COMPARATIVE STUDY OF VARIOUS METHODS USING FUZZY NUMBER

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Abstract

This essay compares alternative approaches for determining the most effective approach to matching fuzzy transportation issues and details how to deal with situations that have transportation costs represented as triangular numbers, i.e. zero point, zero suffix, modified zero suffix method. Fuzzy costs are transformed in to crisp value using graded mean technique, and then we must solve the problem via these existing methods. The objective of this paper is to solve the fuzzy transportation problems (triangular and hexagonal fuzzy numbers) using zero point, zero suffix and modified zero suffix method. The optimum solution of zero point as well as modified zero suffix gave better results as compare to zero suffix, where as allocation value of both (zero point and modified zero suffix) methods will be the same as compare to zero suffix.

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Keywords

Triangular and Hexagonal fuzzy Numbers, Zero Point, Zero Suffix, Modified Zero Suffix methods.

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1 Introduction

A component of the mathematical model is the transportation issue. Hitchcock [4] introduced the transportation issue for the first time in 1941. A fuzzy transportation situation is one in which the demand and supply quantities for the transportation cost are uncertain. The parameters of the transportation issue are not precisely understood and stable in real life. Transportation problems get hazy as a result of this ambiguity. The amount is unknown because of several undisputed factors, including bad weather, transportation hazards, and imprecision in decision-making.

Basirzadeh et. al. [2] in subsequent study from 2011 also suggested a technique for ranking fuzzy numbers using the alpha cut, and they provided rankings for triangular and trapezoidal fuzzy numbers. According to Pukky Ttralian et al. [11] in 2020, research on CV. Ngastiti et. al. [12] had acknowledged about a company, Bintang Elektrik Grace which affiliated with the transportation problem fuzzy acquired the optimum solution that its zero-point method and zero suffix method are the same but that the number of iterations of the zero point method is higher than that of the zero suffix method. In 2012, Sharma et. al. [6] concluded from their research that the method zero point is a symmetrical procedure for transportation problems that can be easily applied and utilized for all kinds of transportation problems to optimize problems in terms of maximums or minima to make decisions regarding different kinds of logistical problems and provide optimal solutions to transportation problems. The procedure developed in this paper, according to Sujatha, et. al. [16] in 2016 provides the best fuzzy solutions and the best fuzzy objective value, both of which are non-negative fuzzy numbers. As a result, the method developed is a crucial tool for the decision maker when handling the transportation problem in a fuzzy environment. In 2022, Jeyaseeli et. al. [8] a came to the conclusion that the issue might be solved optimally by employing the zero suffix and heuristic approaches using two different fuzzy numbers. In 2021, Keerthivasan et. al. [3] are solved fuzzy transportation problem with modern zero suffix.

In this paper, we introduce a Fuzzy transportation problem in which all the parameters are taken in triangular and hexagonal form. The optimum solution is obtained by various methods.

2 Preliminaries

2.1 Fuzzy Transportation Problems

The following formula may be used to express the fuzzy transportation problem, where a decision-maker is unsure of the exact values of transportation costs from the i^{th} source to the j^{th} destination but is certain about the supply and demand of the product.

$$\boldsymbol{Z = min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}},$$

$\boldsymbol{c_{ij}}$
$= the fuzzy\ cost\ of\ transportaion\ onr\ unit\ of\ goods\ from\ origin\ (i)\ to\ destination\ (j)$

$\boldsymbol{x_{ij}} = the\ goods\ transported\ from\ origin\ (i)\ to\ destination\ (j)$

Subject to

$$\boldsymbol{\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, 3, \dots, n}$$

$$\boldsymbol{\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, 3, \dots, m}$$

2.2 Triangular Fuzzy Number

The most frequent tool for defining fuzzy information that exists in the actual world is the fuzzy number, which contains the features of both fuzzy sets and numbers. We are talking about fuzzy triangular numbers when we speak about them in mathematical terms.

If *A*= *(a,b,c;1)* is a vague fuzzy set as well as its membership function is defined by, then *A* is an extended fuzzy number.

$$\mu_{\tilde{A}}^{TFN}(x) = \begin{cases} 0, & if \quad 0 \le x \le a \\ \frac{x-a}{b-a}, & if \quad a \le x \le b \\ 1, & if \quad x = b \\ \frac{x-c}{b-c}, & if \quad b \le x \le c \\ 0, & if \quad x \ge c \end{cases}$$

2.3 Hexagonal Fuzzy Number

If *A* is a generalized hexagonal fuzzy number of the form *A*=$(a_1, a_2, a_3, a_4, a_5, a_6)$, then the effectiveness of membership $\mu_A(x)$of a hexagonal fuzzy number has the following feature:

$$\mu_{\tilde{A}}^{HFN}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & if\ a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & if a_2 \le x \le a_3 \\ 1 & if\ a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right) & if\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & if\ a_5 \le x \le a_6 \\ 0 & otherwise \end{cases}$$

2.4 Graded Mean Ranking Technique

Let $\tilde{A}$ and $\tilde{B}$ are two trapezoidal and hexagonal fuzzy numbers such that
$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$

Then $\tilde{A} \leq \tilde{B}$

If the following inequalities hold

$a_1 \leq b_1, a_2 \leq b_2, a_3 \leq b_3, a_4 \leq b_4, a_5 \leq b_5, a_6 \leq b_6,$

Then graded mean of triangular $M(A)$ and hexagonal $P(A)$ fuzzy number representation of A becomes

$M(A) = \frac{a_1+a_2+a_3}{3}$ and $P(A) = \frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}$.

3 Methodology

For the purpose of finding the most effective optimal solution for a set of triangular and hexagonal fuzzy numbers, we use the existing methods.

1. Zero Point Method,
2. Zero Suffix Method,
3. Modified Zero Suffix.

4 Numerical Examples

4.1 Consider the fuzzy triangular dilemma, in which a corporation has three destinations (*Q*, *R*, and *S*), four sources (*M*, *N*, *O*, and *P*), and a fuzzy transportation cost for a unit quantity of the product from the source (*i*) to the destination (*j*).

Table 4.1- fuzzy triangular transportation problem

	M	*N*	*O*	*P*	Source
Q	(1650, 1700, 1750)	(2000, 2200, 2100)	(1500, 1450, 1400)	(2100, 2000, 2150)	(45, 52, 53)
R	(3250, 3450, 3400)	(4600, 4400, 4500)	(1900, 1800, 2000)	(2700, 2600, 2750)	(25, 45, 50)
S	(3250, 3550, 3400)	(4500, 4400, 4550)	(1750, 1600, 1800)	(2000, 2100, 2300)	(45, 62, 73)
Destination	(15, 35, 45)	(20, 30, 55)	(25, 45, 50)	(32, 45, 58)	

Solution. We use a simple mean technique for the fuzzification procedure. Finally, we compare the results using the modified zero suffix and zero-point zero suffix techniques.

By mean method $M(A) = \frac{a_1+a_2+a_3}{3}$.

$M(1650, 1700, 1750) = 1700$, $M(2000, 2200, 2100)$ =2100, $M(1500, 1450, 1400) = 1450$,
$M(2100, 2000, 2150)$=2083.3, $(3250, 3450, 3400) = 3,350$, $M(4600, 4400, 4500) = 1900$

$M(1900, 1800, 2000) = 1900$, $M(2700, 2600, 2750)$ =2, 683.3, $M(3250, 3550, 3400) = 3, 400$,
$M(4500, 4400, 4550) = 4, 483.3$, $M(1750, 1600, 1800)$ =1,716.6, $M(2000, 2100, 2300) = 1, 533.3$.

Crisp value of supply and demand

$M(15, 35, 45) = 30$, $M(20, 30, 55) = 35$, $M(25, 45, 50) = 40$, $M(32, 45, 58) = 45$, $M((45, 52, 53) = 50$
$M(25, 45, 50) = 40$, $M(32, 45, 58)$ =60.

Table 4.2- Crisp value of fuzzy TP

	M	*N*	*O*	*P*	Source
Q	1700	2100	1450	2083.3	50
R	3,350	4500	1900	2,683.3	40
S	3400	4,483.3	1,716.6	1,533.3	60
Destination	30	35	40	45	

Table 4.3- Row and Column reduction from row reduction table

	M	*N*	*O*	*P*	Source
Q	0	0	0	633.3	50
R	1200	1950	0	783.3	40
S	1,616.7	2300	183.7	0	60
Destination	30	35	40	45	

Table 4.4 Allocate the fully fuzzy transportation using zero point method

	M	*N*	*O*	*P*	Source
Q	**[15]** 1700	**[35]** 2100	1450	2083.3	50
R	**[15]** 3,350	4500	**[25]** 1900	2,683.3	40
S	3400	4,483.3	**[15]** 1,716.6	**[45]** 1,533.3	60
Destination	30	35	40	45	

Optimum solutions of zero point method
=1700*15+2100*35+15*3350+25*1900+15*1716.6+45*1533.3=**291,497.5**

Table 4.5 Using steps 3, method of 2.2 (Zero suffix method)

	M	*N*	*O*	*P*	Source
Q	0	0	0	633.3	50
R	1200	1950	0	783.3	40
S	1,616.7	2300	183.7	0	60
Destination	30	35	40	45	

Now Find the suffix value of all zeros using steps of 3.2 methods is 5 are S_1, S_2, S_3, S_4, & S_5,
$S_1 = 1200$, S_2 =1950, $S_3 =$ 633.3, $S_4 = \frac{1950+783.3+183.3}{3}$, =972.2, $S_5 = \frac{783.3+183.3}{2} = 483.3$

Table 4.6 Allocate the value of crisp table using zero suffix method

	M	*N*	*O*	*P*	Source

Q	**[30]** 1700	**[20]** 2100	1450	2083.3	50
R	3,350	4500	**[40]** 1900	2,683.3	40
S	3400	**[15]** 4,483.3	1,716.6	**[45]** 1,533.3	60
Destination	30	35	40	45	

Optimum solution using method 2.2 is = 30*1700+20*2100+40*1900+15*4,483.3+45*1533.3 **=305,248**

Table 4.7 Allocate the value of crisp table using modified zero suffix

	M	*N*	*O*	*P*	Source
Q	**[15]** 1700	**[35]** 2100	1450	2083.3	50
R	**[15]** 3,350	4500	**[25]** 1900	2,683.3	40
S	3400	4,483.3	**[15]** 1,716.6	**[45]** 1,533.3	60
Destination	30	35	40	45	

Optimum solutions of modified zero suffix method =1700*15+2100*35+15*3350+25*1900+15*1716.6+45*1533.3**=291,497.5**

4.2 Using Hexagonal fuzzy problem by Jeyaseeli [9]

Table 4.8 Hexagonal fuzzy transportation

	Position 1	Position 2	Position 3	Source
Delhi	(2 ,4, 6, 8, 10, 12)	(2, 6, 8, 12, 14, 16)	(8, 10, 12, 14,16,18)	(16, 20, 24, 24,12,8)
Lucknow	(6, 12, 10, 8, 6, 4)	(4, 6, 10, 12, 14, 10)	(8, 14, 12, 10, 4, 2)	(4, 6, 10, 12, 4, 2)
Prayagraj	(2, 10, 12, 14, 12, 24)	(2,16,14,12,10,12)	(10, 18, 8, 12,14,12)	(10,20,24,34,22,20)
Designation	(10, 16, 16, 14, 10, 8)	(10, 2, 12, 14, 10 ,4)	(4, 6, 2, 6, 10, 14)	

Convert the hexagonal number in crisp value using graded mean ranking function; add the dummy column for balancing the crisp table with cost 17.3

Table 4.9 Crisp value table

	Position 1	Position 2	Position 3	Position 4	Source
Delhi	7	9.6	13	0	17.3
Lucknow	7.6	9.3	8.3	0	6.3
Prayagraj	9	11	12.3	0	21.6
Designation	12.3	8.6	7	17.3	

The allocation tables for the zero point technique and the modified zero suffix method will be identical, as can be seen in the table that follows.

Table 4.10 Allocate the fully fuzzy crisp table using zero point and modified zero suffix

	Position 1	Position 2	Position 3	Position 4	Source

Delhi	**[12.3]** 7	**[5]** 9.6	3	0	17.3
Lucknow	7.6	9.3	**[6.3]** 8.3	0	6.3
Prayagraj	9	**[3.6]** 11	**[0.7]** 12.3	**[17.3]** 0	21.6
Designation	12.3	8.6	7	17.3	

Optimal Solution of both methods are =12.3*7+5*9.5+6.3*8.3+3.6*11+0.7*12.3+0*17.3**=234.6**

Table 4.11 Allocate the fully fuzzy crisp table using zero suffix

	Position 1	Position 2	Position 3	Position 4	Source
Delhi	**[10.3]** 7	9.6	**[7]** 13	0	17.3
Lucknow	7.6	**[6.3]** 9.3	8.3	0	6.3
Prayagraj	**[2]** 9	**[2.3]** 11	12.3	**[17.3]** 0	21.6
Designation	12.3	8.6	7	17.3	

Optimum Solution of zero suffix method =10.3*7+7*13+6.3*9.3+2*9+2.3*11+0*17.3=**264.9**

5 Comparison and Decision Table of These Methods:

In the below table analogy of existing algorithms are given following.

Table 5.1 Result analysis

Methods	**Fuzzy Numbers**	**Optimum Cost**	**Allocation Value**
Zero Point	Triangular fuzzy numbers	291,497.5	X_{11}=15, X_{12}= 35, X_{21}=15, X_{23}=25, X_{33}=15, X_{34}=45
Zero suffix		305,248	X_{11}=30, X_{12}= 20, X_{23}=40, X_{32}=15, X_{34}=45
Modified zero suffix		291,497.5	X_{11}=15, X_{12}= 35, X_{21}=15, X_{23}=25, X_{33}=15, X_{34}=45
Zero Point	Hexagonal fuzzy numbers	234.6	X_{11}=12.3, X_{12}=5, X_{23}=6.3, X_{32}=3.6, X_{33}=0.7, X_{34}=17.3
Zero suffix		264.9	X_{11}=10.3, X_{13}=7, X_{22}=6.3, X_{31}=2, X_{32}=2.3, X_{34}=17.3
Modified zero suffix		234.6	X_{11}=12.3, X_{12}=5, X_{23}=6.3, X_{32}=3.6, X_{33}=0.7, X_{34}=17.3

6 Result

In this paper solve the fuzzy triangular and hexagonal fuzzy number, then convert the fuzzy numbers in crisp value using the graded mean value techniques after than we solve this problem using existing algorithm. The optimum solution of triangular and hexagonal fuzzy number is same (zero point and modified zero suffix methods) that is (triangular 291,497.5 and hexagonal solutions 234.6). But the optimum solution of zero suffix method of triangular and hexagonal is 305,248 and 264.9. That means it is greater than in compare to other both algorithms. Allocation value will also be same of both methods (zero point and modified zero suffix) rather than as compare to zero suffix. Jeyaseeli [9] Optimum solution obtained by zero suffix method of hexagonal fuzzy number is 980.75 using robust ranking technique.

7 Conclusion

Earlier much research that solve the problem with this existing method that the solution found by zero-point method gave beater results as compare to zero suffix. But in this paper we compare problems using hexagonal and triangular fuzzy numbers. fuzzy optimal solution of triangular and hexagonal numbers by the existing algorithms is show in Table 5.1, from this table it is clear that the solutions of both numbers(triangular and hexagonal) obtained by zero point method and modified zero suffix methods gave same result, but zero suffix method will gives different optimum solution in both examples.

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