PRESENCE OF FIXED POINT THEOREMS IN FUZZY BANACH SPACE USING COMMON LIMIT RANGE AND E.A PROPERTY

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Abstract

The purpose of this paper is to obtain common fixed point theorems for weakly compatible mappings satisfying the property E.A and CLR_S using implicit relation in Fuzzy Banach Space. Property E.A and CLR_S plays a major role in fixed point theorems and by using Common Limit Range property and E.A we can obtain fixed points in Fuzzy Banach Space.

Keywords: Common fixed point; Weakly Compatible Maps; Fuzzy Banach Space; Property E.A and CLR_S.

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1. Introduction

The concept of Fuzzy Sets was introduced by Zadeh in 1965, which plays a major role in almost all branches of Science and Engineering. Katsaras (1984) and Congxin and Ginxuan (1984) independently introduced the definition of fuzzy norms. Sintunavarat and Kumam introduced the new concept called common limit range property. M.Aamri and D.El Moutawakil defined a property E.A for self maps. For the reader convenience, we recall some terminology from the theory of fuzzy Banach space.

Preliminaries

Definition 2.1. [4]

Let D be a vector space over a field K (where K is R or C) and * be a continuous t-norm. A fuzzy set N in $DX[0,\infty]$ is called a fuzzy norm on D if it satisfies the following conditions:

(i) $N(p,0) = 0 \; \forall \; p, \in D$

(ii) $N(p,t) = 1$ for every t > 0 iff p = 0.

(iii) N(λp,t) = N(q ,$\frac{t}{|\lambda|}$) fo every p Є D,t > 0 and λ Є K

(iv)$N(p+q,t+s) \geq N(p,t) * N(q,s) \forall p,q \in D$ and $t,s \geq 0$

(v) For every $p \in D, N(p,.)$ is left continuous and $\lim_{t\to\infty} N(p,t) = 1$

The triple $(D,N,*)$ will be called fuzzy normed linear space ($FNLS$)

Definition 2.2.[4]

A sequence $\{P_n\}$ in a $FNLS(D,N,*)$ is converge to $p \in D$ if and only if $\lim_{n\to\infty} N(P_n - p,t) = 1 \forall t > 0$

Definition 2.3. [4]

Let $(D,N,*)$ be a $FNLS$. A sequence $\{P_n\}$ in D is called a fuzzy Cauchy sequence if and only if $\lim_{m,n\to\infty} N(P_m,P_n,t) = 1 \forall t > 0$

Definition 2.4. [4]

A linear fuzzy normed space which is complete is called a fuzzy Banach Space.

Definition 2.5.[4]

Self mappings A and S of a fuzzy Banach Space $(D,N,*)$ are said to be weakly commuting if $N(ASp,SAp,t) \geq N(Ap - Sp,t) \forall p \in D$ and $t > 0$.

Definition 2.6.[4]

Self mapping A and S of a fuzzy Banach Space $(D,N,*)$ are said to be compatible if and only if $\lim_{n\to\infty}(ASp_n,SAp_n,t) = 1 \forall t > 0$. Whenever $\{P_n\}$ is a sequence in D such that $Ap_n, Sp_n \to p$ for some $p \in D$ as $n \to \infty$.

Definition 2.7.[4]

Two Self maps A and S are said to be commuting if $ASp = SAp$ for all $p \in D$.

Definition 2.8.[4]

Let A and S be two self maps on a set D, if $Ap = Sp$ for some $p \in D$ then p is called a coincidence point of A and S.

Definition 2.9.[4]

Two Self maps A and S of a fuzzy Banach Space $(D,N,*)$ are said to be weakly compatible if they commute at their coincidence points. That is if $Ap = Sp$ for some $p \in D$ then $ASp = SAp$.

Definition 2.10.[4]

Suppose A and S be two Self mappings of fuzzy Banach Space $(D,N\,*)$. A point $p \in D$ is called a coincidence point of A and S if and only if $Ap = Sp$, then $w = Ap = Sp$ is called a point of Coincidence of A and S.

Definition 2.11.[9]

A pair (A,S) of Self mapping of a fuzzy Banach Space $(D,N,*)$ is said to satisfy (briefly, (CLR_S) property) if there exist a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = z$ where $z \in S(D)$.

Definition 2.12.[9]

Two pairs (A,S) and (B,T) of a self mappings of a fuzzy Banach Space $(D,N,*)$ is said to satisfy the common limit range property with respect to mapping S and T (briefly, (CLR_S) property), if there exist a sequence $\{p_n\},\{q_n\}$ in D. Such that $\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = \lim_{n\to\infty} Bq_n = \lim_{n\to\infty} Tq_n = z$ where $z \in S(D) \cap T(D)$.

Definition 2.12.[1]

A pair (A,S) of Self mapping of a fuzzy Banach Space $(D,N,*)$ is said to satisfy E.A property if there exist a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = z$ Where $z \in D$.

Definition 2.13.[1]

Two pairs (A,S) and (B,T) of self-mappings of a fuzzy Banach Space $(D,N,*)$ is said to satisfy E.A property , if there exists a sequence $\{p_n\},\{q_n\}$ in D. Such that
$\lim_{n\to\infty} Ap_n = \lim_{n\to\infty} Sp_n = \lim_{n\to\infty} Bq_n = \lim_{n\to\infty} Tq_n = z$ where $z \in D$.

Implicit relations : [5]
Let $\{\emptyset\}$ be the set of all real continuous function $\emptyset: (R^+)^6 \to R^+$ satisfying the following condition:
(i) $\emptyset(u,v,u,v,v,u) \geq 0$ imply $u \geq v$ for all $u,v \in [0,1]$
(ii) $\emptyset(u,v,v,u,u,v) \geq 0$ imply $u \geq v$ for all $u,v \in [0,1]$
(iii) $\emptyset(u,u,v,v,u,u) \geq 0$ imply $u \geq v$ for all $u,v \in [0,1]$

Theorem 3.1.
Let $(D,N,*)$ be a fuzzy Banach space and $K,L: D \to D$ be two self-mapping satisfying the following conditions;
(i) Pair (K,L) satisfies property (CLR_L).
(ii) For some $\phi \in (\phi)$ and for all $p,q \in D$ and every $t > 0$, $N(Kp,Lq,kt) \geq \emptyset[\min\{N(Kp,Kq,t),N(Kq,Lq,t),N(Kp,Lp,t),N(Lp,Lq,t)\}]$
(iii) If $K(D)$ is a closed subset of D then pairs (K,L) has a coincidence point in D.
Moreover, if (K,L) is weakly compatible, then mapping K and L have a unique common fixed point in D.
Proof:
Suppose that (K,L) satisfies the property (CLR_L)., then there exists a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} Lp_n = z$ for some $z \in D$.
Since $K(D)$ is a closed subset of D, there exists a point $u \in D$ such that $Ku = z$ for some $z \in D$.
$\therefore Kp_n = Lp_n = z = Ku$
To prove $Lu = z$

$$\phi\{N(Kp,Lq,t),N(Kp,Kq,t),N(Kq,Lq,t),N(Kp,Lp,t),N(Kq,Lp,t),N(Lp,Lq,t)\} \geq 0$$

Put $p = p_n$ and $q = u$ in equation (ii)
$\therefore Lu = z$
Now we have $Lu = Ku = z$
Since (K,L) is weakly compatible,
$LKu = KLu.$
$\Rightarrow Lz = Kz$
$\therefore z$ is a coincidence point of (K,L).
Put $p = p_n$ and $q = z$ in equation (ii),We have
$\therefore Lz = z$
$\therefore Kz = z$ $[\because Kz = Lz = z]$
$\therefore z$ is a common fixed point in D.
Next, to prove the uniqueness
Let w be another common fixed point in D.
Put $p = z$ and $q = w$ in (ii) we have
$\Rightarrow z = w$
Hence z is a common unique fixed point of K,L in D.

Theorem 3.2.
Let $(D,N,*)$ be a fuzzy Banach space and $K,L,R: D \to D$ be three self-mapping satisfying the following One of the pairs (K,R) and (L,R) satisfies property (CLR_R).and $A(D) \subseteq R(D)$ and $L(D) \subseteq R(D)$.
(i) For some $\phi \in (\phi)$ and for all $p,q \in D$ and every $t > 0$, $N(Kp,Lq,kt) \geq \emptyset[\min\{N(Kp,Rp,t),N(Rq,Lq,t),N(Lq,Rp,t),N(Kp,Rq,t)\}]$
(ii) If $R(D)$ is a closed subset of D then pairs (K,R) and (L,R) have a coincidence point in D.
Moreover, if (K,R) and (L,R) are weakly compatible, then mapping K,L and R have a unique common fixed point in D.
Proof:
Suppose that (K,R) satisfies the property (CLR_R).,then there exists a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} Rp_n = z$ for some $z \in D$.
Since $L(D) \subseteq R(D)$, there exists a sequence $\{q_n\}$ in D such that $Lq_n = Rp_n$
$\therefore Kp_n = Rp_n = Lq_n = z$
Since $R(D)$ is a closed subset of D, then there exists a point $u \in D$ such that $Ru = z$.
$\therefore Kp_n = Rp_n = Lq_n = z = Ru$
To prove $Lu = z$

Put $p = p_n$ and $q = u$ in equation (ii), we have
$\therefore Lu = z$
$\therefore Lu = Ru = z$
$\therefore z$ is a coincidence point of (K, R).
Since (L, R) is weakly compatible, we have
$LRu = RLu.$
$\Rightarrow Lz = Kz$
Put $p = p_n$ and $q = z$ in equation (ii), we have
$\therefore Rz = z$
$\therefore Lz = z$
Put $p = z$ and $q = u$ in (ii) we have
$Kz = z$
Hence z is a common fixed point of K, L, R in D.
To prove the uniqueness
Let w be another common fixed point in D.
Put $p = z$ and $q = w$ in (ii) we have
$w = z$
Hence z is a unique common fixed point of K, L, R in D.

Theorem 3.3.
Let $(D, N, *)$ be a fuzzy Banach space and $K, M, T, R, S: D \to D$ be five self-mapping satisfying the following conditions;

(i) One of pairs (K, TR) and (K, MS) satisfies property (CLR_{TR}).and (CLR_{MS}). such that $K(D) \subseteq MS(D)$ and $K(D) \subseteq TR(D)$.
(ii) For some $\phi \in (\phi)$ and for all $p, q \in D$ and every $t > 0$,
$$\phi\left\{\begin{matrix} N(Kp, Kq, t), N(TRp, MSq, t), N(Kq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), \\ N(TRp, Kq, t) \end{matrix}\right\} \geq 0$$
(iii) If (D) ,$MS(D)$ and $TR(D)$ are closed subsets of D such that the pairs (K, TR) and (K, MS) has a coincidence point in D.

Moreover, if (K, S), (K, R), (MS, R) and (TR, S) are commuting pairs, then K, M, T, R and S have a unique common fixed point in D.

Proof:
Suppose that the pair (K, TR) satisfies the property (CLR_{TR}), so there exists a sequence $\{P_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = TRp$ for some $TRp \in D$.
Since $K(D) \subseteq MS(D)$, there exists a sequence $\{q_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} MSq_n$
$$\therefore \lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = TRp$$
We show that $\lim_{n\to\infty} Lq_n = TRp$
Suppose that $\lim_{n\to\infty} Lq_n = z$
$$\phi\left\{\begin{matrix} N(Kp_n, Lq_n, t), N(TRp_n, MSq_n, t), N(Lq_n, MSq_n, t), N(Kp_n, TRp_n, t), N(Kp_n, MSq_n, t), \\ N(TRp_n, Lq_n, t) \end{matrix}\right\} \geq 0$$
$$\phi\{N(TRp, z, t), N(TRp, TRp, t), N(Z, TRp, t), N(TRp, TRp, t), N(TRp, TRp, t), N(TRp, z, t)\} \geq 0$$
$$\phi\{N(TRp, z, t), 1, N(Z, TRp, t), 1, 1, N(TRp, z, t)\} \geq 0$$
$$\therefore TRp = z \quad [\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$$
Hence $\lim_{n\to\infty} Lq_n = z$
$\therefore \lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = z$
Put $p = p_n$ and $q = u$ in equation (ii), we have
$$\phi\left\{\begin{matrix} N(Kp_n, Ku, t), N(TRp_n, MSu, t), N(Ku, MSu, t), N(Kp_n, TRp_n, t), N(Kp_n, MSu, t), N \\ (TRp_n, Ku, t) \end{matrix}\right\} \geq 0$$
$\phi\{N(Ku, z, t), N(z, z, t), N(Ku, z, t), N(z, z, t), N(z, z, t), N(Ku, z, t)\} \geq 0$
$\therefore Ku = z \quad [\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v]$
$\therefore Ku = MSu = z$
Since $TR(D)$ is a closed subset of D, there exists a point $u \in D$ such that $TRu = z$.
$\therefore Ku = MSu = TRu = z$
Let (K, MS) be weakly compatible, we have $Ku = MSu$
$\Rightarrow MSKu = KMSu.$
$\Rightarrow MSz = Kz$

Let (K,TR) be weakly compatible.

$\therefore KTRu = TRKu.$

$\Rightarrow Kz = TRz$

$\therefore z$ is a coincidence point of each pair (K,MS) and (K,TR).

To prove z is the common fixed point of K,T,R,S,M in D.

Put $p = p_n$ and $q = z$ in equation (ii), we have

$$\phi\left\{\begin{matrix} N(Kp_n,Kz,t), N(TRp_n,MSz,t), N(Kz,MSz,t), N(Kp_n,TRp_n,t), N(Kp_n,MSz,t), N \\ (TRp_n,Kz,t) \end{matrix}\right\} \geq 0$$

$\phi\{N(MSz,z,t), N(MSz,z,t), N(Kz,Kz,t), N(z,z,t), N(MSz,z,t), N(MSz,z,t)\} \geq 0$

$\therefore MSz = z \ \ [\because \phi(u,u,v,v,u,u) \geq 0 \ \Rightarrow u \geq v]$

$\therefore Kz = z \ [\because Kz = MSz]$

$\therefore TRz = z \ [\because Kz = MSz = TRz = z]$

Since (K,S), (K,R), (MS,R) and (TR,S) are commuting pairs, we have

$KSz = SKz = Sz$

$KRz = RKz = Rz$

$MSRz = RMSz = Rz$

$TRSz = STRz = Sz$

Put $p = Sz$ and $q = z$ in (iii) we have

$$\phi\left\{\begin{matrix} N(KSz,Kz,t), N(TRSz,MSz,t), N(Kz,MSz,t), N(KSz,TRSz,t), N(KSz,MSz,t), N \\ (TRSz,Kz,t) \end{matrix}\right\} \geq 0$$

$\phi\{N(Sz,z,t), N(Sz,z,t), N(z,z,t), N(Sz,Sz,t), N(Sz,z,t), N(Sz,z,t)\} \geq 0$

$\Rightarrow Sz = z \ \ [\because \phi(u,u,v,v,u,u) \geq 0 \ \Rightarrow u \geq v]$

$\therefore MSz = z \ \Rightarrow Mz = z$

$\therefore Mz = z, Sz = z, Kz = z, TRz = z$

Put $p = z$ and $q = Rz$ in (iii) we have

$$\phi\left\{\begin{matrix} N(Kz,KRz,t), N(TRSz,MSRz,t), N(KRz,MSRz,t), N(Kz,TRz,t), N(Kz,MSRz,t), \\ N(z,z,t) \end{matrix}\right\} \geq 0$$

$\phi\{N(Rz,z,t), N(Rz,z,t), N(Rz,Rz,t), N(z,z,t), N(Rz,z,t), N(Rz,z,t)\} \geq 0$

$\therefore Rz = z \ \ [\because \phi(u,u,v,v,u,u) \geq 0 \ \Rightarrow u \geq v]$

$\therefore Rz = z$

$\therefore Tz = z$

Hence $Sz = Tz = Rz = Kz = Mz = z$

$\therefore z$ is a common fixed point in D.

To prove the uniqueness

Let w be another common fixed point in D.

Put $p = z$ and $q = w$ in (ii) we have

$$\phi\{N(Kz,Kw,t), N(TRz,MSw,t), N(Kw,MSw,t), N(Kz,TRz,t), N(Kz,MSw,t), N(TRz,Kw,t)\} \geq 0$$

$\phi\{N(z,w,t), N(z,w,t), N(w,w,t), N(z,z,t), N(z,w,t), N(z,w,t)\} \geq 0$

$\Rightarrow z = w [\because \phi(u,u,v,v,u,u) \geq 0 \ \Rightarrow u \geq v]$

Hence z is the unique common fixed point of K,T,R,M,S in D.

Teorem 3.4.

Let $(D,N,*)$ be a fuzzy Banach space and $K,L: D \to D$ be two self-mapping fulfilling the following conditions; Pair (K,L) satisfies property (E.A),

(i) A few $\phi \in (\phi)$ and for all $p,q \in D$ and every $t > 0$, $\phi\{N(Kp,Lq,t), N(Kp,Kq,t), N(Kq,Lq,t), N(Kp,Lp,t), N(Kq,Lp,t), N(Lp,Lq,t)\} \geq 0$

(ii) If $K(D)$ is a closed subset of D pairs (K,L) has a coincidence point in D.

Moreover, if (K,L) is weakly compatible, then mapping K and L have a unique common fixed point in D.

Proof:

Suppose that (K,L) satisfies the property (*E.A*), then there be a sequence $\{P_n\}$ in D thus and so $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} Lp_n = z$ a few $z \in D$.

Since $K(D)$ is a closed subset of D, there be a point $u \in D$ thus and so $Ku = z$ a few $z \in D$.

$\therefore Kp_n = Lp_n = z = Ku$

To prove $Lu = z$ $\phi\{N(Kp,Lq,t), N(Kp,Kq,t), N(Kq,Lq,t), N(Kp,Lp,t), N(Kq,Lp,t), N(Lp,Lq,t)\} \geq 0$

Put $p = p_n$ and $q = u$ in equation (ii) then $\phi\{N(z,Lu,t), N(z,z,t), N(z,Lu,t), N(z,z,t), N(z,z,t), N(z,Lu,t)\} \geq 0$

$\therefore Lu = z.$

Now we have $Lu = Ku = z$

Now(K, L) is weakly compatible,

$LKu = KLu.$

$\Rightarrow Lz = Kz$

$\therefore z$ is a coincidence point of (K, L).

Put $p = p_n$ and $q = z$ in equation (ii), We have

$$\phi\{N(Kp_n, Lz, t), N(Kp_n, Kz, t), N(Kz, Lz, t), N(Kp_n, Lp_n, t), N(Kz, Lp_n, t), N(Lp_n, Lz, t)\} \geq 0$$

$\therefore Lz = z.$

$\therefore Kz = z[\because Kz = Lz = z].$

$\therefore z$ is a common fixed point in D.

Next, to prove the singularity.

Let w be another common fixed point in D.

Put $p = z$ and $q = w$ in (ii) we have

$$\phi\{N(Kz, Lw, t), N(Kz, Kw, t), N(Kw, Lw, t), N(Kz, Lz, t), N(Kw, Lz, t), N(Lz, Lw, t)\} \geq 0$$

$$\phi\{N(z, w, t), N(z, w, t), N(w, w, t), N(z, z, t), N(w, z, t), N(z, w, t)\} \geq 0$$

$\Rightarrow z = w.$

Hence z is a common single fixed point of K, L in D.

Theorem 3.5. Let $(D, N,*)$ be a fuzzy Banach space and $K, L, R: D \to D$ be three self-mapping fulfilling the following conditions;

(i) One of the pairs (K, R) and (L, R) satisfies property (E.A) and $A(D) \subseteq R(D)$ and $L(D) \subseteq R(D)$.

(ii) A few$\phi \in (\phi)$ and for all $p, q \in D$ and every $t > 0$,

$$\phi\{N(Kp, Lq, t), N(Rq, Rp, t), N(Kp, Rp, t), N(Rq, Lq, t), N(Lq, Rp, t), N(Kp, Rq, t)\} \geq 0$$

(iii) If $R(D)$ is a closed subset of D the pairs (K, R) and (L, R) have a coincidence point in D.

Moreover, if (K, R) and $(L, R)$are weakly compatible, then mapping K, L and R have a unique common fixed point in D.

Proof:

Suppose that (K, R) satisfies the property (*E.A*), then there be a sequence $\{P_n\}$ in D thus and so$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} Rp_n = z$ a few$z \in D$.

Now$L(D) \subseteq R(D)$, there be a sequence $\{q_n\}$ in Dthus and so$Lq_n = Rp_n$

$\therefore Kp_n = Rp_n = Lq_n = z$

Now $R(D)$ is a closed subset of D, then there be a point $u \in D$ thus and so $Ru = z$.

$\therefore Kp_n = Rp_n = Lq_n = z = Ru$

To prove $Lu = z$

Put $p = p_n$ and $q = u$ in equation (ii), we have

$$\phi\{N(Kp_n, Lu, t), N(Ru, Rp_n, t), N(Kp_n, Rp_n, t), N(Ru, Lu, t), N(Lu, Rp_n, t), N(Kp_n, Ru, t)\} \geq 0$$

$$\phi\{N(Lu, z, t), N(z, z, t), N(z, z, t), N(Lu, z, t), N(Lu, z, t), N(z, z, t)\} \geq 0$$

$\therefore Lu = z \ \ [\because \phi(u, v, v, u, u, v) \geq 0 \ \Rightarrow u \geq v]$

$\therefore Lu = Ru = z$

$\therefore z$ is a coincidence point of (K, R).

Now(L, R) is weakly compatible, we have

$LRu = RLu.$

$\Rightarrow Lz = Kz$

Put $p = p_n$ and $q = z$ in equation (ii), we have

$$\phi\{N(Kp_n, Lz, t), N(Rz, Rp_n, t), N(Kp_n, Rp_n, t), N(Rz, Lz, t), N(Lz, Rp_n, t), N(Kp_n, Rz, t)\} \geq 0$$

Rz = z

$\therefore Lz = z$

Put $p = z$ and $q = u$ in (ii) we have

$$\phi\{N(Kz, Lu, t), N(Ru, Rz, t), N(Kz, Rz, t), N(Ru, Lu, t), N(Lu, Rz, t), N(Kz, Ru, t)\} \geq 0$$

$\Rightarrow Kz = z$

Hence z is a common fixed point of K, L, R in D.

To prove the singularity

Let w be another common fixed point in D.

Put $p = z$ and $q = w$ in (ii) we have

$$\phi\{N(Kz, Lw, t), N(Rw, Rz, t), N(Kz, Rz, t), N(Rw, Lw, t), N(Lw, Rz, t), N(Kz, Rw, t)\} \geq 0$$

$\Rightarrow w = z$

Hence z is a unique common fixed point of K, L, R in D.

Theorem 3.6

Suppose Let $(D, N, *)$ be a fuzzy Banach space and $K, M, T, S, R: D \to D$ be three self-mapping fulfilling the following conditions;

(iv) If $K(D) \subseteq MS(D)$ and $K(D) \subseteq TR(D)$.
(v) A few $\phi \in (\phi)$ and for all $p, q \in D$ and every $t > 0$,

$$\phi\begin{Bmatrix} N(Kp, Kq, t), N(TRp, MSq, t), N(Kq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), \\ N(TRp, Kq, t) \end{Bmatrix} \geq 0$$

(vi) If (D) ,$MS(D)$ and $TR(D)$ are closed subsets of D thus and so the pairs (K, TR) and (K, MS) has a coincidence point in D.

Moreover, if (K, S), (K, R), (MS, R) and (TR, S) are commuting pairs, then K, M, T, R and S have a unique common fixed point in D.

Proof:

Suppose that the pair (K, TR) satisfies the property (*E.A*), so there be a sequence $\{P_n\}$ in D thus and so $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = z$ a few $z \in D$.

Now $K(D) \subseteq MS(D)$, there be a sequence $\{q_n\}$ in D thus and so $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} MSq_n$

$$\therefore \lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = z$$

Put $p = p_n$ and $q = u$ in equation (ii), we have

$$\phi\{N(Kp_n, Ku, t), N(TRp_n, MSu, t), N(Ku, MSu, t), N(Kp_n, TRp_n, t), N(Kp_n, MSu, t), N(TRp_n, Ku, t)\} \geq 0$$

$Ku = z$

$\therefore Ku = MSu = z$

Now $TR(D)$ is a closed subset of D, there be a point $u \in D$ thus and so $TRu = z$.

$$\therefore Ku = MSu = TRu = z$$

Let (K, MS) be weakly compatible, we have $Ku = MSu$

$$\Rightarrow MSKu = KMSu.$$
$$\Rightarrow MSz = Kz$$

Let (K, TR) be weakly compatible.

$$\therefore KTRu = TRKu.$$
$$\Rightarrow Kz = TRz$$

$\therefore z$ is a coincidence point of each pair (K, MS) and (K, TR).

To prove z is the common fixed point of K, T, R, S, M in D.

Put $p = p_n$ and $q = z$ in equation (ii), we have

$$\phi\{N(Kp_n, Kz, t), N(TRp_n, MSz, t), N(Kz, MSz, t), N(Kp_n, TRp_n, t), N(Kp_n, MSz, t), N(TRp_n, Kz, t)\} \geq 0$$

MSz = z

$\therefore Kz = z[\because Kz = MSz]$

$\therefore TRz = z[\because Kz = MSz = TRz = z]$

Now (K, S), (K, R), (MS, R) and (TR, S) are commuting pairs, we have

$KSz = SKz = Sz$

$KRz = RKz = Rz$

$MSRz = RMSz = Rz$

$TRSz = STRz = Sz$

Put $p = Sz$ and $q = z$ in (iii) we have

$$\phi\{N(KSz, Kz, t), N(TRSz, MSz, t), N(Kz, MSz, t), N(KSz, TRSz, t), N(KSz, MSz, t), N(TRSz, Kz, t)\} \geq 0$$

$\Rightarrow Sz = z$

$\therefore MSz = z \Rightarrow Mz = z$

$\therefore Mz = z, Sz = z, Kz = z, TRz = z$

Put $p = z$ and $q = Rz$ in (iii) we have

$$\phi\{N(Kz, KRz, t), N(TRSz, MSRz, t), N(KRz, MSRz, t), N(Kz, TRz, t), N(Kz, MSRz, t), N(z, z, t)\} \geq 0$$

$\therefore Rz = z$

$\therefore Rz = z$

$\therefore Tz = z$

Hence $Sz = Tz = Rz = Kz = Mz = z$

$\therefore z$ is a common fixed point in D.

To prove the singularity

Let w be another common fixed point in D.

Put $p = z$ and $q = w$ in (ii) we have

$$\phi\{N(Kz, Kw, t), N(TRz, MSw, t), N(Kw, MSw, t), N(Kz, TRz, t), N(Kz, MSw, t), N(TRz, Kw, t)\} \geq 0$$

$\Rightarrow z = w$

Hence z is the single common fixed point of K, T, R, M, S in D.

Theorem 3.7.
Let $(D, N, *)$ be a fuzzy Banach space and $K, L, M, T, R, S : D \to D$ be hexadic self-mappings satisfy the following conditions:
(i) one of the pairs (K, TR) and (L, MS) satisfies the property CLR with respect to mapping TR and MS such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For every $p, q \in D$ and for some $\phi \in (\phi)$ and every $t > 0$,
$\phi\{N(Kp, Lq, t), N(TRp, MSq, t), N(Lq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), N(TRp, Lq, t)\} \geq 0$
(iii) If one of $MS(D)$ and $TR(D)$ are closed subset of D.
(iv) Pairs (K, TR) and (L, MS) are weakly compatible.
(v) Each pair of pairs (K, TR) and (L, MS) has a coincidence point in D.
(iv) If $(K, S), (L, R), (MS, R)$ and (TR, S) are commuting pairs than K, L, M, T, R, S have a unique common fixed point in D.

Proof:
Let pairs (K, TR) satisfy CLR property, so there exists a sequence $\{p_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = z$ for some z belongs to TR(D).
Since $K(D) \subseteq MS(D)$, there exists $\{q_n\}$ in D such that $Kp_n = MSq_n$ and we get
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = TR(z) = z$
We claim that $\lim_{n\to\infty} Lq_n = z$
Put $p = p_n$ and $q = q_n$.
$\phi\{N(Kp_n, Lq_n, t), N(TRp_n, MSq_n, t), N(Lq_n, MSq_n, t), N(Kp_n, TRp_n, t), N(Kp_n, MSq_n, t),$
$N(TRp_n, Lq_n, t)\} \geq 0$
$\phi\{N(z, Lq_n, t), N(z, z, t), N(Lq_n, z, t), N(z, z, t), N(z, z, t), N(z, Lq_n, t)\} \geq 0$
$\phi\{N(Lq_n, z, t), N(z, z, t), N(Lq_n, z, t), N(z, z, t), N(z, z, t), N(Lq_n, z, t)\} \geq 0$
$\phi\{N(Lq_n, z, t), 1, N(Lq_n, z, t), 1,1, N(Lq_n, z, t)\} \geq 0$
$\lim_{n\to\infty} Lq_n = z \qquad [\phi\ (u, v, u, v, v, u) \geq 0\ \Rightarrow u \geq v]$
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = \lim_{n\to\infty} Lq_n = z$
Since $MS(D)$ is a closed subset of D, there exists $u \in D$ such that $MSu = z$ and w get
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = \lim_{n\to\infty} Lq_n = MSu$
We assert that $Lu = z$
Put $p = p_n$ and $q = u$
$\phi\{N(Lu, MSu, t), N(Kp_n, TRp_n, t), N(Kp_n, Lu, t), N(TRp_n, MSu, t), N(Kp_n, MSu, t),\ N(TRp_n, Lu, t)\} \geq 0$
$\phi\{N(Lu, z, t), N(z, z, t), N(Lu, z, t), N(z, z, t), N(z, z, t),\ N(Lu, z, t)\} \geq 0$
$\therefore Lu = z \quad [\because \phi(u, v, u, v, v, u) \geq 0 \Rightarrow u \geq v$
Thus $Lu = z$ and $MSu = z$
$\therefore Lu = MSu = z$
Since $L(D) \subseteq TR(D)$, there exists $v \in D$ such that $Lu = TRv$ and we get that $Lu = MSu = TRv = z$
To Prove $Kv = z$
Put $p = v$ and $q = u$ $\phi\{N(Kv, Lu, t), N(TRv, MSu, t), N(Lu, MSu, t), N(Kv, TRv, t), N(Kv, MSu, t),$
$N(TRv, Lu, t)\} \geq 0$
$\phi\{N(Kv, z, t), N(z, z, t), N(z, z, t), N(Kv, z, t), N(Kv, z, t), N(z, z, t)\} \geq 0$
$\therefore Kv = z$
Thus $Kv = z$ and $TRv = z$
$\therefore Kv = TRv = z$
We get $Lu = MSu = TRv = Kv = z$
Let (K, TR) and (L, MS) be weakly compatible pairs.
We have, $Kv = TRv\ \Rightarrow TRKv = LMSv$
$\Rightarrow TRz = Kz$
$\Rightarrow Kz = TRz$
Also, $Lu = MSu\ \Rightarrow MSLu = LMSu$
$\Rightarrow MSz = lz$
$\Rightarrow Lz = MSz$
Hence z is the coincidence point of each pair (K, TR) and (L, MS).
Next we have to show that z is the common fixed point of K, L, M, T, R and S.

For this, we claim that $Kz = z$
Put $p = z, q = u$
$\phi\{N(Kz, Lu, t), N(TRz, MSu, t), N(Lu, MSu, t), N(Kz, TRz, t), N(Kz, MSu, t), N(TRz, Lu, t)\} \geq 0$
$\phi\{N(Kz, z, t), N(Kz, z, t), N(z, z, t), N(Kz, Kz, t), N(Kz, z, t), N(Kz, z, t)\} \geq 0$
$\phi\{N(Kz, z, t), N(Kz, z, t), 1,1, N(Kz, z, t), N(Kz, z, t)\} \geq 0$
$\therefore Kz = z\ [\because \phi(u, u, v, v, u, u) \geq 0\ \Rightarrow u \geq v]$
$\therefore TRz = z\ [\because Kz = TRz]$
$\therefore TRz = Kz = TRz$
We claim that $Lz = z$
Put $p = v, q = z$.
$\phi\{N(Kz, Lz, t), N(TRv, MSz, t), N(Lz, MSz, t), N(Kv, TRv, t), N(Kv, MSz, t), N(TRv, MSz, t)\} \geq 0$
$\phi\{N(Lz, z, t), N(Lz, z, t), N(Lz, Lz, t), N(z, z, t), N(z, Lz, t), N(z, Lz, t)\} \geq 0$
$\phi\{N(Lz, z, t), N(Lz, z, t), 1,1, N(z, Lz, t), N(z, Lz, t)\} \geq 0$
$\therefore Lz = z\ [V(u, u, v, v, u, u) \geq 0\ \Rightarrow u \geq v]$
$Lz = MSz = z$
$\because Kz = MSz = Lz = TRz = z$
Since (K, S) and (TR, S) are commuting pairs, we get $K(Sz) = S(Kz) = Sz$.
Also $TR(Sz) = S(TRz) = Sz$.
From here it follows that , $L(Rz) = TR(Sz) = Sz$.
Since (L, R) and (MS, R) are commuting pairs we have,
$L(Rz) = R(Lz) = Rz$ and $MS(Rz) = R(MSz) = Rz$;
From here it follows that , $L(Rz) = MS(Rz) = Rz$
Put $P = Sz$ and $q = z$
$\phi\{N(KSz, Lz, t), N(TRSz, MSz, t), N(Lz, MSz, t), N(KSz, TRSz, t), N(KSz, MSz, t),\ N(TRSz, Lz, t)\} \geq 0$
$\phi\{N(Sz, z, t), N(Sz, z, t), N(z, z, t), N(Sz, Sz, t), N(Sz, z, t),\ N(Sz, z, t)\} \geq 0$
$\phi\{N(Sz, z, t), N(Sz, z, t), 1,1, N(Sz, z, t),\ N(Sz, z, t)\} \geq 0$
$\therefore Sz = z\ [\because \phi(u, u, v, v, u, u) \geq 0\ \Rightarrow u \geq v]$
Now $MSz = z$
$\Rightarrow Mz = z\ [\because Sz = z]$
$\Rightarrow Kz = Lz = Mz = Sz = TRz = z$
Put $p = z$ and $q = Rz$
$\phi\{N(Kz, LRz, t), N(TRz, MSRz, t), N(LRz, MSRz, t), N(Kz, TRz, t), N(Kz, MSRz, t),\ N(TRz, MSRz, t)\} \geq 0$
$\phi\{N(z, Rz, t), N(z, Rz, t), N(Rz, Rz, t), N(z, z, t), N(z, Rz, t),\ N(z, Rz, t)\} \geq 0$
$\phi\{N(Rz, z, t), N(Rz, z, t), 1,1, N(Rz, z, t),\ N(Rz, z, t)\} \geq 0$
$\therefore Rz = z$
Also $Tz = z$ as $TRz = z$
$\therefore Kz = Lz = Mz = Tz = Rz = Sz = z$
z is the common fixed point of K, L, M, T, R and S in D.
Similarly if (L, MS) satisfies property CLR and $TR(D)$ is closed subset of D, then we prove that z is a common fixed point of K, L, M, T, R and S in Din the same argument as above.
Uniqueness:
w is also a common fixed point in D.
Put $p = z$ and $q = w$
$\phi\{N(Kz, Lw, t), N(TRz, MSw, t), N(Lw, MSw, t), N(Kz, TRz, t), N(Kz, MSw, t),\ N(TRz, Lw, t)\} \geq 0$
$\phi\{N(z, w, t), N(z, w, t), N(w, w, t), N(z, z, t), N(z, w, t),\ N(z, w, t)\} \geq 0$
$\phi\{N(z, w, t), N(z, w, t), 1,1, N(z, w, t),\ N(z, w, t)\} \geq 0$
$\therefore z = w\ [\because \phi(u, u, v, v, u, u) \geq 0\ \Rightarrow u \geq v]$
Hence z is the unique common fixed point of K, L, M, T, R and S in D respectively.

Theorem 3.8.
Let $(D, N,*)$ be a fuzzy Banach space and $K, L, M, T, R, S : D \to D$ be six self-mappings satisfy the following conditions:
(i) If (K, TR) satisfies the property CLR such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For some $\phi \in (\phi)$ and for every $p, q \in D$ and every $t > 0$,
$\phi\{N(Kp, Lq, t), N(TRp, MSq, t), N(Lq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), N(TRp, Lq, t)\} \geq 0$
(iii) If $MS(D)$ is a closed subset of D.
(iv) Pairs (K, TR) and (L, MS) are weakly compatible.
(v) Each pair of pairs (K, TR) and (L, MS) has a coincidence point in D.

(iv) If $(K,S),(L,R),(MS,R)$ and (TR,S) are commuting pairs than K,L,M,T,R,S have a unique common fixed point in D.

Theorem 3.9.
Let $(D,N,*)$ be a fuzzy Banach space and $K,L,M,T,R,S : D \to D$ be six self-mappings satisfy the following conditions:
(i) If (L,MS) satisfies the property CLR such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For some $\phi \in (\phi)$ and every $t > 0$ and for every $p,q \in D$.
$\phi\{N(Kp,Lq,t),N(TRp,MSq,t),N(Lq,MSq,t),N(Kp,TRp,t),N(Kp,MSq,t),N(TRp,Lq,t)\} \geq 0$
(iii) If $TR(D)$ is a closed subset of D.
(iv) Pairs (K,TR) and (L,MS) are weakly compatible.
(v) Each pair of pairs (K,TR) and (L,MS) has a coincidence point in D.
(iv) If $(K,S),(L,R),(MS,R)$ and (TR,S) are commuting pairs than K,L,M,T,R,S have a unique common fixed point in D.

Theorem 3.10.
Let $(D,N,*)$ be a fuzzy Banach space and $K,L,M,T,R,S : D \to D$ be hexadic self-mappings satisfy the following conditions:
(i) one of the pairs (K,TR) and (L,MS) satisfies the property E.A such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For every $p,q \in D$ and for some $\phi \in (\phi)$ and every $t > 0$,
$\phi\{N(Kp,Lq,t),N(TRp,MSq,t),N(Lq,MSq,t),N(Kp,TRp,t),N(Kp,MSq,t),N(TRp,Lq,t)\} \geq 0$
(iii) If one of $MS(D)$ and $TR(D)$ are closed subset of D.
(iv) Pairs (K,TR) and (L,MS) are weakly compatible.
(v) Each pair of pairs (K,TR) and (L,MS) has a coincidence point in D.
(iv) If $(K,S),(L,R),(MS,R)$ and (TR,S) are commuting pairs than K,L,M,T,R,S have a unique common fixed point in D.

Proof:
Let pairs (K,TR) satisfy E.A property, so there exists a sequence $\{p_n\}$ in D such that $\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = z$ for some z belongs to D.
Since $K(D) \subseteq MS(D)$, there exists $\{q_n\}$ in D such that $Kp_n = MSq_n$ and we get
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = TR(z) = z$
We claim that $\lim_{n\to\infty} Lq_n = z$
Put $p = p_n$ and $q = q_n$.
$\phi\{N(Kp_n,Lq_n,t),N(TRp_n,MSq_n,t),N(Lq_n,MSq_n,t),N(Kp_n,TRp_n,t),N(Kp_n,MSq_n,t),$
$N(TRp_n,Lq_n,t)\} \geq 0$
$\phi\{N(z,Lq_n,t),N(z,z,t),N(Lq_n,z,t),N(z,z,t),N(z,z,t),N(z,Lq_n,t)\} \geq 0$
$\phi\{N(Lq_n,z,t),N(z,z,t),N(Lq_n,z,t),N(z,z,t),N(z,z,t),N(Lq_n,z,t)\} \geq 0$
$\phi\{N(Lq_n,z,t),1,N(Lq_n,z,t),1,1,N(Lq_n,z,t)\} \geq 0$
$\lim_{n\to\infty} Lq_n = z \qquad [\phi\,(u,v,u,v,v,u) \geq 0 \Rightarrow u \geq v]$
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = \lim_{n\to\infty} Lq_n = z$
Since $MS(D)$ is a closed subset of D, there exists $u \in D$ such that $MSu = z$ and w get
$\lim_{n\to\infty} Kp_n = \lim_{n\to\infty} TRp_n = \lim_{n\to\infty} MSq_n = \lim_{n\to\infty} Lq_n = MSu$
We assert that $Lu = z$
Put $p = p_n$ and $q = u$
$\phi\{N(Lu,MSu,t),N(Kp_n,TRp_n,t),N(Kp_n,Lu,t),N(TRp_n,MSu,t),N(Kp_n,MSu,t),\ N(TRp_n,Lu,t)\} \geq 0$
$\phi\{N(Lu,z,t),N(z,z,t),N(Lu,z,t),N(z,z,t),N(z,z,t),\ N(Lu,z,t)\} \geq 0$
$\therefore Lu = z \quad [\because \phi(u,v,u,v,v,u) \geq 0 \Rightarrow u \geq v$
Thus $Lu = z$ and $MSu = z$
$\therefore Lu = MSu = z$
Since $L(D) \subseteq TR(D)$, there exists $v \in D$ such that $Lu = TRv$ and we get that $Lu = MSu = TRv = z$
To Prove $Kv = z$
Put $p = v$ and $q = u$ $\phi\{N(Kv,Lu,t),N(TRv,MSu,t),N(Lu,MSu,t),N(Kv,TRv,t),N(Kv,MSu,t),$
$N(TRv,Lu,t)\} \geq 0$
$\phi\{N(Kv,z,t),N(z,z,t),N(z,z,t),N(Kv,z,t),N(Kv,z,t),N(z,z,t)\} \geq 0$

$\therefore Kv = z$
Thus $Kv = z$ and $TRv = z$
$\therefore Kv = TRv = z$
We get $Lu = MSu = TRv = Kv = z$
Let (K, TR) and (L, MS) be weakly compatible pairs.
We have, $Kv = TRv \Rightarrow TRKv = LMSv$
$\Rightarrow TRz = Kz$
$\Rightarrow Kz = TRz$
Also, $Lu = MSu \Rightarrow MSLu = LMSu$
$\Rightarrow MSz = lz$
$\Rightarrow Lz = MSz$
Hence z is the coincidence point of each pair (K, TR) and (L, MS).
Next we have to show that z is the common fixed point of K, L, M, T, R and S.
For this, we claim that $Kz = z$
Put $p = z, q = u$
$\phi\{N(Kz, Lu, t), N(TRz, MSu, t), N(Lu, MSu, t), N(Kz, TRz, t), N(Kz, MSu, t), N(TRz, Lu, t)\} \geq 0$
$\phi\{N(Kz, z, t), N(Kz, z, t), N(z, z, t), N(Kz, Kz, t), N(Kz, z, t), N(Kz, z, t)\} \geq 0$
$\phi\{N(Kz, z, t), N(Kz, z, t), 1,1, N(Kz, z, t), N(Kz, z, t)\} \geq 0$
$\therefore Kz = z$ $[\because \phi(u, u, v, v, u, u) \geq 0 \Rightarrow u \geq v]$
$\therefore TRz = z$ $[\because Kz = TRz]$
$\therefore TRz = Kz = TRz$
We claim that $Lz = z$
Put $p = v, q = z$.
$\phi\{N(Kz, Lz, t), N(TRv, MSz, t), N(Lz, MSz, t), N(Kv, TRv, t), N(Kv, MSz, t), N(TRv, MSz, t)\} \geq 0$
$\phi\{N(Lz, z, t), N(Lz, z, t), N(Lz, Lz, t), N(z, z, t), N(z, Lz, t), N(z, Lz, t)\} \geq 0$
$\phi\{N(Lz, z, t), N(Lz, z, t), 1,1, N(z, Lz, t), N(z, Lz, t)\} \geq 0$
$\therefore Lz = z$ $[V(u, u, v, v, u, u) \geq 0 \Rightarrow u \geq v]$
$Lz = MSz = z$
$\because Kz = MSz = Lz = TRz = z$
Since (K, S) and (TR, S) are commuting pairs, we get $K(Sz) = S(Kz) = Sz$.
Also $TR(Sz) = S(TRz) = Sz$.
From here it follows that , $L(Rz) = TR(Sz) = Sz$.
Since (L, R) and (MS, R) are commuting pairs we have,
$L(Rz) = R(Lz) = Rz$ and $MS(Rz) = R(MSz) = Rz$;
From here it follows that , $L(Rz) = MS(Rz) = Rz$
Put $P = Sz$ and $q = z$
$\phi\{N(KSz, Lz, t), N(TRSz, MSz, t), N(Lz, MSz, t), N(KSz, TRSz, t), N(KSz, MSz, t), N(TRSz, Lz, t)\} \geq 0$
$\phi\{N(Sz, z, t), N(Sz, z, t), N(z, z, t), N(Sz, Sz, t), N(Sz, z, t), N(Sz, z, t)\} \geq 0$
$\phi\{N(Sz, z, t), N(Sz, z, t), 1,1, N(Sz, z, t), N(Sz, z, t)\} \geq 0$
$\therefore Sz = z$ $[\because \phi(u, u, v, v, u, u) \geq 0 \Rightarrow u \geq v]$
Now $MSz = z$
$\Rightarrow Mz = z$ $[\because Sz = z]$
$\Rightarrow Kz = Lz = Mz = Sz = TRz = z$
Put $p = z$ and $q = Rz$
$\phi\{N(Kz, LRz, t), N(TRz, MSRz, t), N(LRz, MSRz, t), N(Kz, TRz, t), N(Kz, MSRz, t), N(TRz, MSRz, t)\} \geq 0$
$\phi\{N(z, Rz, t), N(z, Rz, t), N(Rz, Rz, t), N(z, z, t), N(z, Rz, t), N(z, Rz, t)\} \geq 0$
$\phi\{N(Rz, z, t), N(Rz, z, t), 1,1, N(Rz, z, t), N(Rz, z, t)\} \geq 0$
$\therefore Rz = z$
Also $Tz = z$ as $TRz = z$
$\therefore Kz = Lz = Mz = Tz = Rz = Sz = z$
z is the common fixed point of K, L, M, T, R and S in D.
Similarly if (L, MS) satisfies property E.A and $TR(D)$ is closed subset of D, then we prove that z is a common fixed point of K, L, M, T, R and S in Din the same argument as above.
Uniqueness:
w is also a common fixed point in D.
Put $p = z$ and $q = w$
$\phi\{N(Kz, Lw, t), N(TRz, MSw, t), N(Lw, MSw, t), N(Kz, TRz, t), N(Kz, MSw, t), N(TRz, Lw, t)\} \geq 0$
$\phi\{N(z, w, t), N(z, w, t), N(w, w, t), N(z, z, t), N(z, w, t), N(z, w, t)\} \geq 0$

$\phi\{N(z,w,t), N(z,w,t), 1,1, N(z,w,t),\ N(z,w,t)\} \geq 0$
$\therefore z = w\ [\because \phi(u,u,v,v,u,u) \geq 0\ \Rightarrow u \geq v]$
Hence z is the unique common fixed point of K, L, M, T, R and S in D respectively.

Theorem 3.11.
Let $(D, N, *)$ be a fuzzy Banach space and $K, L, M, T, R, S : D \to D$ be six self-mappings satisfy the following conditions:
(i) If (K, TR) satisfies the property E.A such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For some $\phi \in (\phi)$ and for every $p, q \in D$ and every $t > 0$,
$\phi\{N(Kp, Lq, t), N(TRp, MSq, t), N(Lq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), N(TRp, Lq, t)\} \geq 0$
(iii) If $MS(D)$ is a closed subset of D.
(iv) Pairs (K, TR) and (L, MS) are weakly compatible.
(v) Each pair of pairs (K, TR) and (L, MS) has a coincidence point in D.
(iv) If $(K, S), (L, R), (MS, R)$ and (TR, S) are commuting pairs than K, L, M, T, R, S have a unique common fixed point in D.

Theorem 3.12.
Let $(D, N, *)$ be a fuzzy Banach space and $K, L, M, T, R, S : D \to D$ be six self-mappings satisfy the following conditions:
(i) If (L, MS) satisfies the property E.A such that $K(D) \subseteq MS(D)$ and $L(D) \subseteq TR(D)$.
(ii) For some $\phi \in (\phi)$ and every $t > 0$ and for every $p, q \in D$.
$\phi\{N(Kp, Lq, t), N(TRp, MSq, t), N(Lq, MSq, t), N(Kp, TRp, t), N(Kp, MSq, t), N(TRp, Lq, t)\} \geq 0$
(iii) If $TR(D)$ is a closed subset of D.
(iv) Pairs (K, TR) and (L, MS) are weakly compatible.
(v) Each pair of pairs (K, TR) and (L, MS) has a coincidence point in D.
(iv) If $(K, S), (L, R), (MS, R)$ and (TR, S) are commuting pairs than K, L, M, T, R, S have a unique common fixed point in D.

Example 3.13.
Let $D = [0, \infty)$ be a fuzzy banach space.
Define $K, L: D \to D$ by

$$KD = \begin{cases} p & if\ p = 3 \\ 3 & if\ p > 3 \\ 0 & if\ p < 3 \end{cases} ;\ LD = \begin{cases} 6 - p & if\ p = 3 \\ 3 & if\ p > 3 \\ 0 & if\ p < 3 \end{cases}$$

Consider a sequence $\{p_n\} = \{3 + \frac{1}{n}\}$.
Here (K, L) is weakly compatible and satisfies the (CLR_L) and E.A property and have a coincidence point 3 in D. All the conditions of the above theorem are satisfied.
Hence 3 is their unique common fixed point of K, L in D.
Example 3.14.
Let D= $[\mathbf{0}, \infty)$ be the fuzzy banach space.
Define $K, L, R: D \to D$ by

$$KD = \begin{cases} 0 & if\ p < 2 \\ p & if\ p = 2 \\ 2 & if\ p > 2 \end{cases} ;\ LD = \begin{cases} 0 & if\ p < 2 \\ \frac{p+2}{2} & if\ p = 2 \\ 2 & if\ p > 2 \end{cases} ;\ RD = \begin{cases} 0 & if\ p < 2 \\ \frac{p+4}{3} & if\ p = 2 \\ 2 & if\ p > 2 \end{cases}$$

Consider a sequence $\{p_n\} = \{2 + \frac{3}{n}\}$ and $\{q_n\} = \{2 + \frac{1}{n}\}$.
Here (K, R) satisfies the (CLR_R) and E.A property and are weakly compatible and have a coincidence point 2 in D. All the conditions of the above theorem are satisfied.
Hence 2 is their unique common fixed point of K, L, R in D.

Example 3.15.
Let $D = [0, \infty)$ be a fuzzy banach space.
Define $K, M, T, R, S: D \to D$ by

$$KD = \begin{cases} p & if\ p = 2 \\ 2 & if\ p > 2 \\ 0 & if\ p < 2 \end{cases} ;\ MD = \begin{cases} \frac{p+2}{2} & if\ p = 2 \\ 2 & if\ p > 2 \\ 0 & if\ p < 2 \end{cases} ;\ TD = \begin{cases} \frac{p+4}{3} & if\ p = 2 \\ 2 & if\ p > 2 \\ 0 & if\ p < 2 \end{cases} ;\ RD = \begin{cases} 4 - p & if\ p = 2 \\ 2 & if\ p > 2 \\ 0 & if\ p < 2 \end{cases} ;$$

$SD = \begin{cases} 6 - p^2 & if \ \ p = 2 \\ 2 & if \ \ p > 2 \\ 0 & if \ \ p < 2 \end{cases}$ Consider a sequence $\{p_n\} = \{2 + \frac{3}{n}\}$ and $\{q_n\} = \{2 + \frac{1}{n}\}$.

Here (K, TR) satisfies the CLR_{TR} property and E.A property and weakly compatible and 2 is the coincidence point in D. All the conditions of the above theorem are satisfied.
Hence 2 is their unique common fixed point of K, M, T, R, S in D.

Example 3.16.
Let $D = [0, \infty)$ be the fuzzy Banach space.
Define $K, L, M, T, R, S, : D \to D$ by

$KD = \begin{cases} p \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$; $LD = \begin{cases} \frac{3-p}{2} \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$; $RD = \begin{cases} \frac{4-p}{2} \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$

$SD = \begin{cases} \frac{5-p}{2} \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$; $MD = \begin{cases} \frac{p+1}{2} \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$; $TD = \begin{cases} \frac{p+2}{2} \ if \ p = 1 \\ 1 \ if \ p > 1 \\ 0 \ if \ p < 1 \end{cases}$

Let $p \in D$.
Consider the sequence $\{p_n\} = \{1 + \frac{1}{n}\}$ and $\{q_n\} = \{1 + \frac{2}{n}\}$.
Here (K, TR) and (L, MS) satisfies the CLR and E.A property and are weakly compatible.
All the conditions of the above theorem are satisfied.
1 is the coincidence point in D.
Hence 1 is the unique common fixed point of K, L, M, T, R and S in D respectivly.

2. Conclusion

The target of this paper is to emphasize the role of Common limit range and E.A property in fuzzy banach space. The existence of common fixed points are proved for the pair of weak compatible mapping along with Common limit range and E.A property.

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