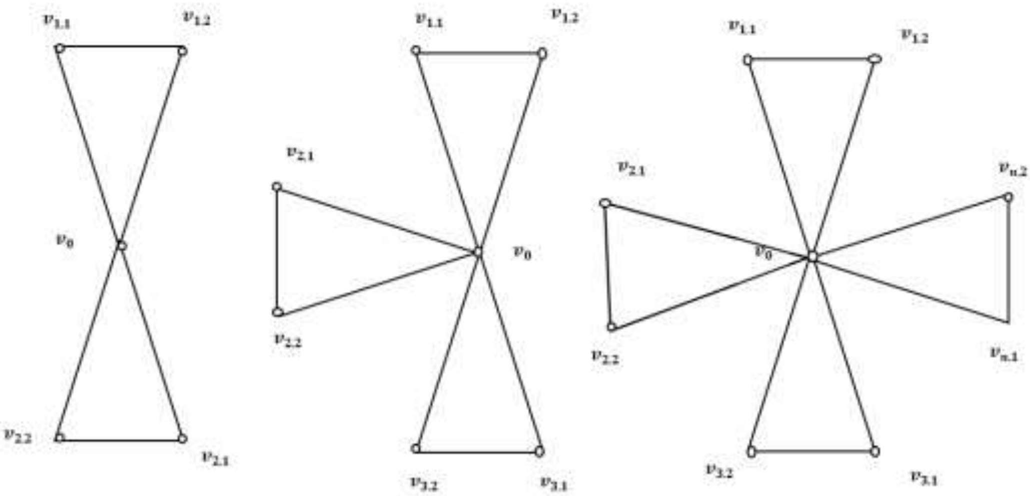

Figure 4. Friendship Graph

Let W_{j-1} be the weight of the $(j-1)^{th}$ value of GF_r

$$\begin{aligned} \mathrm{W_{j-1}} &= 1 + r + i - (j - 1) + 2r + (j - 1) - i \\ &= 3\mathrm{r} + 1 \end{aligned} \quad (7)$$

Let W_j be the weight of the j^{th} value of GF_r

$$\begin{aligned} \mathrm{W_j} &= 1 + r + i - j + 2r + j - i \\ &= 3\mathrm{r} + 1 \end{aligned} \quad (8)$$

Let W_j be the weight of the $(j+1)^{\text{th}}$ value of GF_r

$$\begin{aligned} \mathrm{W_{j-1}} &= 1 + r + i - (j + 1) + 2r + (j + 1) - i \\ &= 3\mathrm{r} + 1 \end{aligned} \quad (9)$$

From equation (7), (8) and (9)

$$\mathrm{W_{j-1}} = W_j = W_{j+1}$$
$$\mathrm{W_j} = k \text{ for all values of } 1 \le \mathrm{j} \le \mathrm{r}$$

since $\rho_1, \rho_2, \ldots, \rho_r$ are algebraic integers, so is their sum.

Hence $(\rho_1 + \rho_2 + \cdots + \rho_n)$ must be an integer if $RE(GF_r)$ is rational. Therefore, $\mathrm{RE}(GF_r) \equiv \mathrm{trace}(\mathrm{X}(GF_r))(\ \mathrm{mod} 2)$.

Theorem 3.6

Let GP_r be a face magic labeled prism graph. If the randić energy $R\mathrm{E}(GP_r)$ is rational, then $RE(GP_r) \equiv \mathrm{trace}(\mathrm{X}(GP_r))(\mathrm{mod}2)$.

Proof:

Let GP_r be a prism graph,
Let $\rho_1, \rho_2, \ldots, \rho_n (n \in \{1,2, \ldots, r\})$ be positive, and the remaining eigenvalues be non-positive.

The vertex set of G is represented by

$$\mathrm{V}(GP_r) = \{v_{il}/1 \le \mathrm{i} \le 2, 1 \le \mathrm{j} \le \mathrm{r}$$

Then,

$$\begin{aligned} \mathrm{RE}(GP_r) &= \sum_{i=1}^{r} |\rho_i| \\ &= (\rho_1 + \rho_2 + \cdots + \rho_n) - (\rho_{n+1} + \rho_{n+2} + \cdots + \rho_r) \\ &= 2(\rho_1 + \rho_2 + \cdots + \rho_n) - \mathrm{trace}(\mathrm{X}(GP_r)) \end{aligned}$$

The labeling of GP_r can be represented by

$$L(v_{ij}) = \begin{cases} j/ & i = 1, \quad 1 \leq j \leq r \\ 2r + 1 - j & /i = 2, \quad 1 \leq j \leq r \end{cases}$$

Let W_j be the weight oh the j th face of GP_r containing the vertices $v_{1,j}, v_{1,j+1}, v_{2,j}, v_{2,j+1}$ We have to prove that W_i is true for any three continuous value of $j-1, j, j+1$.
Let W_{j-1} be the weight of the $(j-1)^{th}$ face value of GP_r

$$\begin{aligned} W_{j-1} &= (j-1) + (j-1) + 2r + 1 - (j-1) + 2r + 1 - (j-1) \\ &= 2r + 1 + 2r + 1 \\ &= 4r + 1 \end{aligned} \quad (10)$$

Let W_j be the weight of the j^{th} face value of GP_r

$$\begin{aligned} W_j &= j + j + 2r + 1 - j + 2r + 1 - j \\ &= 2r + 1 + 2r + 1 \\ &= 4r + 2 \end{aligned} \quad (11)$$

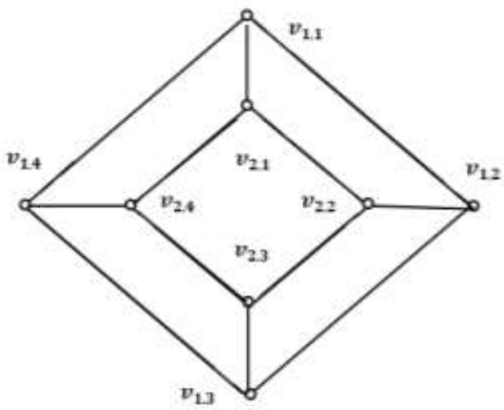

Figure 5. Prism graph

Let W_{j+1} be the weight of the $(j+1)^{th}$ face value of GP_r

$$\begin{aligned} W_{j+1} &= (j+1) + (j+1) + 2r + 1 - (j+1) + 2r + 1 - (j+1) \\ &= 2r + 1 + 2r + 1 \\ &= 4r + 2 \end{aligned} \quad (12)$$

From (10), (11) and (12)

$$W_{j-1} = W_j = W_{j+1} \text{ for all j}$$

That is $W_j = k$ for all values of $1 \leq j \leq r$
since $\rho_1, \rho_2, \ldots, \rho_r$ are algebraic integers, so is their sum.

Hence $(\rho_1 + \rho_2 + \cdots + \rho_n)$ must be an integer if $RE(GP_r)$ is rational. Therefore, $\mathrm{RE}(GP_r) \equiv \mathrm{trace}(\mathrm{X}(GP_r))(\bmod 2)$.

4. Conclusion

In this paper, we presented Randić energy for the face magic labelled graphs. In the above results we have proved that the Ladder graph, Para chain Hexagon, Friendship graph and Prism graphs are accepting the face magic labelling with randić energy. In future, we include the investigation of Randić energy of another special graphs, connections between graph energy and another graph parameters on face magic labelling.

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