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OPTIMAL INVENTORY SYSTEM FOR DETERIORATING ITEMS WITH NON-LINEAR TIME-DEPENDENT DEMAND AND VARYING HOLDING COST UNDER ACCEPTABLE PAYMENT DELAY

Palanivelu Saranya[1], Ekambaram Chandrasekaran[1]

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Abstract

In practical monetary transactions, the supplier typically offers a maximum tolerable settlement delay to its retailer to stimulate sales volume. This paper examines a time-dependent nonlinear ramp curve demand and trade credit model for products having maximum life duration while accounting for these fluctuations and assuming time-varying holding costs. Besides that, many products constantly degrade and should not be sold after their expiration dates. The non-linear ramp curve demand rate is deterministic, varies with time up to a certain point in time (the breakthrough point), and then becomes constant. The breakthrough point is assumed to have occurred within the cycle length. Three situations are considered: (i) the payment settlement delay period is less than the breakthrough point of demand; (ii) the payment settlement is greater than the breakthrough point of demand; and (iii) the payment settlement is greater than the cycle length. The best optimality is compared through the numerical examples. The problem of generating inventory is presented as a nonlinear constraint optimization problem. The numerical examples are also discussed in order to make the results clearer, and the applicability span of the inventory parameters is then demonstrated using the sensitivity analysis.

Key Words: Inventory, non-linear ramp-curve demand rate, time varying holding cost, acceptable delay in payment, economic order quantity, deterioration.

1 Department of Mathematics, Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology, Chennai 600062, India;

psaranya@veltech.edu.in (P.S.); drchandrasekarane@veltech.edu.in(E.C.)

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1 Introduction

Demand plays a significant role in selecting the best stocking policy, which is a crucial component of our infrastructure for production, transportation, and retail. Researchers were hired to create the inventory models under the premise that the demand for the goods would be constant, linearly increasing or decreasing, or exponentially increasing or decreasing with time, stock-dependent, etc. Later, it was discovered that the aforementioned demand patterns did not accurately reflect the demand for some products, such as recently released fashion items, apparel, cosmetics, automobiles, etc. for which the demand increases over time as they are launched to the market and eventually becomes constant. The idea of ramp-type demand, that rises up to a certain point before stabilising and being consistent.

In the supply line of almost any business organisation, maintaining inventories of goods that are deteriorating is a significant issue. Over time, a lot of the tangible products deteriorate or degrade. Due to its crucial relationship with frequently used items in everyday living, the inventory lot-size issue for deteriorating items is significant. Examples of degrading goods include fruits, veggies, livestock, photographic films, and more. Items that are susceptible to deteriorate are frequently categorised according to how long they will last or be useful after being purchased.

The suppliers frequently employ the common payment method of a permissible payment delay, which typically results in higher sales and eventually, higher revenue. In a scenario of deteriorating products, this technique is important. Due to the trade credit offered by suppliers, retailers are typically urged to make large purchases. If the account is resolved within this time frame, there will be no interest assessed. However, interest is applied if the purchase is not completed within the given time.

Holding cost is predictable and fixed in the majority of models. But because of the regular changes in the temporal worth of money and the price index, holding costs might not always be consistent. Holding costs might not stay constant over time in this age of globalisation and intense economic rivalry.

1.1 **Aim of this study**

The aim of the present article's inventory model development is to elaborate inventory model which includes: (i) Non-linear ramp curve pattern of demand rate function, (ii) holding cost as a continually increasing function of time, (iii) constant rate of deterioration (iv)the permissible delay in payments.

1.2 Contribution

Many researchers deal with the ramp-type demand rate function with a linear increase in time and then becoming constant. This research contributes to the ramp-type demand rate function with a nonlinear function of increasing in time and then becoming constant.

1.3 Flow of the paper

The instantaneous condition of the on-hand inventory is calculated using differential equations. The total cost function and economic order quantity can be derived by taking appropriate cost factors into account. By optimising the procurement time, the model objective is to reduce the order quantity and average total cost. Also, the required and adequate conditions for the existence of the ideal solutions are given. To demonstrate the proposed approach, various examples are given. Lastly, a sensitivity analysis of the ideal solution is undertaken in combination with different factors.

2 Literature Review

Goyal (1985) is believed to be the first who developed an EOQ model under the condition of permissible delay in payments. He computed the interest earned on the sales revenue on unit purchase price. Aggarwal and Jaggi (1995) developed the optimal ordering policy under permissible delay in payments. For a seller whose end demand is price-dependent, Abad and Jaggi (2003) investigated a joint method for determining unit price and credit period length. Huang (2003) examined the best retailer purchasing procedures for trade credit financing. A number of models for acceptable payment delays have been studied by Teng (2002) and Chung & Liao (2004). Chung (2009) observed a model for decaying items with partial backlogs and allowable payment delays. In the EOQ model with payments by discount and trade credit, researchers Jammal et al. (1997), Sarker et al. (2000), Jaggi et al. (2008), Liao et al. (2007), Shah and Huang, Chung (2003), and others investigated the optimal replenishment and payment policies. Shah (2006) employed a variety of models to examine the legal limits for payment delays and concluded that the retailer can produce interest-based income by delaying payments all the way up to the last day of the permitted grace period. Shah (2010) considered the pricing and ordering practices of retailers using trade credit in a down market and concluded that the retailer should purchase damaged units at a discount and dispatch them as soon as possible. For three parameter Weibull deterioration under trade credit, Tripathy and Pradhan (2011) explored a model of the best pricing and ordering strategy. An EOQ model for linearly deteriorating rates with shortages and allowable payment delays was explored by Tripathy Mishra (2010). An inventory model with a general ramp type demand rate, a constant rate of deterioration, a partial backlog of unmet demand, along with standards for acceptable payment delays was developed by Skouri (2011). T.Singh and pattnayak (2013) investigated an EOQ model for perishable goods with time-dependent quadratic demand and variable deterioration under permitted payment delay. A pharmaceutical inventory model was developed by Uthayakumar and Karuppasamy (2017) for a quadratic demand rate and variable holding cost with acceptable late payment.

Y Shi and others (2019) developed the optimal ordering policies with ramp-type demand rate under the permissible delay in payment with different possible cases. Singh.T and others (2020) explored the inventory model taking into the variation in the rate of deterioration and the rate of demand for quadratic function depending on time. Shah,N.H. et al(2020) deals with supply chain model with constant rate of deterioration and credit period dependent demand. Mondal,B. et al(2021) presented a generalized order-level inventory system with time dependent demand and two parameter Weibull deterioration rate of deterioration under permissible delay in payment across various trade-credit intervals. Shrama.A and Kaushik.J(2021) created the inventory model under trade credit policy for the deteriorating items with price and time-dependent ramp type demand and with shortages allowed. Palanivelu, S., and Chandrasekaran, E developed the deterministic inventory model for deteriorating giffen goods with linear time-dependent demand.

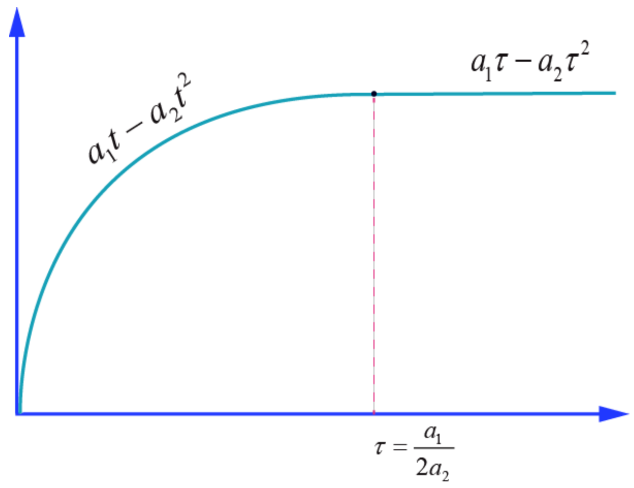

Fig.1 Two-phase demand rate function

3 Notations and Assumptions

3.1 Assumptions

- Inventory system considers only one item.
- Demand rate is characterized by time-dependent function of the non-linear ramp curve.

Fig.1 represents the demand rate curve.

- Time horizon is infinite.
- Replacement rate is unbounded.
- There has been no lead time.
- Deterioration rate is constant and no replacement during the cycle.
- Shortages are not allowed.
- Permissible delay in the payments is provided by the suppliers to the retailers.
- Permissible delay period occurs within the cycle length of time in model I
- Permissible delay period exceeds the cycle length of time in model II

3.2 Notations

- A – The ordering cost/order.
- R (t) – Deterministic Demand. $R(t) = \begin{cases} a_1t - a_2t^2 & , 0 < t \le \tau \\ D(\tau) & , \tau \le t < T \end{cases}$

Where $D(\tau) = a_1\tau - a_2\tau^2$ and $\tau \le \frac{a_1}{2a_2}$.

- η - Constant rate of deterioration.
- $q_{1,}q_{2,}q_3$ - The economic order quantities.
- $h_1t + h_2$ - Holding cost per unit per time.
- $I_1(t), I_2(t), I_3(t)$ - the inventory on the level of time t.
- OC – Ordering cost per cycle.
- ICC- Inventory carrying cost per cycle.
- IDC – Deterioration Cost per cycle.
- IE_1, IE_2, IE_3 - Interest earned during the period of permissible delay in payments.

- IC_1, IC_2, IC_3 - Interest charged after the period of permissible delay in payments.
- TIC_1, TIC_2, TIC_3 - Total inventory cost per unit time.
- T_1, T_2, T_3 - Replenishment period of cycle time.

4 Mathematical Model and Methodology

4.1 Mathematical Model

$$\frac{dI(t)}{dt} = -\eta I(t) - R(t) \tag{1}$$

$$\frac{dI(t)}{dt} = -\eta I(t) - (a_1 t - a_2 t^2), \qquad 0 < t \le \tau \tag{2}$$

$$\frac{dI(t)}{dt} = -\eta I(t) - (a_1 t - a_2 t^2), \qquad 0 < t \le \tau \tag{3}$$

With initial and boundary conditions,

$I(0) = q$ And $I(T) = 0$
(4)

On solving (2),(3) and (4)

$$I(t) = \frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2 + a_1\tau + \eta q\right) - a_1\tau + a_2\tau^2\right), \qquad 0 < t \le \tau \tag{5}$$

$$I(t) = e^{-\eta T} q - \frac{2a_2 + a_1\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 a_2}{\eta} - \frac{T\left(2a_2 + a_1\eta\right)}{\eta^2}, \qquad \tau \le t < T \tag{6}$$

With the boundary condition we can get,

$$q = \frac{1}{\eta}\left(a_2\tau^2\left(e^{\eta T} - 1\right) - a_1\tau\left(e^{\eta T} + 1\right)\right) \tag{7}$$

(5) and (6) will become,

$$I(t) = \frac{2a_2 + a_1\eta}{\eta^3}\left(1 - e^{-\eta t}\right) + \frac{e^{-\eta t}}{\eta}\left(\left(a_2\tau^2 - a_1\tau\right)\left(1 - e^{\eta T}\right)\right) + \frac{a_2 t^2}{\eta} - \frac{t\left(2a_2 + a_1\eta\right)}{\eta^2} \tag{8}$$

$$I(t) = \frac{1}{\eta}\left(a_2\tau^2 - a_1\tau - e^{-\eta(t+T)}\left(a_2\tau^2 - a_1\tau\right)\right) \tag{9}$$

Inventory Carrying Cost

$$ICC = \int_0^T \left(h_1 t + h_2\right) I(t)\, dt$$

$$= \int_0^\tau \left(h_1 t + h_2\right) I(t)\, dt + \int_\tau^T \left(h_1 t + h_2\right) I(t)\, dt$$

$$= \int_0^{\tau} \left(h_1 t + h_2 \right) \left(\frac{1}{\eta} \left(e^{-\eta T} \left(-a_2 \tau^2 + a_1 \tau + \eta q \right) - a_1 \tau + a_2 \tau^2 \right) \right) dt$$

$$+ \int_{\tau}^{T} \left(h_1 t + h_2 \right) \left(e^{-\eta T} q - \frac{2a_2 + a_1 \eta}{\eta^3} \left(1 + e^{-\eta T} \right) + \frac{T^2 a_2}{\eta} - \frac{T \left(2a_2 + a_1 \eta \right)}{\eta^2} \right) dt$$

(10)

Inventory Deterioration Cost

$$IDC = d \left(q - \int_0^T R(t)\, dt \right)$$

$$= d \left(q - \int_0^{\tau} R(t)\, dt - \int_{\tau}^{T} R(t)\, dt \right)$$

$$= d \left(q - \int_0^{\tau} \left(a_1 t - a_2 t^2 \right) dt - \int_{\tau}^{T} \left(a_1 \tau - a_2 \tau^2 \right) dt \right) \tag{11}$$

MODEL -I $P_D \le T$

The retailer starts to pay interest for the products in storage after some time P_D with the rate I_C because the credit term P_D in this instance is less than or equal to the restocking cycle time. There are two chances for the permissible delay period P_D to occur: one is before the breakthrough point, and the other is after the breakthrough point (τ). The allowable delay duration in model-I for both of the cases is shown in Fig.2 in relation to the point at which demand breakthroughs and the length of the inventory cycle.

Case (i) $\tau < P_D$

If the permissible delay period occurs after the breakthrough point of the demand,

Earned Interest during $\left[0, P_D \right]$,

$$EI_1 = PI_e \int_0^{P_D} R(T)\, dt$$

$$= PI_e \int_0^{\tau} R(t)\, dt + \int_{\tau}^{P_D} R(t)\, dt$$

$$= PI_e \int_0^{\tau} \left(a_1 t - a_2 t^2\right) dt + \int_{\tau}^{P_D} \left(a_1 \tau - a_2 \tau^2\right) dt \tag{12}$$

$Model\ I\ :\ P_d < T$

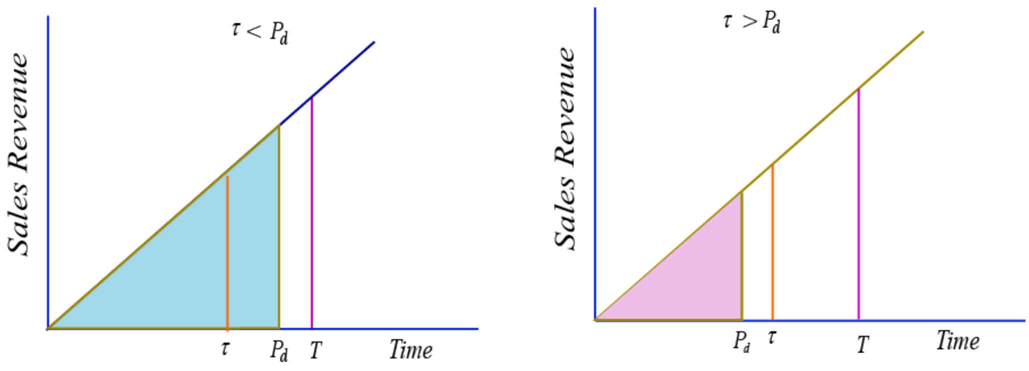

Fig.2 Acceptable delay period is less than cycle length

Total interest charges payable by the retailer during $\left[P_D, T\right]$

$$CI_1 = cI_c \int_{P_D}^{T} I(t)\, dt$$

$$= cI_c \left\{ \int_{P_D}^{\tau} \frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2 + a_1\tau + \eta q\right) - a_1\tau + a_2\tau^2\right) dt + \int_{\tau}^{T} \left(e^{-\eta T} q - \frac{2a_2 + a_1\eta}{\eta^3}\left(1 + e^{-\eta T}\right) + \frac{T^2 a_2}{\eta} - \frac{T\left(2a_2 + a_1\eta\right)}{\eta^2} \right) dt \right\} \tag{13}$$

Case (ii) $\tau > P_D$

If the breakthrough point of the demand exceeds the permissible delay period then

Earned Interest during $\left[0, P_D\right]$,

$$EI_2 = PI_e \int_0^{P_D} R(T)\, dt$$

$$= PI_e \int_0^{P_D} \left(a_1 t - a_2 t^2\right) dt \tag{14}$$

Total interest charges payable by the retailer during $\left[P_D, T\right]$

$$CI_2 = cI_c \int_{P_D}^{T} I(t)\, dt$$

$$= cI_c \left\{ \int_{P_D}^{\tau} I(t)\, dt + \int_{\tau}^{T} I(t)\, dt \right\}$$

$$= cI_c\left\{\int_{P_D}^{\tau}\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)dt+\int_{\tau}^{T}\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt\right\} \tag{15}$$

Inventory Total Cost

$$ITC_1=\frac{1}{T_1}\left(OC+ICC+IDC-EI_1+CI_1\right)$$

$$=\frac{1}{T_1}\left(\begin{array}{l} k+\int_0^{\tau}\left(h_1t+h_2\right)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)\right)dt \\ \quad+\int_{\tau}^{T}\left(h_1t+h_2\right)\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt+ \\ d\left(q-\int_0^{\tau}\left(a_1t-a_2t^2\right)dt-\int_{\tau}^{T}\left(a_1\tau-a_2\tau^2\right)dt\right) \\ -PI_e\int_0^{\tau}\left(a_1t-a_2t^2\right)dt+\int_{\tau}^{P_D}\left(a_1\tau-a_2\tau^2\right)dt \\ +cI_c\left\{\int_{P_D}^{\tau}\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)dt+\int_{\tau}^{T}\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt\right\} \end{array}\right) \tag{16}$$

$$ITC_2=\frac{1}{T_2}\left(OC+ICC+IDC-EI_2+CI_2\right)$$

$$=\frac{1}{T_2}\left(\begin{array}{l} k+\int_0^{\tau}\left(h_1t+h_2\right)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)\right)dt \\ \quad+\int_{\tau}^{T}\left(h_1t+h_2\right)\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt+ \\ d\left(q-\int_0^{\tau}\left(a_1t-a_2t^2\right)dt-\int_{\tau}^{T}\left(a_1\tau-a_2\tau^2\right)dt\right) \\ -PI_e\int_0^{P_D}\left(a_1t-a_2t^2\right)dt \\ +cI_c\left\{\int_{P_D}^{\tau}\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)dt+\int_{\tau}^{T}\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T_2{}^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt\right\} \end{array}\right) \tag{17}$$

Model –II $P_D > T_3$

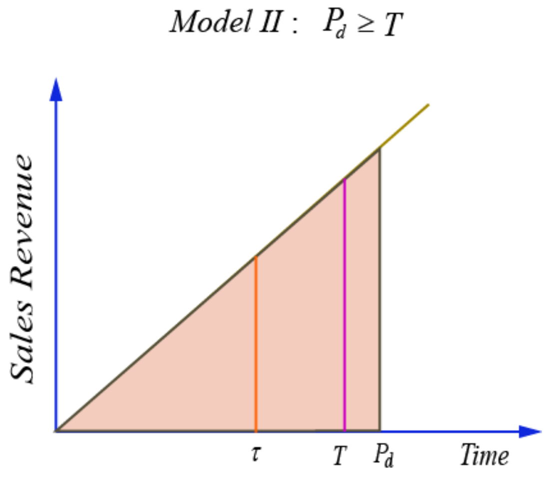

Fig.3 Permissible delay period greater than the cycle length

The permissible delay duration in relation to the point at which demand breakthroughs and the length of the inventory cycle are represented in Fig.3.

If the permissible delay period exceeds the replenishment cycle length, then the charged interest is zero.

Earned interest during this period $[0, P_D]$,

$$EI_3 = PI_e \left\{ \int_0^{T_3} tR(t)\,dt + (P_D - T) \int_{T_3}^{P_D} R(t)\,dt \right\}$$

$$= PI_e \left\{ \int_0^{\tau} tR(t)\,dt + \int_{\tau}^{T_3} tR(t)\,dt + (P_D - T) \int_{T_3}^{P_D} R(t)\,dt \right\} \quad (18)$$

$$EI_3 = PI_e \left\{ \int_0^{T_3} tR(t)\,dt + (P_D - T) \int_0^{T_3} R(t)\,dt \right\}$$

$$= PI_e \left\{ \int_0^{\tau} t\left(a_1 t - a_2 t^2\right) dt + \int_{\tau}^{T_3} t\left(a_1 \tau - a_2 \tau^2\right) dt + (P_D - T) \int_{T_3}^{P_D} \left(a_1 \tau - a_2 \tau^2\right) dt \right\} \quad (19)$$

Charged interest during this period $[0, P_D]$, $CI_3 = 0$ (20)

$$ITC_3 = \frac{1}{T_3}\left(OC + ICC + IDC - EI_3 + CI_3\right)$$

$$=\frac{1}{T_3}\left(\begin{array}{l} k+\int_0^{\tau}\left(h_1t+h_2\right)\left(\frac{1}{\eta}\left(e^{-\eta T}\left(-a_2\tau^2+a_1\tau+\eta q\right)-a_1\tau+a_2\tau^2\right)\right)dt \\ +\int_{\tau}^{T}\left(h_1t+h_2\right)\left(e^{-\eta T}q-\frac{2a_2+a_1\eta}{\eta^3}\left(1+e^{-\eta T}\right)+\frac{T^2a_2}{\eta}-\frac{T\left(2a_2+a_1\eta\right)}{\eta^2}\right)dt+ \\ d\left(q-\int_0^{\tau}\left(a_1t-a_2t^2\right)dt-\int_{\tau}^{T}\left(a_1\tau-a_2\tau^2\right)dt\right) \\ -PI_e\left\{\int_0^{\tau}t\left(a_1t-a_2t^2\right)dt+\int_{\tau}^{T_3}t\left(a_1\tau-a_2\tau^2\right)dt+\left(P_D-T_3\right)\int_{T_3}^{P_D}\left(a_1\tau-a_2\tau^2\right)dt\right\} \end{array}\right)$$

(21)

4.2 Methodology and Algorithm

The aim is to attain the minimum total inventory cost in this work. The necessary condition to for the minimization of total inventory cost for the all cases of models respectively is

$$\frac{dTIC_1}{dT_1}=0,\quad \frac{dTIC_2}{dT_2}=0\ ,\ \frac{dTIC_3}{dT_3}=0\ . \tag{22}$$

In order to attain the minimal cost mathematical model to be satisfied the sufficient condition,

$$\frac{d^2TIC_1}{dT_1^2}\geq 0,\quad \frac{d^2TIC_2}{dT_2^2}\geq 0\ ,\ \frac{d^2TIC_3}{dT_3^2}\geq 0\ . \tag{23}$$

Since the expression (22) is highly non-linear in ‘t’, it is very difficult to solve analytically. By assigning numerical values to the various parameters of the mathematical model, the equation (22) can be solved numerically. The following algorithm is followed in MATLAB software to solve the problem in numerical way.

4.2.1 Algorithm:

- Define the decision variables the mathematical model.
- Assign the appropriate numerical values to the different parameters of the developed model.
- Solve the governing differential equation of the model.
- Obtain the initial order quantity ‘q’ by using the initial and boundary conditions.
- Obtain the inventory level I(t) at any time ‘t’.
- Calculate the inventory holding cost, inventory deterioration cost through the obtained inventory level I(t).
- Calculate the interest payable and the interest earned through the permissible delay in payments offered by the supplier.
- Write down the total inventory cost per unit time
- Apply the optimization tools (e.g. fminsearch, fmincon, etc.,) to get the optimal total inventory cost and optimal cycle length of time.

5 Numerical Examples and Sensitivity Analysis

Numerical examples are provided assuming with different values of model parameters.

5.1 Numerical Examples

Example: 1 Model – I: Case –I ($\tau \le P_d \le T_1$)

Assuming the values $a_1 = 200$, $a_2 = 90$, $\eta = 0.02$, $d = 20$, $\tau = 0.25$, $h_1 = 0.5$, $h_2 = 3$, $k = 500$, $P_d = 0.3$, $i_e = 0.08$, $c = 30$, $i_c = 0.25$, $p = 50$, We get the optimal cycle length $T_1^* = 1.5489$

By using this optimum cycle length value, we have obtained optimal inventory cost $TIC_1^* = 685.12$ and economic order quantity $q_1^* = 69.81$.

Example: 2 Model – I: Case –II ($P_d \le \tau \le T_1$)

Assuming the values $a_1 = 200$, $a_2 = 90$, $\eta = 0.02$, $d = 20$, $\tau = 0.5$, $h_1 = 0.5$, $h_2 = 3$, $k = 500$, $P_d = 0.3$, $i_e = 0.08$, $c = 30$, $i_c = 0.25$, $p = 50$ we get the optimal cycle length $T_1^* = 1.5489$

By using this optimum cycle length value, we have obtained optimal inventory cost and economic order quantity.

Example:3 Model – II: ($\tau \le T_1 \le P_d$)

Assuming the values $a_1 = 200$, $a_2 = 100$, $\eta = 0.02$, $d = 20$, $\tau = 0.5$, $h_1 = 0.5$, $h_2 = 3$, $k = 500$, $P_d = 1.8$, $i_e = 0.08$, $c = 30$, $i_c = 0.25$, $p = 50$ we get the optimal cycle length $T_3^* = 1.7989$

By using this optimum cycle length value, we have obtained optimal inventory cost $TIC_3^* = 461.46$ and economic order quantity $q_3^* = 137.37$.

By the numerical values total inventory cost, holding cost and deterioration cost are plotted for the developed inventory models. Fig.4 represents the convexity of total inventory cost function for the model-I: case(i) permissible delay period is less than the cycle length and the breakthrough point. Fig.5 explains the convexity of the total inventory cost for the model-I: case(ii) permissible delay period is less than the cycle length and greater than the demand breakthrough point. Fig.6 clarifies that the convexity of total inventory cost function for the model-II permissible delay period ends after the cycle length of time.

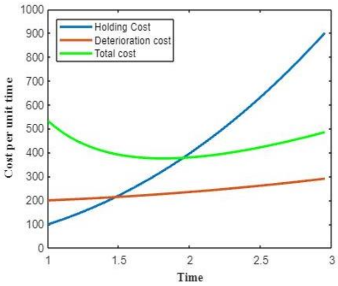

Fig.4 Total Cost Curve case(i)

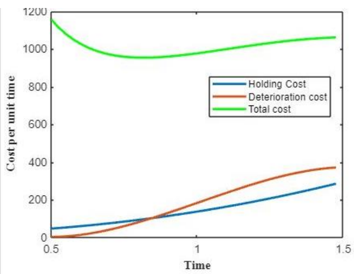

Fig.5 Total Cost Curve case (ii)

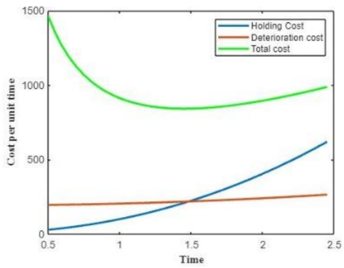

Fig.6 Total Cost Curve Model-II

5.2 Sensitivity Analysis

Sensitivity Analysis: Model I: Case –(i)

Parameter	Value	T^*	TIC^*	q*
a_1	160	1.7141	599.11	59.94
	180	1.6233	643.10	64.97
	200	1.5489	685.12	69.81
	220	1.4866	725.56	74.51
	240	1.4337	764.70	79.09
a_2	70	1.5336	696.08	71.06
	80	1.5412	690.61	70.43
	90	1.5489	685.12	69.81
	100	1.5568	679.61	69.18
	110	1.5649	674.08	68.55
ɳ	0.01	1.5685	676.80	70.15
	0.015	1.5586	680.97	69.98
	0.02	1.5489	685.12	69.81
	0.025	1.5393	689.26	69.64
	0.03	1.5298	693.40	69.47
h_1	0.3	1.5627	681.50	70.44
	0.4	1.5557	683.32	70.12
	0.5	1.5489	685.12	69.81
	0.6	1.5422	686.91	69.50
	0.7	1.5357	688.69	69.20
h_2	1	1.6977	611.35	76.63
	2	1.6182	649.07	72.98
	3	1.5489	685.12	69.81
	4	1.4878	719.73	67.02
	5	1.4335	753.06	64.53

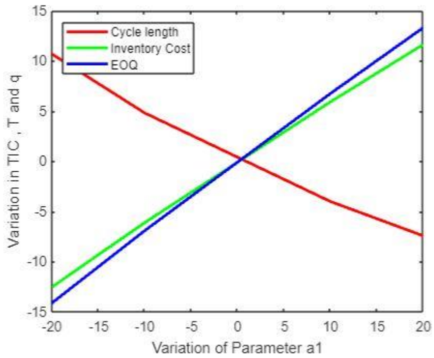

Fig.7 Influence of a_1

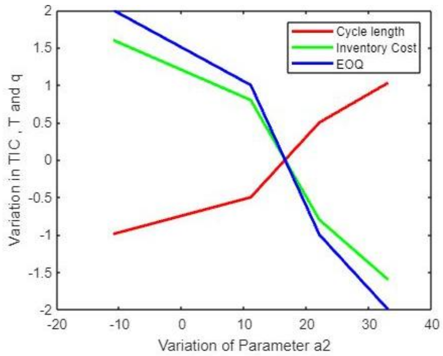

Fig.8 Influence of a_2

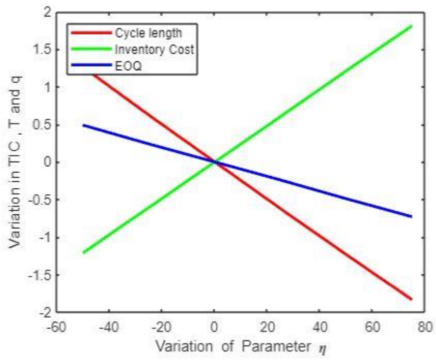

Fig.9 Influence of η

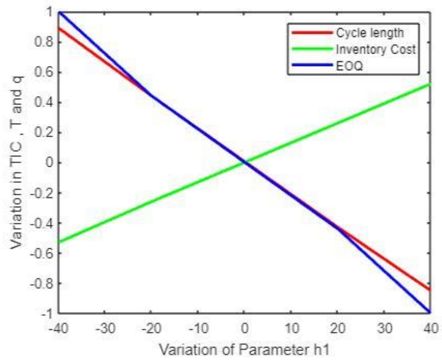

Fig.10 Influence of h_1

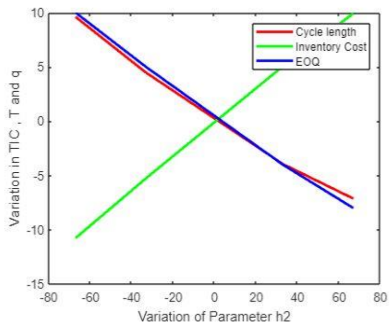

Fig.11 Influence of h_2

The influence of model parameters a1, a2, η, h1, and h2 on the decision variable cycle length, economic order quantity and total inventory cost is shown in Fig.7,8,9,10, and 11 for model-I case-(i).

Model- I - Case- (ii)

Parameter	Value	T^*	TIC^*	q*
a_1	180	1.001	836.18	57.38
	200	0.7818	872.61	45.65
	220	0.7196	906.48	44.95
	240	0.6683	935.72	44.02
a_2	80	0.7140	900.69	41.66
	90	0.7446	887.74	43.65
	100	0.7818	872.61	45.65
	110	0.8162	852.95	47.16
	120	0.9101	823.12	60.75
η	0.01	0.9999	922.90	67.10
	0.015	0.9992	927.47	67.21
	0.02	0.7818	872.61	45.65
	0.025	0.7624	875.38	43.89
	0.03	0.7615	878.55	43.90
h_1	0.3	0.7523	930.76	43.52
	0.4	0.7664	871.75	44.16
	0.5	0.7818	872.61	45.65
	0.6	0.8232	878.91	49.70
	0.7	0.9421	934.86	51.72
h_2	1	0.7965	825.37	47.07
	2	0.7843	849.20	45.89
	3	0.7818	872.61	45.65
	4	0.7626	894.89	43.81
	5	0.7430	916.14	41.94

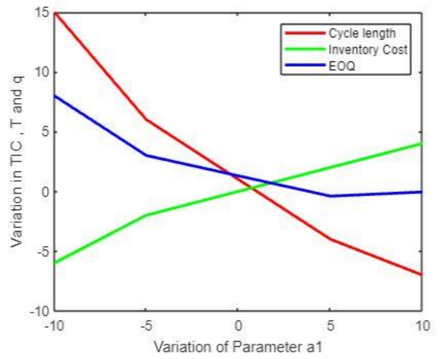

Fig.12 Influence of a_1

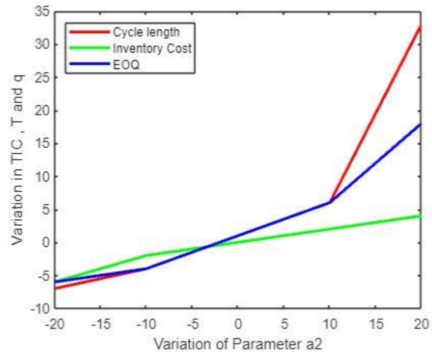

Fig.13 Influence of a_2

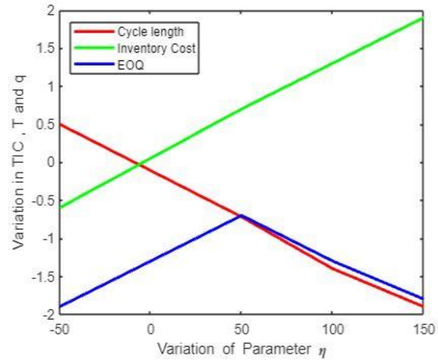

Fig.14 Influence of η

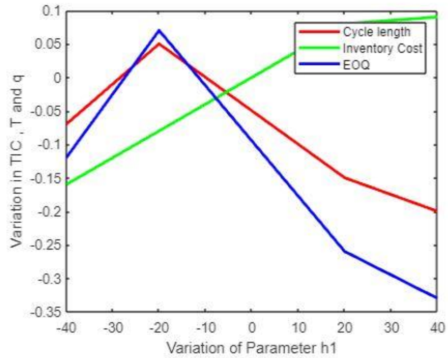

Fig.15 Influence of h_1

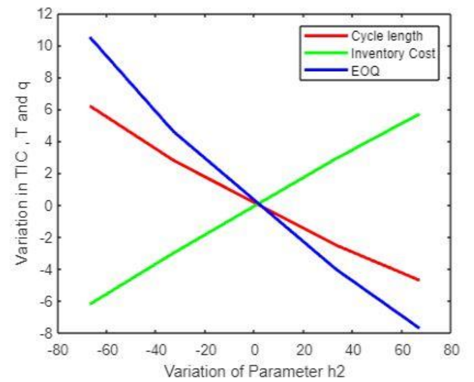

Fig.16 Influence of h_2

The influence of model parameters a1, a2, ŋ, h1, and h2 on the decision variable cycle length, economic order quantity and total inventory cost is shown in Fig.12,13,14,15, and 16.

Model-II

Parameter	Value	T^*	TIC^*	q*
a_1	180	1.8572	433.11	122.99
	200	1.8262	447.42	130.20
	220	1.7989	461.46	137.37
	240	1.7746	475.27	144.51
	260	1.7528	488.88	151.63
a_2	80	1.7846	480.39	145.35
	90	1.7915	470.94	141.36
	100	1.7989	461.46	137.37
	110	1.8067	451.97	133.38
	120	1.8150	442.46	129.38
ɳ	0.01	1.8359	446.30	138.20
	0.02	1.7989	461.46	137.37
	0.03	1.7725	476.56	136.53
	0.04	1.7468	491.57	135.69
	0.05	1.7218	506.51	134.86
h_1	0.3	1.8254	452.99	139.44
	0.4	1.8119	457.26	138.39
	0.5	1.7989	461.46	137.37
	0.6	1.7863	465.62	136.40
	0.7	1.7742	469.71	135.45
h_2	1	2.0538	309.82	157.25
	2	1.9134	387.98	146.29
	3	1.7989	461.46	137.37
	4	1.7033	531.11	129.95
	5	1.6221	597.56	123.65

The influence of model parameters a1, a2, ɳ, h1, and h2 on the decision variable cycle length, economic order quantity and total inventory cost is shown in Fig.12,13,14,15, and 16 for model-II.

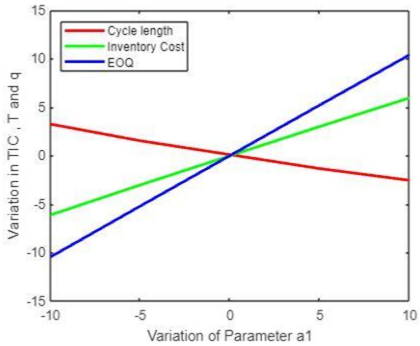

Fig.17 Influence of a_1

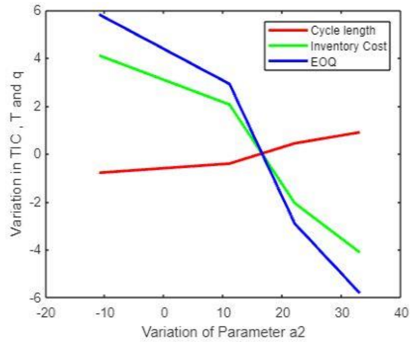

Fig.18 Influence of a_2

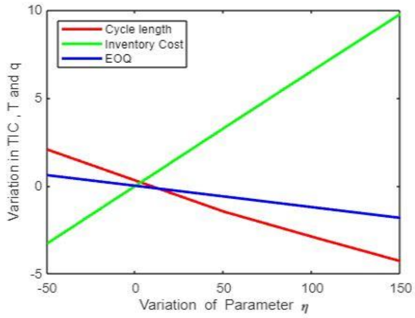

Fig.19 Influence of η

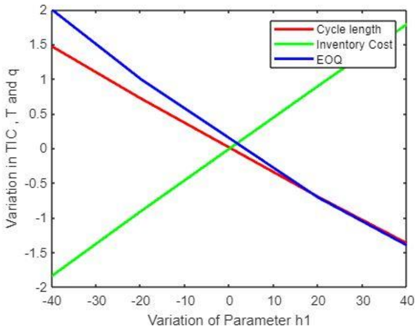

Fig.20 Influence of h_1

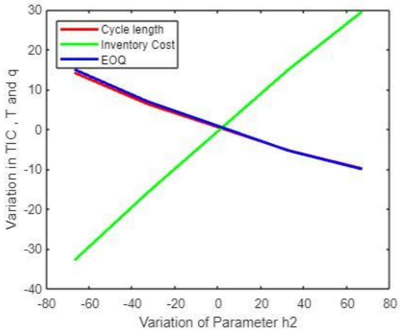

Fig.21 Influence of h_2

5.3 Discussion of Results

Model-I

- T_1^* ,TIC_1^*,q_1^* are highly sensitive with the change of parameter a_1.
- T_1^* ,TIC_1^*,q_1^* are low sensitive with the change of parameter a_2.
- T_1^* ,TIC_1^*,q_1^* are highly sensitive with the raise in the deterioration parameter η.
- As h_1 increases, T_1^* and q_1^* decrease. But TIC_1^* increases.
- T_1^* ,TIC_1^*,q_1^* are highly sensitive with the holding cost parameter h_2.
- The permissible delay period in payments plays the vital role in total inventory cost. As the period P_d increases, total inventory cost get decreased.
- The demand breakthrough point increases total inventory cost also increase in order to satisfy the demand.

Model-II

- T_1^* , TIC_1^* , q_1^* are moderately sensitive with the change of demand parameters a_1 and a_2.
- Impact on T_1^* , TIC_1^* , q_1^* is very high with the raise in the deterioration parameter η.
- T_1^* , TIC_1^* , q_1^* are highly sensitive with the holding cost parameters h_1 and h_2 .
- T_1^* , TIC_1^* , q_1^* are highly sensitive with the permissible delay period in payments.
- T_1^* , TIC_1^* , q_1^* meet the high impact with the change in the demand breakthrough point τ .

6. Conclusion

This study can significantly assist merchants or purchasers in selecting when to make payments by considering the advantages of a reasonable payment delay. The model taken into consideration above is appropriate for goods with time-varying demand and a constant rate of deterioration. This model can be applied to things like the most recent technological innovations, pharmaceutical products, fruits, and vegetables. The optimal replenishment time is found using this methodology. The total inventory cost can be calculated using calculus techniques, incorporating the optimal cycle length that was obtained. The feasibility of the proposed model is demonstrated by the various numerical examples.

The approach suggested can be expanded to include goods with linearly rising demand, demand that is dependent on stock, and demand that is dependent on price and allows for shortages. The Weibull or Gamma distribution of the deterioration rate might be added to this model in order to further proceed. The investigation of two-phase demand (first phase non-linear time-dependent and second phase constant) under acceptable payment delay will be initiated by this study.

References

1. Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *Journal of the operational Research Society*, *46*(5), 658-662.
2. Abad, P. L., & Jaggi, C. K. (2003). A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. *International Journal of Production Economics*, *83*(2), 115-122.
3. Huang, Y. F. (2003). Optimal retailer's ordering policies in the EOQ model under trade credit financing. *Journal of the Operational research society*, *54*(9), 1011-1015.
4. Teng, J. T. (2002). On the economic order quantity under conditions of permissible delay in payments. *Journal of the operational research society*, *53*(8), 915-918.
5. Huang, Y. F. (2003). Optimal retailer's ordering policies in the EOQ model under trade credit financing. *Journal of the Operational research society*, *54*(9), 1011-1015.
6. Teng, J. T. (2002). On the economic order quantity under conditions of permissible delay in payments. *Journal of the operational research society*, *53*(8), 915-918.
7. Chung, K. J., & Liao, J. J. (2004). Lot-sizing decisions under trade credit depending on the ordering quantity. *Computers & Operations Research*, *31*(6), 909-928.
8. Chung, K. J. (2009). A complete proof on the solution procedure for non-instantaneous deteriorating items with permissible delay in payment. *Computers & Industrial Engineering*, *56*(1), 267-273.

9. Jamal, A. M. M., Sarker, B. R., & Wang, S. (1997). An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *Journal of the operational research society*, *48*(8), 826-833.
10. Sarker, B. R., Jamal, A. M. M., & Wang, S. (2000). Optimal payment time under permissible delay in payment for products with deterioration. *Production Planning & Control*, *11*(4), 380-390.
11. Jaggi, C. K., Goyal, S. K., & Goel, S. K. (2008). Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. *European Journal of Operational Research*, *190*(1), 130-135.
12. Liao, J. J. (2007). On an EPQ model for deteriorating items under permissible delay in payments. *Applied Mathematical Modelling*, *31*(3), 393-403.
13. Chung, K. J., & Huang, Y. F. (2003). The optimal cycle time for EPQ inventory model under permissible delay in payments. *International Journal of Production Economics*, *84*(3), 307-318.
14. Shah, N. H. (2006). Inventory model for deteriorating items and time value of money for a finite time horizon under the permissible delay in payments. *International Journal of Systems Science*, *37*(1), 9-15.
15. Tripathy, P. K., & Pradhan, S. (2011). An integrated partial backlogging inventory model having Weibull demand and variable deterioration rate with the effect of trade credit. *International Journal of Scientific and Engineering Research*, *2*(4), 01-05.
16. Tripathi, R. P., Misra, S. S., & Shukla, H. S. (2010). A cash flow oriented EOQ model under permissible delay in payments. *International Journal of Engineering, Science and Technology*, *2*(11).
17. Skouri, K., Konstantaras, I., Papachristos, S., & Teng, J. T. (2011). Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments. *Expert Systems with Applications*, *38*(12), 14861-14869.
18. Singh, T., & Pattnayak, H. (2013). An EOQ model for a deteriorating item with time dependent quadratic demand and variable deterioration under permissible delay in payment. *Applied Mathematical Science*, *7*(59), 2939-2951.
19. Uthayakumar, R., & Karuppasamy, S. K. (2017). A pharmaceutical inventory model for variable demand and variable holding cost with partially backlogged under permissible delay in payments in healthcare industries. *International Journal of Applied and Computational Mathematics*, *3*, 327-341.
20. Shi, Y., Zhang, Z., Zhou, F., & Shi, Y. (2019). Optimal ordering policies for a single deteriorating item with ramp-type demand rate under permissible delay in payments. *Journal of the Operational Research Society*, *70*(10), 1848-1868.
21. Singh, T., Muduly, M. M., Asmita, N., Mallick, C., & Pattanayak, H. (2020). A note on an economic order quantity model with time-dependent demand, three-parameter Weibull distribution deterioration and permissible delay in payment. *Journal of Statistics and Management Systems*, *23*(3), 643-662.
22. Shah, N. H., Chaudhari, U., & Cárdenas-Barrón, L. E. (2020). Integrating credit and replenishment policies for deteriorating items under quadratic demand in a three-echelon supply chain. *International Journal of Systems Science: Operations & Logistics*, *7*(1), 34-45.
23. Mondal, B., Garai, A., & Roy, T. K. (2021). Optimization of generalized order-level inventory system under

fully permissible delay in payment. *RAIRO-Operations Research*, *55*, S195-S224.

24. Sharma, A., & Kaushik, J. (2021). Inventory model for deteriorating items with ramp type demand under permissible delay in payment. *International Journal of Procurement Management*, *14*(5), 578-595.

25. Palanivelu, S., & Chandrasekaran, E. (2022). A Deterministic Replenishment Policy for Constant Deteriorating Giffen Goods with Time-Dependent Demand. In *Proceedings of 2nd International Conference on Mathematical Modeling and Computational Science: ICMMCS 2021* (pp. 271-280). Singapore: Springer Nature Singapore.