Analysis of Bacteria Using Fractals

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ABSTRACT

Bacteria or single–celled organisms, which are vital to luminous body in ecosystem. Integration of Bacterial population will shape complexity based on the constructive Environment. Many species of Bacteria have been set up in the form of Fractals Colonies. Birth rate, Death rate and Doubling period of Bacterial population can be picked out by Mathematical Modelling. Lacunarity is formed between the Bacterial cells which are calculated from Histo Stretched Software. The single – celled organisms is collated with as Julia set which is most famous in Fractals set.

Keywords: Fractals, Ecology, Bacteria, Complexity

AMS classification: 28A80, 92D40, 92D30, 90C60

I. INTRODUCTION

1.1 ECOLOGY

The word ecology was first proposed in the year 1869 by *Ernst Haeckel*, although many contributions to this subject were done. However 1900's ecology was recognized. *Eugene Odum* is lionized throughout science as the father of modern ecology. The study of ecology explains the different organisms in environment and their interaction around them. Complex relationships between organisms and each other and organisms and their non-living environment. Ecology come up with new knowledge which tie in between people and nature i.e., Essential for offering food Prolonging clean air and water comfort habitat in swapping weather. The two main factors which are

responsible for shaping the environment are Biotic and Abiotic. Biotic are living factors of ecosystems. Instance like Bacteria, Animals, Birds, Fungi, and Plants etc. Abiotic are non-living chemical and physical factors of Atmosphere, Lithosphere, and Hydrosphere. Instance like Soil, Sunlight, Air, Moisture and Minerals etc., [10].

1.2 FRACTALS

"A fractal is generally a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced- size copy of the whole, a property called self–similarity"[2]. Benoit Mandelbrot the father of Fractals coined the term in 1975 and was derived from the Latin word fractus meaning *broken* or *fractured*. The Occurrence of Recursion defines the mathematical fractal at different scale. Main characters of Fractal patterns are fractal dimensions, considering these will estimate complexity. Since nature is full of Fractals, The Study will give us a countable language to outline the self-similar shapes created in the nature. Therefore Fractal is extremely familiar. Some of instance of Fractals in nature split of trees, animal cardiovascular systems, snowflakes, thunder stroke and hydro power, plants and leaves, landform and catchment area, clouds, crystals. To express some complexity in nature fractal geometry will provide better tools. Fractal Geometry experiment to outline the messy complexity of shapes we see around us. The most famous Fractal set are Julia sets and Mandelbrot set. [2, 6].

II. BACTERIA

Bacteria play a part in our ecosystem in numerous ways. Mainly bacteria are considered as decomposers because they break down dead materials and recycle it. But the truth is Bacteria are the producers in our ecosystem as well. In Earth there would be no life without producers. Producers are organisms they make their own food which we consider as green plants. The producers make food from sunlight such as photosynthesis bacteria or chemicals such as chemosynthetic Bacteria. Like plants, Bacteria also carry out photosynthesis using carbon dioxide and sunlight and also chemosynthesis, here chemical is used to make food. Bacterial outspread from cell to cell is hold up by shaping of membrane projection that explore into the cytosol of adjoining cells [12]. Bacteria in Human body carry out only one set of chromosomes instead of two disparately it is more complex forms of life. The process of reproducing the Bacteria by splitting into two cells is called *binary fission*.The reproduced cell is called offspring which are alike, essential clones with the matching genetic material. Bacteria gesture from one place to another for obtaining warmer with no brain to furnish incentive a bacterium ideally must rely on chemical cues from its environment to come up with an impetus to move. A single mother cell replicate to shape a class of genetically identical cell form a pile of cells derived from the mother cell is known asa Bacterial colony.

2.1 TYPES OF BACTERIA

There are wide ranges of bacteria that are present in our ecosystem which can be classified by their shapes. They are

- *Spherical*: Bacteria which are in the form of circle are called *cocci*, and a single bacterium is a *coccus*. Examples Streptococcus pyogenes.
- *Rod-shaped*: Rod shaped bacteria are known as bacilli and single bacterium is called bacillus. Some might be in curve shape and they are known as vibrio.
  Examples E. coli bacteria
- *Spiral*: spiral shape bacteria are known as spirilla and single bacterium is known as spirillus. If the coil of spiral bacteria is compact then it is said as spirochetes. Some of disease caused by spiral shaped bacteria is Leptospirosis, Lyme disease, and syphilis [12].

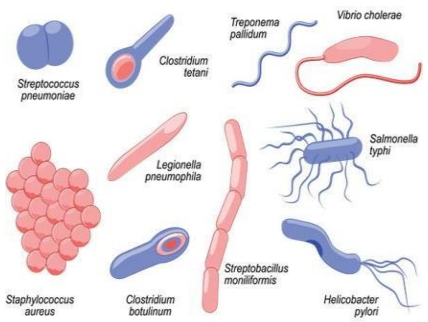


Fig. 1 Types of Bacteria

Since bacteria is present everywhere in environment. A common rod shaped bacteria is consider as a model whose count has been calculated from different place in various time.

Escherichia coli (E. coli) is a bacteria that are found in the intestines of people and animal and in the environment they can also be found in food and unrated water. Some bacteria are more harmful.

Table1. E. coli bacteria count from different places of Beaver and Kelly creek in United States.

Month & Year	**Greenway Footbridge**	**Division Road**	**Downstream Pond**	**Dogwood lane**
January 2017	15	65	15	15
May 2017	45	98	130	98
July 2017	25	95	98	15
December 2017	15	95	15	96
January 2018	95	98	260	175
April 2018	15	125	20	175
July 2018	75	25	15	280
October 2018	40	160	175	20
February 2019	25	180	30	50
Mean $\bar{x}$	38.8	104.5	84.2	102.6
Variance σ^2	0.26	0.16	0.06	0.2
Lacunarity	1.000176	1.000015	1.000009	1.000019

Mean and variance has been calculated for E.coli Bacteria Count, from that Lacunarity can be calculated using the formula L(r) = 1+ [var (r) / $mean^2$(r)]. Lacunarity asses the texture pattern of Cells. Gaps between the cells can determine using L(r). Here all the gap size is same for E.coli Bacteria.

Here the same Mean and Variance for E.coli Bacteria is calculated from HistoStretched Software by using the image analysis. At different factor from 0 to 1 stretched image for same bacteria image can be calculated using HistoStretched Software. In addition to the fractal dimension, the Lacunarity is used for the purpose of fractal analysis of a cell or a tissue. The Lacunarity quantifies the deviation from homogeneity in the texture. It is measured by competing the size of the holes on the fractals. i.e., counting the white pixels in the fractal image.

III. GENERAL THEORY OF JULIA SET

Julia set fractals are clearly engender by initializing a complex number z = x + iy where i^2= -1 and x and y are image pixel coordinates in the range of about -2 to 2. Then, z is frequently updated using: z = z^2 + c where c is additional complex number that offer a peculiar Julia set. Julia sets may be defined in terms of the action of the iterates f^k (z) for large k. Initially define the filled in Julia sets of the polynomial f,

$$k\ (f) = \{z \in a: f^k(z) \nrightarrow \infty\}$$

The Julia set of f is the boundary of the filled-in Julia sets J(f) = ∂k(f). Thus z ∈ J(f) if in every neighbourhood of z there are points w and v with $f^k(w) \to \infty$ and $f^k(v) \nrightarrow \infty$ The complement of the Julia set is called the Fatou set or stable set F(f). The symmetry and frame of the

Julia sets of polynomials in peculiar J is usually a Fractal. A set is said to be closed whose complement is an open set. In a topological space, a closed set can be describe as a set which Suppress all its limit points. A function f expounds on some set X with real or complex values are called bounded if the set of its values are bounded. Taking $f(z) = z^2 + c$ where c is a small complex number. It is easy to see that $f^k(z) \to w$ if z is small, where w is the fixed point of f close to 0, and that $f^k(z) \to \infty$ if z is large. Therefore the Julia set is the boundary between these two types of behaviour and now J is a fractal curve.

- ***Lemma 3.1*** *Given a polynomial $f(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_0$ with $n \geq 2$ and a_nG0 there exists a number r such that if $|z| \geq r$, then $|f(z)| \geq 2|z|$. In particular if $|f^m(z)| \geq r$ for some $m \geq 0$ then $f^k(z) \to \infty$ as $k \to \infty$. Thus either $f^k(z) \to \infty$ or the set $\{f^k(z): k = 0, 1, 2\ldots\}$ is bounded.*

Proof

Let r be any arbitrary number in the polynomial, Choose r sufficiently large to ensure that if $|z| \geq r$ then

$$\frac{1}{2}|a_n||z|^n \geq 2|z| \quad \text{and}$$

$$(|a_{n-1}||z|^{n-1} + \ldots + |a_1||z| + |a_0|) \leq \frac{1}{2}|a_n||z|^n.$$

Then if $|z| \geq r$,

$$|f(z)| \geq |a_n||z|^n - (|a_{n-1}||z|^{n-1} + \ldots + |a_1||z| + |a_0|)$$
$$\geq \frac{1}{2}|a_n||z|^n \geq 2|z|.$$

Furthermore, if $|f^m(z)| \geq r$ for some m, then applying this inductivity $|f^{m+k}(z)| \geq 2^m |f^k(z)| \geq r$, so $f^k(z) \to \infty$. Hence the proof. Based on the lemma 3.1 Consider the polynomial f(z) as a bacteria k represent real numbers using power law to this polynomial $f^k(z)$ represents the integration of bacteria which will lead to infinity. From this each bacteria cell are bounded.

- ***Proposition 3.2*** *Let f (z) be a polynomial. Then the filled in Julia set k(f) and the Julia set J(f) are non-empty and compact, with $J(f) < k(f)$.*

Proof

Based on this lemma 3.1 that k is contained in the disc B (0, r) and so is bounded, as its boundary J. If z Ø k, then $f^k(z) \to \infty$, so $|f^m(z)| > r$ for some integer m. By continuity of f^m, $|f^m(w)| > r$ for all w in a sufficiently small disc centred at z, so far such w, $f^k(w) \to \infty$ by the lemma 3.1 giving that w Ø k. Thus the complement of k is open, so k is closed. As the boundary of k, the Julia set J is closed and contained in k. Thus k and J are closed and bounded, and so are compact. The equation $f(z) = z$ has at least one solution z_0, say so $f^k(z_0) = z_0$ for all k, so $z_0 \in k$ and k is non-empty. Let $z_1 \in a\backslash k$. Then the point $\lambda z_0 + (1-\lambda) z_1$ on the line joining z_0 and z_1 will lie in the boundary of k for some $0 \leq \lambda \leq 1$ taking λ as the infimum value for which $\lambda z_0 + (1-\lambda) z_1 \in k$ will do. Thus $J = \partial K$ is non- empty [2].

3.1 PROPERTIES OF BACTERIA IN FRACTALS

Each bacterium extends in a flat fare, streak or order of cells is formed, but these orders are unbalanced to small disturbances. As multiplicity of cells hustle will opposed to each other, habitual instability leads to twist and overlap of cell order. If this continuous as the cells continue to enlarge and split, leading to the evolution of barrel of aligned cells display in self-similar subdivided patterns, or fractals. Since, some living organisms are tiny, it can be viewed through microscope. Some species are classified as single-celled or unicellular microbes or microorganisms. Bacterial colonies usually form curl shapes as they grow, their cells configure inner divisions which are tremendously asymmetrical and offshoot. These branched cells are self- similar to each other forms a fractal patterns which is mainly due to physical factors and local uncertainty that are typical part of bacterial cell maturation. If the cell continue to grow and divide in this manner will lead to form a raft of aligned cells display in self-similar subdivided patterns, or fractals. [5, 3]. By the theorem 3.1 and 3.2 growths of bacteria can be compared with Julia set where bacteria grow in exponential order 2^n, $n \geq 2$ the boundary of each bacteria cell are closed and bounded and so each cell is compact within itself. In fig 2.1 and 2.2 the single cell organisms started to multiply and completely fill the space alike Fractal Julia set as one zooms down finer and finer scales will resemble self - similarity property. These aligned cells of bacteria form a cluster pattern which can be collated with Julia set.

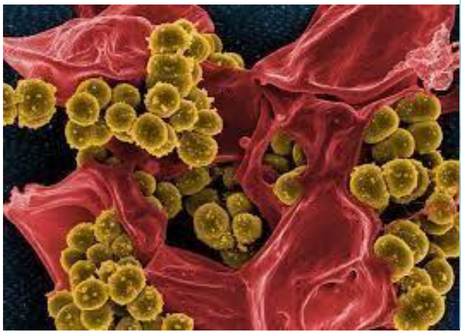


Fig. 2.1Bacteria in Human body

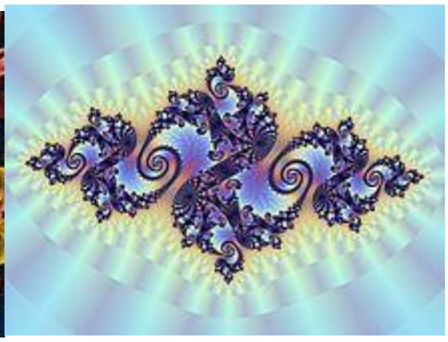


Fig. 2.2 Julia set

IV. EFFECT OF BACTERIAL GROWTH

The physical factors like temperature, osmotic pressure, pH, and oxygen concentration are main reason for the rate of growth or death of a particular microbial species. Every bacterial growth takes up to 4 to 20 minutes. Contagious bacteria can compel you to unwell. They replicate rapidly in human body. Numerous bacteria give off enzymatic called bane, which can harm flesh and make you out of sorts. Integration of bacteria is mainly due to changes in environment based on time. This generates complexity which can be analyzed by Mathematical modelling [7].

4.1 Mathematical Modelling

Mathematical Modelling in terms of differential equations arises when the situation modelled involves some continuous variables varying with respect to some other continuous variables and we have some reasonable hypotheses about the rates of change of dependent variables with respect to independent variables. One dependent variable x (say population size) depending on one independent variable (say time t) we get a mathematical model in terms of an ordinary differential equation of the first order if the hypothesis is about the rate of change dx/dt. The model will be in terms of an ordinary differential equation of thesecond order, if the hypothesis involves the rate of change dx/dt [7, 8].

4.2 Population Growth Model

Let x(t) be the population size at time t and let b and d be the birth and death rate, i.e., the number of individuals born or dying per individual per unit time, then in time interval $(t, t+\Delta t)$, the number of births and deaths would be $bx\Delta t + 0(\Delta t)$ and $dx\Delta t + 0(\Delta t)$, where $0(\Delta t)$ is an infinitesimal which approaches zero as Δt approaches zero, so that

$$x(t, t+\Delta t) - x(t) = (bx(t) - dx(t))\,\Delta t + 0(\Delta t) \quad (1)$$

so that dividing by Δt and proceeding to the limit as $\Delta t \to 0$, we get

$\frac{dx}{dt}$ = (b-d) x = ax (say) (2)

Integrating Equation. (2) we get

$x(t)=x(0)e^{at}$ (3)

so that the population grows exponentially if a $>$ 0 decays exponentially if a $<$0 and remains constant if a =0

(i) If a $>$0, the population will become double its present size at time T, where

$2x(0) = x(0)\, e^{at}$ or $e^{at} = 2$ or

$T = \frac{1}{a} \log 2 = (0.69314118)^{-1}$ (4)

T is called the doubling period of the population and it may note that this doubling period is independent of x (0). It depends only on a and is such that the greater the value of a (i.e., greater the difference between birth and death rates), the smaller is the doubling period.

(ii) If a $<$ 0 the population will become half its present size in time **T'** when

$\frac{1}{2}x(0) = x(0)\, e^{aT'}$ or $e^{aT'} = \frac{1}{2}$ or

$T' = \frac{1}{a} \log \frac{1}{2} = -\,(0.69314118)\, a^{-1}$ (5)

It may be noted that **T'** is also independent of x (0) and since a $<$ 0, **T'**>0. **T'** may be called the half-life period of the population and it decreases as the excess of death rate over birth rate increases [9].

Population Growth Model explains the growth of bacteria in exponential manner varying with time which is illustrated graphically

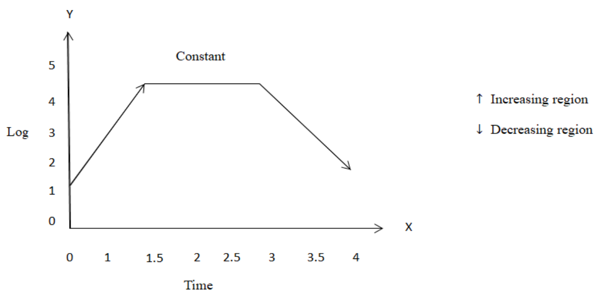


Fig.3 Graphical representation of Bacterial growth

Growth is shown as Log (numbers) where numbers represent the bacteria colony forming units per ml, along with Time. Bacteria growth varies for different timing, it remains constant for some time, and in some cases, if the temperature is low it will reduce the growth rate in bacteria this is mainly due to changes in the environment [7].

From Figure 3, Graphical representation of bacterial growth the bacterial population undergoes exponential growth based on Time. Bacteria usually undergo four stage delay stage, log stage, stationary stage and death stage. In Figure 4, t increasing region represents the *log stage*, constant region represent the *stationary stage* and ↓ decreasing region represent the *death stage* of bacteria [7, 8].

V. BACTERIAL TRANSFORMATION

The bacterial transformation was revealed by Avery et al in 1944. This bacterial transformation was first turned up in Streptococcus pneumonia by Griffith in 1928. Bacterial transformation is a procedure of oblique gene relocates by which bacteria take up external genetic material (naked DNA) from the surrounding. This procedure doesn't requisite any donor cell but the existence of lasting DNA in the surrounding. Once the DNA undergone the transformation enters the cytoplasm. This persistent bacteria is vital for bacteria to encounter transformation is its potential to use up free, intercellular genetic stuff. These types of pathogens are called as competent cells. The land agents that guide the natural competence will range between abundant genes. The nucleus will degrade if the mismatch gene taken up from the bacterial DNA. If the external genetic stuff is same to bacterial DNA it may integrate toward chromosome. Sometimes external genetic stuff coincides as a bacteriophage with genetic DNA. [1,5].

5.1 Reason for Transformation

To defeat great imbalance in population alteration and to defeat the opposition of sustain the population numbers during severe and outermost surrounding changes natural transformation are enabled. In the time of similar circumstance a few bacterial genera voluntarily deliver DNA out of cells into the surrounding free to be consuming by the part of cells. By the natural transformation process competent cells also counter to the exchange in the habitat and dominate the amount of gene addition.

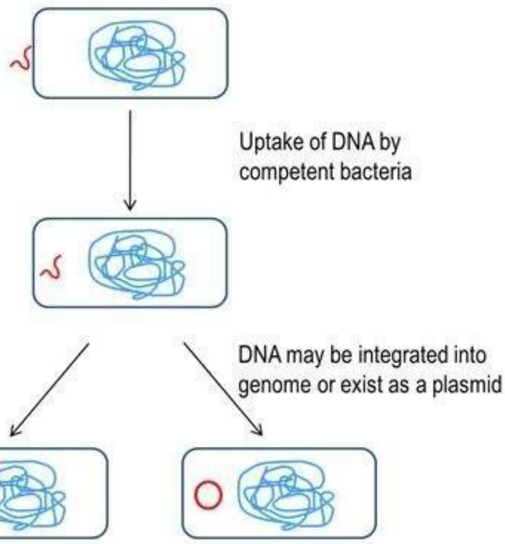
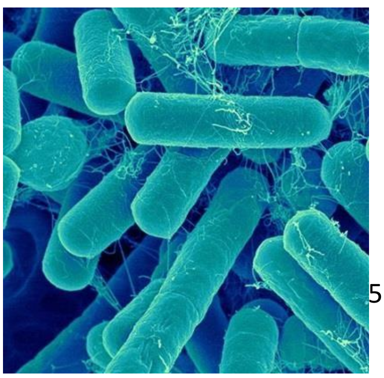


Fig.4.1 Schematic representation of transformation in bacteria



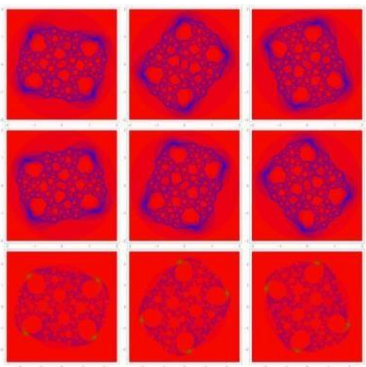


Fig 4.2 Bacterial Transformation **Fig 4.3 Julia set Transformation**

From Fig 4.2 and 4.3 As bacteria undergoes transformation to avoid imbalance in population, In Julia set Mobius transformation is engaged which is applied over and over again, a fractal shape is emerged. The range cells of bacteria remind Fractal set. So bacteria transformation is of the form Julia set.

5.2 Advantages and Disadvantages of Bacterial Growth

Some bacteria that are used as probiotics or those living in human gut are useful bacteria. Such as E.coli, Streptomyces, Lactobacillus etc. out of these E.coli helps in the food digestion. Lactobacillus is found in curd and ferment milk into cheese and yoghurt [4].

Benefits of Microbes

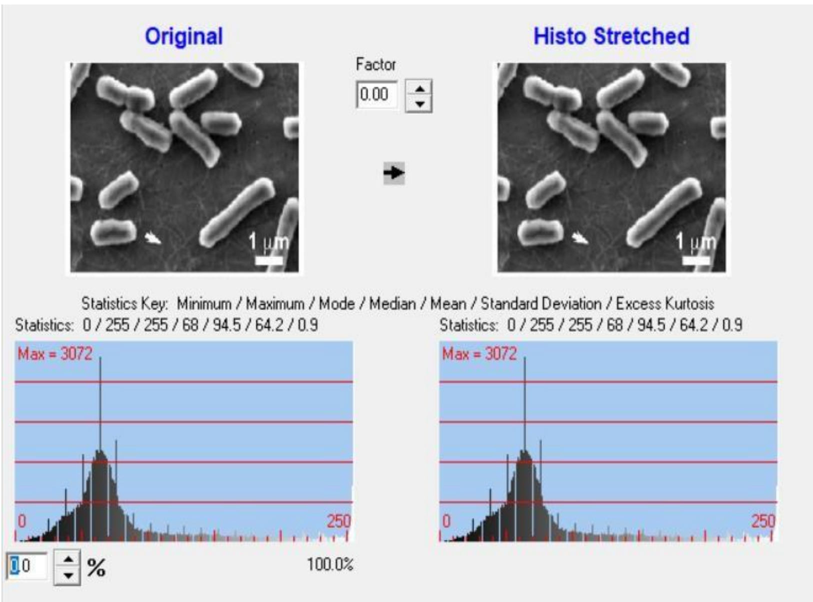
- Abet in the food digestion process.
- Purify of cheese, bread and congeal in foodservice industry
- Manufacture of firewater, citric acid, balsamic and so on for factory purpose.
- Cast – off antibiotic development and Farming of microbes.
- Environment rinse
- Abet in the nitrogen cycle and fixation of nitrogen.

There are number of bacteria that causes various diseases to living organisms, such bacteria are referred to as harmful bacteria. A disease such as cholera, typhoid, pneumonia and tuberculosis etc. human begins caused by bacteria [11].

Limitations of Microbes

- Excess of microbes cause illness in animals plants and humans
- Ruin food products
- Give rise to Tooth decay
- Basis for spreading diseases.

VI. RESULT



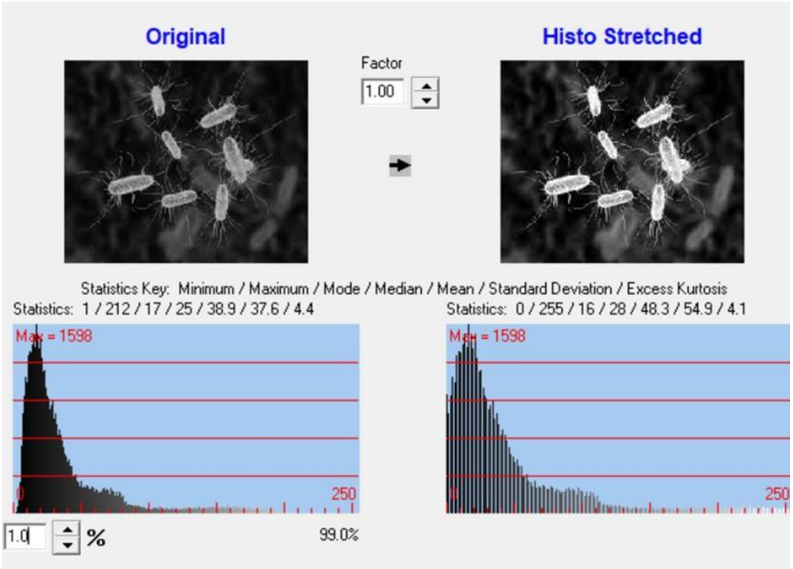


Fig 5. Bacteria image analysing by Histo Stretched Software

Table 2. Data Analysis of Bacterial cells (original) using Lacunarity

Factor	Mean	SD	Variance	1+[Variance/ (mean)^2]
0	38.9	37.6	1413.76	1.93427
1	38.9	37.6	1413.76	1.93427
0	94.5	64.2	4121.64	1.46153
1	94.5	64.2	4121.64	1.46153

Table 2(i). Data Analysis of Bacterial cells (Histo Stretched) using Lacunarity

Factor	Mean	SD	Variance	1+[Variance/ (mean)^2]
0	38.9	37.6	1413.76	1.93427
1	48.3	54.9	3014.01	2.29196
0	94.5	64.2	4121.64	1.46153

1	94.7	64.4	4147.36	1.46245

From Fig 5, By Histo Stretched software for two bacterial image are calculated which is shown in the above table. Lacunarity is calculated for both images using the formula $L(r) = 1+ [var\,(r) / mean^2(r)]$. Comparing both Lacunarity in Table 1, Table 2 and Table 2(i) the values are analogy. The Lacunarity is used for the purpose of fractal analysis of a cell or a tissue. Examine the values in the Table 1 and Table 2, Table 2(i) at the point above 1 Fractal complexity is formed. Analyzing Bacterial image satisfies the Fractal property. Bacteria undergo a transformation at a certain environment changes alike Julia set. Transformation forms a cluster pattern, growth can be analyzed by population growth Mathematical modelling. Integration growth of bacteria forms complexity. Bacteria form a Fractal texture.

VII. CONCLUSION

Presence of Bacteria everywhere in environment like in the air, water, soil, on our body and in our mouth. Spontaneous growth of Bacteria causes infections and diseases if not reduced. Generally bacteriostatic agents are used to control bacterial growth. Since some of the bacteria are helpful in food digestion and environment action, only if the bacteria are limited in growth it will be useful otherwise excess growth will lead to certain disadvantages. Doubling period and growth rate (i.e., birth and death rate) are analyzed by Population Growth model. Formation of Lacunarity is analyzed in Bacterial cell by using Histo Stretched Software as well as data calculation. Every bacterial cell duplicates the nature of Julia sets. From the Julia sets each bacterial cell are closed and bounded and so each cell are compact inside. Residence Fractals describe the complexity in the bacterial cell.

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