Section A-Research paper
ISSN 2063-5346

REGULAR NUMBER OF SUBDIVISION OF MIDDLE GRAPH OF A GRAPH

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Keywords:
Regular number / middle graph /subdivision graph of a graph /Regular number of subdivisions of Middle graph of a graph.

ABSTRACT

The middle graph of a subdivision of a graph, represented by M[S(G)], for any (p, q) graph G, is a graph whose vertex set is $V[S(G)] \cup E[S(G)]$; If two edges of G are next to one other or if one is a vertex and the other is an edge incident with it, those two points are said to be adjacent. The least number of sub-sets into which the edge set of the M[S(G)] should be divided in order to generate a regular subgraph for each subset is known as the regular number of the M[S(G)] and is indicated by the symbol $r_{sm}(G)$. Several findings on the regular number of $r_{sm}(G)$ were made and expressed in terms of G elements in this article.

DOI: 10.48047/ecb/2023.12.si4.1749

1. INTRODUCTION

Here, only non-trivial, simple, finite graphs are taken into consideration. The common notations for a graph's vertex and edge counts are p and q, respectively, while the symbol for a vertex's highest degree in a graph is $\Delta(G)$. If removing a vertex from G results in G having more components, the vertex is said to be a cutvertex. If a graph G contains no edges, it is said to be trivial. A diameter is the shortest possible distance between any two vertices in G, and it is represented by the symbol diam (G). Stanton James and Cown introduced the path and tree

numbers in. Every term in this work that is not defined can be found in [3]. If a tree has one vertex of degree 2 and all the other vertices are of degree 1 or 3, it is referred to as a binary tree.

The intermediate graph of G is designated by the vertex set $V(G) \cup E(G)$, where two vertices are only considered to be near if and only if they are either adjacent edges of G or if one is a vertex and the other is an edge incident with it $M(G)$. The lowest order of dividing (G) into subsets required for each set to generate an independent subgraph is called the edge set independence number $\beta*(G)$. The largest cardinality of an edge independent set in G is represented by the independence number $\beta_1(G)$.

Presume $G = (V, E)$ be a graph. A set $D' \subseteq V$ is supposed to be a dominating set of G, if every vertex in (V – D') is adjacent to some vertex in D'. The vertices are having minimum cardinality represented in a set such a set is called the domination number of G and is denoted by $\gamma(G)$. A dominating set is said to be total dominating set of G, if $N(D') = V$ or equivalently, if for every $v \in V$, there exists a vertex $u \in D'$, $u \neq v$, such that u is adjacent to v.

The middle graph of a subdivision of a graph, represented by M[S(G)], for any (p, q) graph G, is a graph whose vertex set is $V[S(G)] \cup E[S(G)]$; If two edges of G are next to one other or if one is a vertex and the other is an edge incident with it, those two points are said to be adjacent. The least number of sub-sets into which the edge set of the M[S(G)] should be divided in order to generate a regular subgraph for each subset is known as the regular number of the M[S(G)] and is indicated by the symbol $r_{sm}(G)$. Several findings on the regular number of $r_{sm}(G)$ were made and expressed in terms of G elements in this article.

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Subject Classification Number: AMS 05C69, 05C70.

INTRODUCTION

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Theorem [A] = For any tree T, $r(T) = \Delta(T)$

The exact value of a regular Number of a middle graph of star subdivision is determined in the following theorem.

Section A-Research paper
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Theorem 1: For any star $K_{1,p}$ with $p \geq 3$ vertices $r_{sm}(k_{1,p}) = 3$

Proof: Let $G = k_{1,p}$, with $E(G) = \{e_1 = vv_1, e_2 = vv_2, e_3 = vv_3 \cdots\cdots\cdots, e_p = vv_p\}$ with v as center vertex with maximum degree and $v_i \in N(v)$ for $1 \leq i \leq p$ further let $\{v'_1, v'_2, v'_3, \cdots\cdots, v'_p\}$ be the vertices that divides each edge that gives the subdivision $S(G)$ of Gsuch that $v_i v'_i \in V[S(G)]$; Now $V[M(S(k_{1,p})] = \{v'_1, v'_2, v'_3 \cdots\cdots\cdots, v'_{p-1}\} \cup \{v_1, v_2, v_3 \cdots\cdots\cdots, v_{p-1}, v_p\} \cup$
Where $E[M(S(k_{1,p})] = v'_{p-2}v'_{p-1}\} = \{e'_1 = v'_1v'_2, e'_2 = v'_2v'_3, e'_3 = v'_3v'_4 \cdots\cdots\cdots e'_{p-2}\}$
Further in $M[S(k_{1,p})]$, the vertices $v_i \in N(v)$ forms a complete block which is k_{p+1} complete graph with p-regular number and hence constitutes to the one of the regular partition F_1 Now let $\langle F_2 \rangle$ be the another edge partition of $M[S(k_{1,p})]$ such that $v_i v_j \in e_{ij}$ of edge partition of F_1 that forms a closed path C_3 with 2-regularity finally let E' be the end edge set where $e'_j \in E'$; $\forall\, i \leq j$ that contributes to a single partition with 1-regularity Thus we can come to remark that
$r_{sm}(k_{1,p}) = |F_1, F_2, F_3| = 3$

Theorem 2: For any wheel W_p, $p \geq 4$ vertices $r_{sm}(W_p) = 2$, if $p = 4$ $r_{sm}(W_p) = 3$ if $p > 4$

Proof: Let $v_1, v_2, \cdots \ldots, v_p$ be the vertices of W_p, where v_p is the center vertex with degree $p - 1$. Now let $e_i = \{v_i, v_{i+1}\}$; for $i = 1, 2, \cdots\cdots\cdots, p - 2$ be the edges embedded on the plane of $W_p$and let $e'_1 = \{v_i, v_p\}$; for $i = 1, 2, 3, \cdots \ldots . p - 1$ be the interior edges of W_p respectively .Now let us insert an vertex v_j for degree two in between all the edges of $W_p$such that $e_j = \{v_j v_p\}$; $\forall\, j = 1,2, \cdots\cdots\cdots, p - 2$ which corresponds to vertex set in $M[S(W_p)]$, such that $V[M(S_p)] = E(V[S(W_p)) \cup E[S(W_p)]$.
Now we recognize the regular number of $M[S(W_p)]$ we go through with following cases :
Case1: if $p = 4$; Then in $M[S(W_p)]$, $E[S(W_p)] = \{e'_1, e'_2, e'_3, \cdots \ldots \ldots, e'_p\} \cup \{e_1, e_2, e_3, \cdots \ldots, e_{p-1}\}$
Let $E_1 \subseteq E(S(W_p))$ be the set of edge such that

$$E_1 = \{(e_1, e_2, \cdots\cdots e_6), (e_7, e_8, \cdots\cdots e_{12}), (e_{13}, e_{14}, \cdots\cdots e_{18}), (e_{19}, \cdots\cdots e_{24})\}$$

Forms a one partition of 3-regular as F_1. Now and also edge set
$E_2 = \{(e'_1, e'_2, e'_3), (e'_4, e'_5, e'_6), (e'_7, e'_8, e'_9), (e'_{10}, e'_{11}, e'_{12}), (e'_{13}, e'_{14}, e'_{15}), (e'_{16}, e'_{17}, e'_{18})\}$
Form a another 2-regular partition as F_2 Hence $|F_1, F_2| = 2$ see (fig 1 a)
Case 2: For W_p with $p > 4$ vertices we have the subdivision of W_p as $S(W_p)$ and and edge set of $M[S(W_p)] = E_1 \cup E_2 \cup E_3$. Now let E_i for $i = 1, 2, 3$ be the regular partition of $M[S(W_p)]$ each sub graph induced $\langle E_i \rangle$ induced by E_i is required to be regular. Hence each E_i is either 2-regular or 3-regular. Thus the $p - 1$ edges incident with the vertices of $\Delta = 3$, must be the part of edge set E_1.

which introduces one partition F_1 with $(p - 1)$ regualarly. Furthermore, the edge set E_2, if any subgraph $\langle E_i \rangle$ is $2-$regular, forms a minimum partition as F_2. Now the remaining edges of edge $E_2 \notin E_1$ *and* E_2 containing the edges such that $e_i \notin E_i$ for $i = 1, 2$ and adjacent to edge set E_2, forms a third partition of 2-regularity (see fig 1 b).
Thus, finally conclude by considering the description of case 2 we can conclude that each component of set F_1, that $\langle F_1 \rangle$ is complete graph k_p and is $p - 1$ regular, for set F_2 & F_3 each component is C_3 and is $2-$regular where as edge component of $F_2 \notin$ edge component of F_3
Hence $r_{sm}(W_p) = |F_1, F_2, F_3|$ for $p \geq 5$ vertices
$= 3$

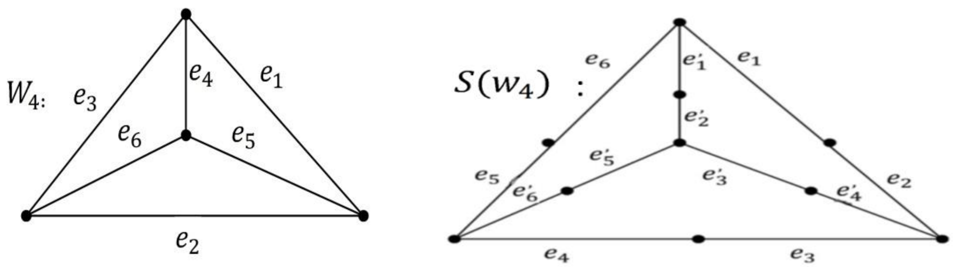


Fig 1(a)

$M[S(W_4)]$:

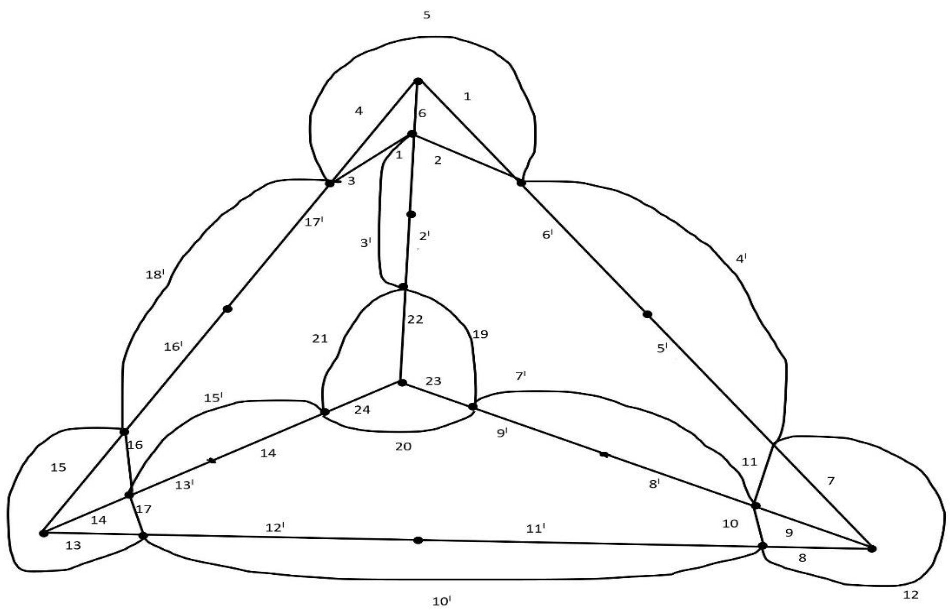


Fig 1 (b)

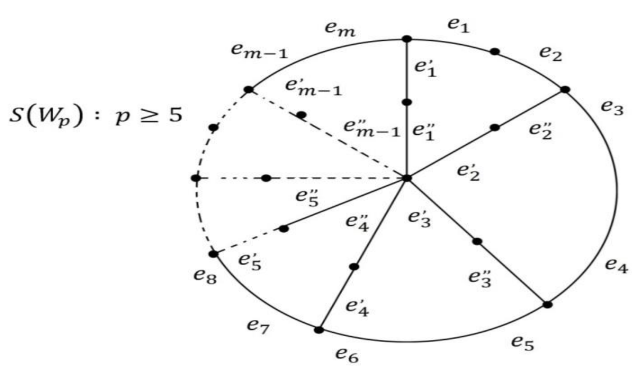
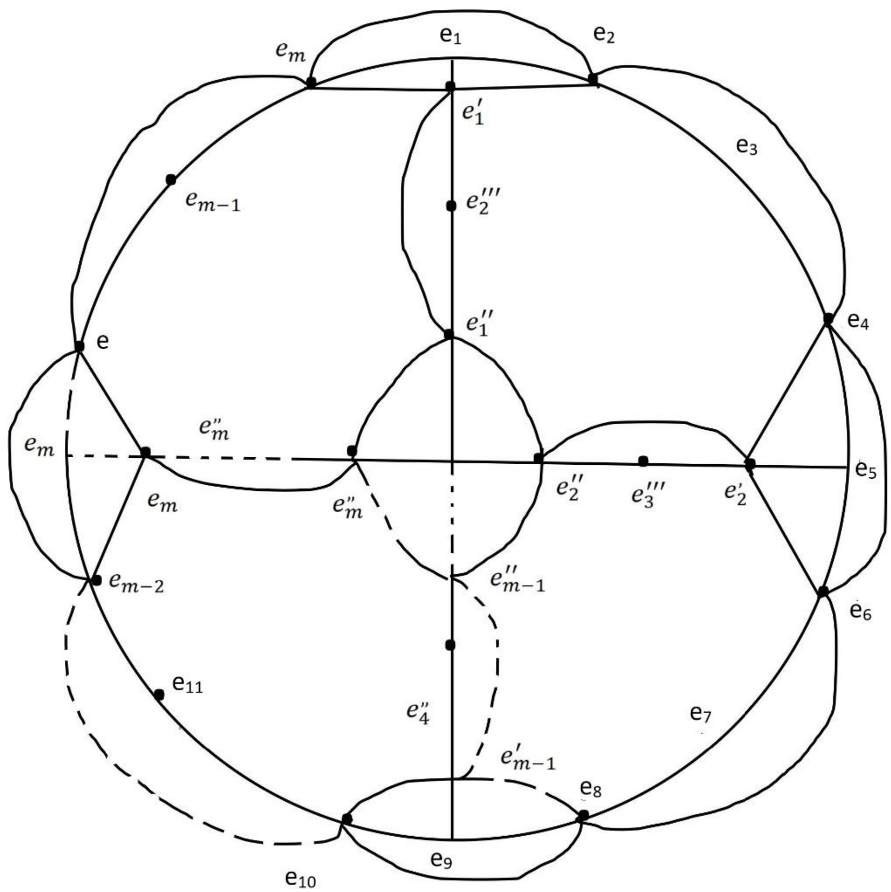


Fig 2(a)

$M[S(W_p)]$: $with\ p > 4$



(Fig 2 b)

Theorem 3: For any graph G, $r_{sm}(G) \leq \beta_1^*[S(G)]$

Proof: Let $E(G) = \{e_1, e_2, e_3, \cdots \ldots . . e_i\}$ be the edge set of G and let $E'(G) = \{e_1', e_2', e_3', \cdots \ldots . e_i'\}$ be the edge set of $S(G)$ such that $E[S(G)] = E(G) \cup E'(G)$ which equivalently gives the twice the number of edges in $S(G)$. Now let $E''(G) = \{e_1'', e_2'', e_3'', e_4'', e_5'', \cdots \ldots e_n''\}$ be the edge set of $M[S(G)]$ which corresponds to elements of $E'(G)$. Now there exists a partition of $E[S(G)]$ as $E_1(G) \cup E_1'(G)$. Where $E_1(G) \subset E(G)$ and $E_1(G) \subset E'(G)$. The sub graph induced by $\langle E_1 \cup E_1' \rangle$ set is independent such that $\left|E_1(G) \cup E_1'(G)\right| = \beta_1^*[S(G)]$. suppose $F_1, F_2, F_3, \cdots \ldots F_n$ be the regular edge sets of $M[S(G)]$ in which F_i, $1 \leq i \leq n$ includes one partitions constituting the edges from $E''(G)$ with different regularity then
$|F_1, F_2, F_3, \cdots \ldots F_n| \leq |E_1(G) \cup E_1'(G)|$ which gives $r_{sm}(G) \leq \beta_1^*[S(G)]$

Secondly, we determine the outcome that reveals the link between

Theorem4: For any graph G, $r_{sm}(G) \leq \Delta(G) + \chi(G)$ where $\Delta(G)$ is maximum degree and χ is the chromatic number of G

Proof: We come up with the following cases:-

Case1: Suppose G is not tree T Now let us consider vertex u_i with $degu_i \geq 2$ Then there exists at least one vertex $u \in u_i$ with maximum degree $\Delta(G)$ $1 \leq i \leq n$ further vertices which are adjacent to maximum degree vertex u together with u_i forms a induced complete regular sub graph which gives one regular partition F_1 and also the sets composing the vertex set as $\{u_1u'_1, u_2u'_2, u_3u'_3, \cdots \ldots . u_ju'_j\}$; $\forall\ 1 \leq j \leq n$ which are the edges of $M[S(G)]$ belongs to the different set of partition sets of F_i ; $\forall\ 2 \leq i \leq n$ respectively Further each edge not incident to u belongs to any other partition of F_n Thus $r_{sm}(G) = |F_1, F_2, F_3, \cdots \ldots \ldots, F_n|$ since for any graph $G \neq T$, the chromatic number $\chi(G) \geq 2$ always and .we can conclude with

$$|F_1, F_2, F_3, \cdots \ldots \ldots, F_n| \leq \chi(T) + \Delta(G)$$

Case2: Suppose G is tree T with and n number cut vertices and and maximum degree $\Delta(T)$

Subcase 2.1: For any tree T, let $V' = \{u_1, u_2, u_3, \cdots \ldots, u_n\}$ be the subset of $V[S(T)]$ be the set of all vertices that are not ends, and each vertices' degree is the same, as shown by $\deg(u'_1) = \deg(u'_2) = \deg(u'_3) = \cdots \ldots \ldots \deg(u'_n) = k$ (say) Then in $M[S(T)]$, the edge set $\{e'_1, e'_2, e'_3, \cdots \ldots . e'_n\}$ divided by the new vertex set $\{u'_1, u'_2, u'_3, \cdots \ldots, u'_{n-1}\}$. respectively and join these vertices by the new edges to the adjacent vertices which also corresponds to the vertex set of $M[S(T)]$ .Now in $M[S(T)]$, $\forall\, u_i \in V'$, such that $1 \leq i \leq n$, gives n-number of k-regular blocks forms a one complete regular partition F_1 with edges incident to vertices of maximum degree $u \in V(T)$. let $u_j \in V'$, $1 \leq i \leq n$ such that $\{N(u_i) \cup u_i\} \in F_1$ and $u_j \in V'$ such that $N(u_j) \cup u_i$ in $S(T)$ which further gives $\{N(u_j) \cup u_j\} \in F_2$ with 2-regular partition . Hence $V[M(S(T))] - \{N(u_i) \cup u_i\} \cup \{N(u_j) \cup u_j\} \in F_3$ Thus $V[M(S(T))] = F_1 \cup F_2 \cup F_3$ which are edge disjoint sub graphs of $[S(T)]$. Since for any tree T, $\chi(T) = 2$ and $v \in v_i = \Delta(T)$. Thus the result follows

Subcase 2.1: Now let us take consider the vertex set $S(T)$ of n −number of cut vertices with distinct degree such that $\deg(v_i) \neq \deg(v_j)$; $\forall\, i \leq i \leq j$ and The edges incident to vertices set v_i *and* v_j $1 \leq i \leq j$ in $M[S(T)]$ forms a block with different regularity corresponding to some regular partition F_i for $i \neq j$, Now the remaining neighborhood blocks which are also bridges of $M[S(T)]$ forms a closed path which is k_3 one of the minimal edge partitions of F_i, where each block is a complete sub graph of $M[S(T)]$ Finally the set of end edges in $M[S(T)]$ forms k_2 −graph with 1 −regularity corresponds to the one more partition of $F_i, \forall\, i \leq i \leq n$ Thus considering all the above cases and theorem [B] can approach to the remark that

Thus

$$\sum_{1=1}^{n} F_i \leq \chi(T) + |\max\{\deg(u)\}|$$

$$r_{sm}(G) \leq \chi(T) + \Delta(G)$$

In the following theorem we find the relationship between, $r_{sm}(G)$ with vertex covering number of G

Theorem 5: For any graph G, $r_{sm}(G) \leq p - \alpha_o(G)$

Proof: Let $E = \{e_1, e_2, e_3, e_4, \cdots \ldots \ldots, e_{p-1}\}$ be edge subdivision of G such that $E \subset S(G)$ which togetherly with $V[S(G)]$ also corresponds forms a vertex set in $M[S(G)]$ such that $V(M[S(G)]) = E[S(G)] \cup V[s(G)] = \{e'_1 = v_1v'_1, e'_2 = v_2v'_2, e'_3 = v_3v'_3 \ldots \ldots . e'_{p-1} = v'_{p-1}v_{p-1}, e_p = v'_pv_p\}$

Let us consider the vertex $v \in V[M(S(G)]$ with maximum edge that forms a one regular partition F_1 with its adjacent vertices in $M[S(G)]$. Next we follow those vertices which forms a closed path. with odd number of vertices that give us next partition F_2 of $M[S(G)]$. We go on continuing this process until all the edges of $M[S(G)]$ belongs to any one of the partitions of $M[S(G)]$ Thus we can proceed to write with $|F_1, F_2, F_3, \ldots \ldots ., F_n| = |F|$ since its known that $V(G) = p$ such that $c = \{v_i, v_{i+1}, v_{i+2}, \ldots \ldots \ldots, v_j\}$ be the maximum set of vertex covering that covers all the edges of G. Such that $j < n$ and $c \subseteq p$, then $|C| = \alpha_o(G)$ and $V(G) \subset VM(SG))]$ Hence it is clear that $|F| \leq p - |c|$ $r_{sm}(G) \leq p - \alpha_o(G)$

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2. Conclusions

We looked at the properties of our concept by applying it to a few typical graphs. Also, we were able to determine the regular number of subdivisions of the centre graph by adding a new vertex to each edge of a few common graphs and linking the newly produced neighbouring vertices. By creating a conventional graph by adding new nearby vertices to each edge and dividing each edge into a new vertex. Nonetheless, many of the outcomes are accurate.

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