A STUDY ON SEMI-T_i, (i = 0,1,2,3,4) AND ALPHA-T_i, (i = 0,1,2,3,4) IN KASAJ TOPOLOGICAL SPACES

[1]P.Sathishmohan, [1]E.Prakash, [2]K.S.Viplavanjali and [3]K.Rajalakshmi
[1]Assistant Professor, [2]Research Scholar, [1and2]Department of Mathematics,
KongunaduArtsandScienceCollege(Autonomous),Coimbatore-641029, TN,India.
[3]AssistantProfessor,DepartmentofScienceandHumanities,
SriKrishnaCollegeofEngineeringandTechonology,Coimbatore-641008, TN,India.
Email:vijay26vipla@gmail.com[2]

ABSTRACT

In the present study, we introduce some kinds of separation axioms in kasaj topological spaces.New classesofseparationaxiomsinkasajtopologicalspace namely, KS-semi and KS_α spaces are introduced by utilizing KS-semiand KS_α-open and closed sets respectively and studied several of their fundamentalcharacterizations and their relationships with other corresponding kinds of spaces are discussed.

KEYWORDS:KS-semi-T_i(i=0,1,2,3,4)spacesand KS_α-T_i(i=0,1,2,3,4)spaces

AMSSubjectClassifications:54A05,54B05,54A99

INTRODUCTION

In 2019, Chandrasekar[1] introduced a new topology namely, micro topology which is an extension of nano topological spaces introduced by Lellis[3].For this extension, he has usedLevine's simple extension method.In 2020, Kashyap.G.Rachchh and Sajeed.I.Ghanchi[8] introduced partial extension of micro topologicalspacenamelykasajtopologicalspaces.Inthesameyear,theyestablishedtheconcept of kasaj-closure,kasaj-interior,kasajsemiclosedandkasajgeneralizedclosedsetsinkasajtopological spaces.In2022,Prakashetal.[2]definedandstudiedthenotionsofKS_{gs}-closed and KS_{sg}-closedsets inkasajtopologicalspace.TheclassofKS-semiand KSα-openandclosedsetsplaysanimportant roleinthedevelopmentofkasajtopologicalspace.Theinvestigationonabovesignificantcontribution tothetheoryofseparationaxioms.Themaingoalofthepresentpaperistoconsiderandstudynew classesofseparationaxiomscalledKS-semiand KS_α separation axiomsbyusingtheaboveKS-open and KS-closed sets respectively.Several properties concerning of these kinds of spaces were presented.Also, the relationships among these spaces were investigated and presented the exemplars where ever necessary.

PRELIMINARIES

Definition0.1.[8]Let $(U,\tau_R(X),KS_R(X))$be a nano topological space.Then kasaj topology is defined by$KS_R(X) = \{(K \cap S) \cup (K' \cap S'): K, K' \in \tau_R(X)$, fixed $S, S' \notin \tau_R(X), S \cup S' = U\}$

The Kasaj topology $KS_R(X)$ satisfies the following postulates:

1. $U, \phi \in KS_R(X)$
2. The union of elements of any subcollection of $KS_R(X)$ is in $KS_R(X)$
3. The intresection of any finite subcollection of elements of $KS_R(X)$ is in $KS_R(X)$

Then $(U, \tau_R(X), KS_R(X))$ is called kasaj topological spaces and the members of $KS_R(X)$ are called kasaj-open (KS-open)set and the complement of a kasaj-open set is called a kasaj-closed (KS-closed)set.

Definition0.2.[8]AsubsetPofKStopologicalspace$(U, \tau_R(X), KS_R(X))$iscalled

(i) KS-semi-openset,ifP⊆$KS_{cl}(KS_{int}(P))$
(ii) KS-semi-closedset,if$KS_{int}(KS_{cl}(P))$⊆P
(iii) KS_α-openset,ifP⊆$KS_{int}(KS_{cl}(KS_{int}(P)))$
(iv) KS_α-closedset,if$KS_{cl}(KS_{int}(KS_{cl}(P)))$⊆P

Definition0.3.[8]ForanytwosubsetsP and QofUinakasajtopologicalspace$(U, \tau_R(X), KS_R(X))$. The Kasaj-semi-closure and the kasaj-α-closure of a set P are denoted by $KS_{scl}(P)$ and $KS_\alpha(P)$, respectively.They are defined

$KS_{scl}(P)$=∩{Q:P⊆Q,QisKS-semi-closed}
$KS_\alpha(P)$=∪{Q:P⊆Q,QisKS-α-closed}

Definition 0.4. Let $(U, \tau_R(X), KS_R(X))$ be a kasaj topological space and let x be a point of U. A subsetPofUiscalledKS-semi-nhbdofUtheiroccuraKS-semi-opensetFsuchthatforx ∈F⊆P.

Definition 0.5. Let$(U, \tau_R(X), KS_R(X))$beakasajtopologicalspaceandletxbeapointofU.Asubset PofUiscalledKSα-nhbdofUtheiroccuraKSα-opensetFsuchthatforx∈F⊆P.

1. KS-SEMI-T_i, (i=0,1,2,3,4)

Inthissection,wedefineanddiscusssomepropertiesofKS-semi-T_i(i=0,1,2,3,4)spacesinkasajtopological spaces.

Definition1.1.AKStopologicalspace$(U, \tau_R(X), KS_R(X))$issaidtobe

(i) KS-semi-T_0spaceifapairofdistinctpointsx,y∈UeitherthereexistsKS-semi-openset G∈$KS_R(X)$suchthatx∈G,y∉GorthereexistsKS-semi-opensetH∈$KS_R(X)$suchthat y∈ H, x ∉H.
(ii) KS-semi-T_1spaceifapairofdistinctpointsx,y ∈Uwith$x \neq y$,thereexistsKS-semi-openset G,H∈$KS_R(X)$suchthatx∈G,y∉G;y∈H,x∉H.
(iii) KS-semi-T_2spaceifapairofdistinctpointsx,y ∈Uwith$x \neq y$,thereexistsKS-semi-openset G,H∈$KS_R(X)$suchthatx∈G,y∈H,G∩H=φ.
(iv) TheKS-semi-T_3spaceifanelementx∈ UandKS-semi-closedsetF⊆ Usuchthatx∉F, thereexistsdisjointKS-semi-opensetsG_1,G_2⊆Usuchthatx ∈G_1,F⊆G_2.
(v) The KS-semi-T_4space if a pair of disjoint KS-semi-closed sets C_1,C_2⊆U, there exists disjoint KS-semi-opensetsG_1,G_2⊆ UsuchthatC_1⊆ G_1,C_2⊆ G_2.

Exemplar1.2. LetU={a,b,c,d,e}withU/R={{a,c},{d,e},{b}}andX={a,b}⊆U.Then$\tau_R(X)$={φ, U,{b},{a,b,c},{a,c}}.IfweconsiderS={c},S'={a,b,d,e}then$KS_R(X)$= {φ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,b,d,e},U}andKS-semi-open={φ,{a},{b},{c}{a,b}, {a,c},{a,d},{a,e},{b,c},{b,d},{b,e},{a,b,c},{a,b,d},{a,b,e},{a,c,d},{a,c,e},{a,d,e},{b,c,d},{b,c,e}, {b,d,e},{a,b,c,d},{a,b,c,e},{a,b,d,e},{a,c,d,e},{b,c,d,e},U}.

(i) Leta,b∈U, $a \neq b$ ∃ KS-semi-open set = {a,c}suchthata∈{a,c}andb∉{a,c}.
(ii) FromExemplar1.2(i)and∃KS-semi-openset={b,e}suchthatb∈{b,e}anda∉{b,e}.
(iii) FromExemplar1.2((i)&(ii))∃{a,c}∩{b,e}=φ.
(iv) Leta∈U,{b,e}=KS-semi-closedsetsanda∉{b,e}∃{a,c,d}and{b,e}=KS-semi-opensets suchthata ∈{a,c,d}and{b,e}⊆{b,e}.
(v) Let{a}and{b}=KS-semi-closedsetswhere$\{a\} \neq \{b\}$∃{a,c,d}and{b,e}=KS-semi-open setswhere$\{a,c,d\} \neq \{b,e\}$suchthat{a}⊆{a,c,d}and{b}⊆{b,e}.

Theorem1.3. EveryKS-T_0spaceisKS-semi-T_0space.

Proof: Let U be a KS-T_0space, a and b be two distinct points of U, as U is KS-T_0space there exists KS-opensetPsuchthata∈Pandb∉P.SinceeveryKS-opensetisKS-semi-openandhencePisKS-semi-opensetsuchthata∈Pandb ∉P.HenceUisKS-semi-T_0space.

Theorem1.4. EveryKS-T_1spaceisKS-semi-T_1space.

Proof: LetUbeaKS-T_1space, xandybetwodistinctpointsofU.ThenthereexistsdistinctKS-open setGandHsuchthatx∈Gandy∉Gandy∈Handx∉H.AseveryKS-opensetisKS-semi-openandhenceGandHaredistinctKS-semi-opensetswithx∈Gandy∈Hsuchthaty∉G,x∉H.HenceUisKS-semi-T_1space.

Theorem1.5. EveryKS-semi-T_1spaceisKS-semi-T_0space.

Proof: Let U be a KS-semi-T_1space and let x and y be two distinct points of U, as U is KS-semi-T_1 spacethereexitsKS-semi-opensetsGandHsuchthatx∈Gandy∉Gandy∈Handx∉H.SinceeveryKS-opensetisKS-semi-openandhenceGisKS-semi-opensetsuchthatx∈Gandy∉G.HenceUisKS-semi-T_0space.

Theorem1.6. EveryKS-T_2spaceisKS-semi-T_2space.

Proof: LetUbeaKS-T_2space, xandybetwodistinctpointsofU.ThenthereexistsdistinctKS-open setGandHsuchthatx∈Gandy∈H.AseveryKS-opensetisKS-semi-openandhenceGandHaredistinctKS-semi-opensetssuchthatx ∈Gandy∈H.HenceUisKS-semi-T_2space.

Theorem1.7. EveryKS-semi-T_2spaceisKS-semi-T_0space.

Proof: Let U be a KS-semi-T_2space and let x and y be two distinct points ofU, as U is KS-semi-T_2 spacethereexitsKS-semi-opensetsGandHsuchthatx∈Gandy∈H.SinceGandHaredisjoint.SinceeveryKS-opensetisKS-semi-openandhenceGisKS-semi-opensetsuchthatx∈Gandy∉G.HenceUisKS-semi-T_0space.

Theorem1.8. EveryKS-semi-T_2spaceisKS-semi-T_1space.

Proof: Suppose U is KS-semi-T_2.Let x,y ∈Uwith $x \neq y$ .Since U is KS-semi-T_2, there exists disjoint KS-semi-opensetsG,Hwithx∈G,y∈H.SinceGandHaredisjoint,wehavex∉Handy∉G.HenceUisKS-semi-T_1space.

Theorem1.9. EveryKS-T3spaceisKS-semi-T3space.

Proof: Let U is KS-T3space and F be a KS closed set not containing x implies F be a KS-semi-closedset not containing x.As U is KS-T3there exists jointly disjoint KS-semi-open sets G,H such that x ∈G,F⊆ H.HenceUisKS-semi-T3 space.

Theorem1.10. EveryKS-semi-T_3spaceisKS-semi-T_0space.

Proof: LetUisKS-semi-T_3.AsUisKS-semi-T_3everysingletonsetxisKS-semi-closedsubsetofUandybeanypointU/xthen$x \neq y$.BydefinitionofKS-semi-T_3thereexiststwojointlydisjoint KS-semi-opensetsGandHsuchthatx⊆Gandy∉H,impliesx∈Gand y∉H.HenceUisKS-semi-T_0space.

Theorem1.11. EveryKS-semi-T_3spaceisKS-semi-T_2space.

Proof: LetUisKS-semi-T_3.AsUisKS-semi-T_3everysingletonsetxisKS-semi-closedsubsetofUandybeanypointU/xthen$x \neq y$.BydefinitionofKS-semi-T_3thereexiststwojointlydisjoint KS-semi-opensetsGandHsuchthatx⊆Gandy∉H,impliesx∈Gand y∉H.HenceUisKS-semi-T_2space.

Theorem1.12. EveryKS-T_4spaceisKS-semi-T_4space.

Proof: LetUbeaKS-T_4spaceandC_1,C_2bepairofdisjointKS-closedsets.AseveryKS-closedset is KS-semi-closed set.$C_1$and $C_2$are KS-semi-closed sets and U is KS-semi-T_4, therefore there exists disjoint KS-semi-open sets G and H such that C_1⊆G and C_2⊆H.Thus, for every pair of disjoint KS closed sets $C_1$and C_2there exists disjoint KS-semi-open sets G and H such that C_1⊆G and C_2⊆H. HenceUisKS-semi-T_4space.

Theorem1.13. EveryKS-semi-T_4spaceisKS-semi-T_3space.

Proof: LetUbeaKS-semi-T_4space,letFbeanyKS-semi-closedsetandletxbeapointofUsuchthatx ∉F.As {x} is KS-semi-closed subset of U such that {x}∩F= φ.Then by KS-semi-T_4, there exists KS-semi-open sets G and H such that {x}⊆G, F⊆Hand G∩H =φ.Also {x}⊆G ⇒x ∈G. Thus, thereexitsKS-semi-opensetsGandHsuchthatx∈G,F⊆HandG∩H=φitfollowsthatthespaceisUisKS-semi-T_3.

The following exemplar shows that that the reverse implications of the theorem(1.3 to 1.13) is nottrue for the above theorems:

Exemplar1.14. LetU={a,b,c,d,e}withU/R={{a,e},{b,d},{c}}andX={b,c}⊆U.
Then$\tau_R(X)$={φ,U,{c},{b,c,d},{b,d}}.IfweconsiderS={c,d},S'={a,b,e}then
$KS_R(X)$= {φ,{b},{a},{c},{d},{b,d},{b,c},{c,d},{a,b,e},{b,c,d},{a,b,c,e},{a,b,d,e},U}and KS-semi-open={φ,{b},{c},{d},{a,b},{b,c},{b,d},{b,e},{c,d},{a,b,c},{a,b,d},{a,b,e},{b,c,d},{b,c,e},{b,d,e},{a,b,c,d},{a,b,c,e},{a,b,d,e},{b,c,d,e},U}.={φ,{b},{c},{d},{a,b},{b,c},{b,d},{b,e},{c,d},{a,b,c},{a,b,d},{a,b,e},{b,c,d},{b,c,e},{b,d,e},{a,b,c,d},{a,b,c,e},{a,b,d,e},{b,c,d,e},U}.

(i) Letb,d∈U,b ≠ d∃KS-semi-openset={a,b,e}suchthatb∈{a,b,e}andd∉{a,b,e}then itisKS-semi-T_0spacebutnotKS-T_0space.

(ii) Leta,c∈U,a ≠ c∃G={a,b}andH={c,d},{a,b}/={c,d}suchthata∈G,c∉G;c∈H,a∉HthenitisKS-semi-T_1spacebutnotKS-T_1space.

(iii) Similarly,by(i)itisKS-semi-T_0spacebutnotKS-T_1space.

(iv) From(ii)itshowsthatG∩H=φthenitisKS-semi-T_2spacebutnotKS-T_2space.

(v) Similarly,by(i)itisKS-semi-T_0spacebutnotKS-semi-T_2space.

(vi) Similarly,by(ii)itisKS-semi-T_1spacebutnotKS-semi-T_2space.

(vii) Lete∈U,KS-semi-closedsetF={c,d};{c,d}⊆U∃G={b,e}andH={c,d}suchthate∈{b,e},{c,d}⊆{c,d}thenitisKS-semi-T_3spacebut notKS-T_3space.

(viii) Similarly,by(i)itisKS-semi-T_0spacebutnotKS-semi-T_3space.

(ix) Similarly,by(iv)itisKS-semi-T_2spacebutnotKS-semi-T_3space.

(x) LetKS-semi-closedsets{a}and{c}where$\{a\} \neq \{c\}$∃G={a,b}andH={c,d}suchthat{a}∈{a,b}and{c}∈{c,d}thenitisKS-semi-T_4spacebutnotKS-T_4space.

(xi) Similarly,by(vii)itisKS-semi-T_3spacebutnotKS-semi-T_4space.

Theorem1.15. IfUisKS-semi-T_0spaceandVisasubspaceofUthenVisalsoKS-semi-T_0.

Proof: LetUbeKS-semi-T_0spaceandVbeasubspaceofU.ToshowthatVisKS-semi-T_0space, letx,y∈Vwithx$\neq y$.SinceV⊆U,wehavex,y∈U.ButUisKS-semi-T_0.Sothereexists KS-semi-openset G such that G contains only one point x ∈G and Y∉G then V∩G is KS-semi-open setinVsuchthatx ∈V∩Gandy∉V∩G.HenceVisKS-semi-T_0.

Theorem1.16.AKStopologicalspace$(U, \tau_R(X),$ $KS_R(X))$isaKS-semi-T_0spaceiffKS-semiclosure of distinct points are distinct.

Proof: LetxandybedistinctpointsofU.SinceUisaKS-semi-T_0spacethereexistKS-semi-open setGsuchthatx∈Gandy∉G.ConsequentlyU/GisaKS-semi-closedcontainingybutnotx.ButKS_{scl}(y)istheintersectionofallKS-semi-closedsetcontainingy.Hencey∈KS_{scl} (y).But x∉KS_{scl} (y)asx∉U-G.ThereforeKS_{scl} (x)$\neq KS_{scl}$ (y).Conversely,letKS_{scl} (x)$\neq KS_{scl}$ (y)forx$\neq y$.Thenthereexistsatleastonepointz∈Usuchthatz∈KS_{scl} (x)butz∉KS_{scl} (y).We claimx∉KS_{scl} (y)becauseifx∈KS_{scl} (y), x⊆KS_{scl} (y)impliesKS_{scl} (x)⊆KS_{scl} (y).So,z∈KS_{scl} (y),whichisacontradiction.Hencex∉KS_{scl} (y),whichimpliesx∈U-KS_{scl} (y),whichisaKS-semi-opensetcontainingxbutnoty.HenceUisaKS-semi-T_0.

Theorem1.17.AKStopologicalspace$(U, \tau_R(X), KS_R(X))$isKS-semi-T_1ifandonlyifeachone-point set is KS-semi-closed.

Proof: AssumethatUisKS-semi-T_1.Letx ∈U.Thenforeachy∈U-{x}thereexistsKS-semi-openset Usuchthaty∈Uandx∉U.Sincex∉U,thesets{x}andUaredisjoint.Thatis,{x}∩U=φ thatimpliesU⊆U-{x}.Thusy∈U⊆U-xthatimpliesU-{x}isKS-semi-opensetthatimplies {x}isKS-semi-closedset.Conversely,assumethateachone-point setisKS-semi-closed.Letx,y∈U with$x \neq y$.So,U-{x} isKS-semi-opensetcontainingyandbutnotx.Also,U-{y} isKS-semi-open containing x but not y.So U is KS-semi-T_1.

Theorem1.18. EverysubspaceofKS-semi-T_1spaceisKS-semi-T_1.

Proof: Let$(U, \tau_R(X), KS_R(X))$be KS-semi-T_1space.Let$(V, \tau_R(Y), KS_{R_R}(Y))$be a subspace of U. Let x,y∈Vwith$x \neq y$.SinceV⊆U,wehavex,y∈U. SinceUisKS-semi-T_1thereexistKS-semi-open setsGandHsuchthatx∈G, y ∉G and y ∈H, x∉H.Let I =V∩G and J =V∩H.Then I and J areKS-semi-opensetsinV.Also,x ∈I,y ∉Iandy ∈J,x ∉J.So,VisKS-semi-T_1.

Theorem 1.19.A KS topological space U is KS-semi-T_1iff every finite subset of U is KS-semi-closedin U.

Proof: AssumethatUisKS-semi-T_1.LetGbeafinitesubsetofU.LetG={x1,x2,...xn}.ThenG={x1}∪{x2}∪...{xn}whichisKS-semi-closed,beginafiniteunionofKS-semi-closedsets.Conversely, leteachfinitesubsetofUisKS-semi-closedinU.Then{x}isKS-semi-closedsinceitisfinite.SinceeachsingletonisKS-semi-closed,UisKS-semi-T_1.

Theorem1.20.A KS topological space UisKS-semi-T_1spaceifftheintersectionofallKS-semineighborhoods of any point x in U is the singleton{x}.

Proof: AssumethatUisKS-semi-T_1.Letx∈U.LetGbetheintersectionofallKS-semineighborhoodsofx.LetybeanypointinUdifferentfromx.SincethespaceUisKS-semi-T_1.Thereexistsa KS-semineighborhoodNofxsuchthaty∉N.Sincey∉N,wehavey∉GsinceGistheintersection of all KS-semi neighborhoods of x.Since y ∉G, no point different from x is in G. Hence G={x}. Conversely,assumethattheintersectionofallKS-semineighborhoodsofpinUisp.Toprovethat,U is KS-semi-T_1, let x,y ∈ Uwith$x \neq y$.Let G = ∩{all KS-semi neighborhoods of x in U}.Then G={x}.Let H =∩{all KS-semi neighborhoods of y in U}.Then G={y}.Since y ≠ x, we have y ∉G thatimplies∃aKS-semi neighborhood P of x with y ∉U. Since$x \neq y$, x ∉Hthat implies a KS-semi neighborhoodQsuchthatywithx ∉Q.HenceUisKS-semi-T_1.

Theorem1.21. ForaKStopologicalspaceU,eachofthefollowingstatementareequivalent:

(i) UisKS-semi-T1space.

(ii) TheintersectionofallKS-semi-opensetscontainingthesetGisG.

(iii) TheintersectionofallKS-semi-opensetscontainingthepointx∈Uis{x}.

Proof:(i)⇒ (ii) Suppose U is KS-semi-T_1space.By Theorem (1.20) each singleton set is KS-semi- closedinU.LetG ⊆U.Thenforeachx ∈U-G,{x} isKS-semi-closedinUandhenceU-{x} is KS-semi-open.Clearly,G ⊆U-{x} foreachx ∈U-G.ThereforeG ⊆∩{U-{x}: x ∈U-G}.On theotherhand,ify∉Gtheny∈U-Gandy∉U-{y}.Thereforey∉∩{U -{x}:x∈U-G}⊆G.Therefore,theintersectionofallKS-semi-opensetscontainingthesetGisG.

(ii)⇒ (iii) Suppose the intersection of all KS-semi-open sets containing the set G is G. Take G ={x}. Then G ={x}=∩{H: H is KS-semi-open and x ∈H}.Therefore, the intersection of all KS-semi-open setscontainingthepointx ∈Uis{x}.
(iii)⇒(i)Let x,y ∈Uand$x \neq y$.Then y ∉{x}=∩{H: H is KS-semi-open and x ∈H}.Hence there existsKS-semi-opensetHcontainingxbutnoty.Similarly,thereexistsKS-semi-opensetHcontainingybutnotx.Thus,UisKS-semi-T_1space.

Theorem1.22. EachsingletonsetinKS-semi-T_2spaceisKS-semi-closed.

Proof: LetUbeKS-semispace.SinceUisKS-semi-T_2.⇒UisKS-semi-T_1.This⇒{x}isKS-semi-closedforx∈U.Hence,eachsingletonsetinKS-semi-T_2.

Theorem1.23. AsubspaceofKS-semi-T_2spaceisKS-semi-T_2space.

Proof: Let V be a subspace of KS-semi-T_2space U. Let p,q ∈Vwith $p \neq q$.Then p,q ∈U. Since U is KS-semi-T_2, there exist KS-semi-open sets G and H such that p ∈G, q ∈Hand G∩H =φ.Thus, we haveG∩H,H∩VareKS-semi-openinV,(G∩V)∩(H∩V)=φ.p ∈G∩Vandq∈H∩V.Hence VisKS-semi-T_2space.

Theorem1.24. InanyKStopologicalspaceU,thefollowingareequivalent:

(i) UisKS-semi-T_2space.

(ii) Foreachx $\neq y$,thereexistsaKS-semi-opensetGsuchthatx∈Gandy∉KS_{scl}(G).

(iii) Foreachx∈U,{x}=∩{KS_{scl}(U):UisaKS-opensetinUandx∈U}

Proof:(i)⇒(ii) Assume (i) holds.Let x,y ∈Uandx $\neq y$,then there exists disjoint KS-semi-open setsG and H such that x ∈G and y ∈H.Clearly U-H is KS-semi-closed set.Since G∩H =φ,G ⊆U-H. ThereforeKS_{scl}(G)⊆KS_{scl}(U-H) = U-H.Nowy∉U-Hthatimpliesy∉KS_{scl}(G).
(ii)⇒(iii)Foreachx≠ythereexistsKS-semi-opensetGsuchthatx∈Gandy∉KS_{scl}(G).So, y ∉∩{KS_{scl}(G):GisKS-semi-opensetinUandx ∈G}={x}.
(iii)⇒(i)Letx,y ∈Uandx≠y.ByhypothesisthereexistsKS-semi-opensetGsuchthatx ∈Gand KS_{scl}(G).ThisimpliesthereexistsKS-semi-closedsetHsuchthaty∉H.Thereforey∈U-Hand U-HisKS-semi-openset.Thus,thereexiststwodisjointKS-semi-opensetGandU-Hsuchthat x ∈Gandy ∈U-H.Therefore,UisaKS-semi-T_2space.

Theorem 1.25. Let the topological space U is semi-T_3space iffKS topological space U is KS-semi-T_3space.

Proof: SupposeUaresemi-T_3space.Letx∈UandA⊆UisKS-semi-closedx∈U-A.Thereforex∈Uand A ⊆U. Since U is semi-T_3, there exists disjoint KS-semi-open sets G,H ∈U. x ∈G and A ⊆H. Thisimpliesthatx ∈GandA ∈H.SinceGandHaredisjointKS-semi-opensets,wehave G∩H = φ. Thus G∩H = φ.Hence G and H are disjoint KS-semi-open sets.This implies that U is KS-semi-T_3. Conversely, assume that U is KS-semi-T_3.Let x∈Uand A be a KS-semi-closed subset of U. Thereforex ∈Uand A is KS-semi-closed in U. Since U is KS-semi-T_3there exists disjoint KS-semi-open sets G andHsuchthatx∈GandA⊆H.Hencex∈GandA⊆H.ThisprovesthatUisKS-semi-T_3space.

Theorem1.26. AsubspaceofKS-semi-T_3spaceisKS-semi-T_3space.

Proof: LetUbeKS-semi-T_3spaceandVbeasubspaceofU.ToprovethatVisKS-semi-T_3.Let P∈VandFbea KS-semi-closed set in V such that P ∉F. So, F =V∩ KS_{scl}(F).Since P ∉F. We seethatP∉KS_{scl}(F).SinceUisKS-semi-T_3,thereexist disjointKS-semi-opensetsGandHinU suchthatKS_{scl}(F) ⊆G,P∈H.NowF⊆ KS_{scl}(F) ⊆G.SinceF⊆V,F⊆V∩G.SinceP∈V andP∈H,P∈Y∩H.Further (V∩G)∩(V∩H)=φ.since G∩H =φ.Thus V∩G, V∩Hare KS-semi-opensets inV,P∈V∩H,F⊆V∩Gand(V∩G)∩(V∩H)=φ.HenceVisKS-semi-T_3.

Theorem 1.27.A KS topological space U is KS-semi-T_3iff for any x∈Uand KS-semi-neighborhood Nofx,thereisKS-semi-opensetGsuchthatx∈ G⊆KS_{scl}(G)⊆N.

Proof: Assume that U is KS-semi-T_3space and N is KS-semi-neighborhood of x. Then $N^c$is a KS- semi-closedsetandx∉N^c.Since U is T_3, there exist disjoint KS-semi-open sets G and H such that x∈GandN^c⊆H.So,H^c⊆N.SinceG∩H=φ, G⊆H^c⇒KS_{scl}(G)⊆H^c.SinceH^{c}isKS-semi-closedset.Thusx∈G⊆KS_{scl}(G)⊆ N.Conversely,assumethatthegivenconditionissatisfied.LetFbeKS-semi-closedset inUandx∉F.SinceF^{c}isKS-semineighborhoodofx,byassumptionthereisKS-semi-opensetGsuchthatx∈G⊆KS_{scl}(G)⊆F^c.Thus,thedisjointKS-semi-opensetsG and $[KS_{scl}(G)]^c$containsxandFrespectively.HenceUisKS-semi-T_3.

Theorem1.28.A KS topological space U is KS-semi-T_3iff for any x∈Uand KS-semi-neighborhood Nofx,thereisKS-semi-opensetGsuchthatx∈ G⊆KS_{scl}(G)⊆N.**Proof:**LetUbe KS-semineighborhood of x, there exists G belong to KS-semi-open in U such that x∈G⊆U.Now G^cbelongs to KS-semi-closed in U and x ∉G^c.From (i) there exist P, Q disjoint KS-semi-open sets suchthatG^c⊆P,x∈ Q,P∩Q=φ.So,Q⊆M^c.NowKS_{scl}(Q)⊆KS_{scl}(P^c)=G^candG^c⊆P. Thisimplies P^c⊆G ⊆U.ThereforeKS_{scl}(Q) ⊆U.Conversely, letKS-semi-closedFinUandx ∉F orx∈F^candUisKS-semi-openandsoF^c isKS-semineighborhoodofx.Byhypothesis,thereexist KS-semi-openneighborhoodNsuch that x∈N, KS_{scl}(N)⊆F^c.This implies F⊆{U-KS_{scl}(N)}and N ∩{(U-KS_{scl}(N)} = φ.Thus,UisKS-semi-T_3.

Theorem1.29. LetUisKS-semi-T_3iffforeveryGbelongsKS-semi-closedinUandpointp∈(U-G) thenx∈U,G⊆NandKS$_{scl}$(N) ∩KS_{scl}(U)=φwhereNandUareopensets.

Proof: Given that U is KS-semi-T3.Let G belongs to KS-semi-closed in U and U∉G.Then p ∈Mand G⊆NandM∩N=φ where M and N are open sets.This implies M∩KS_{scl}(N)=φ.Since U is KS- semi-T_3,p∈Pand KS_{scl}(N)⊆Q, P ∩N =φ where P,Q are KS-semi-open.Also KS_{scl}(P)∩Q =φ. LetV= M∩Pthenp ∈V,G ⊆NandKS$_{scl}$(N)∩KS_{scl}(V) =φwhereN,VareKS-semi-open inU. Conversely, suppose for all G belongs to KS-semi-closed in U and p∈(U/G), we have p∈U, G⊆NandKS$_{scl}$(N)∩KS_{scl}(V)=φ where N, V are KS-semi-open sets.This implies p∈U,G⊆N, and V∩N= φ.Therefore, U is KS-semi-T_3.

Theorem1.30.AKS-semi-closedsubspaceofKS-semi-T_4spaceisKS-semi-T_4space.

Proof: Let V be a KS-semi-closed subspace of KS-semi-T_4space.Let $C_1$and $C_2$are disjoint KS-semi- closedsubsetsofV.SinceVisKS-semi-closedinU,C_1andC_2arealsoKS-semi-closedinU.There existdisjointKS-semi-opensetsGandHinUsuchthatC_1⊆GandC_2⊆H.SinceVcontainsboth C_1andC_2,wehaveC_1⊆V∩G,C_2⊆V∩Hand(V∩G) ∩(V∩H)=φ.SinceGandHare KS-semi-open in U,(V∩G) and (V∩H) are KS-semi-open in V. Thus, in the subspace V, we have disjointKS-semi-opensets(V∩G)containing$C_1$and(V∩H)containingC_2.HencethesubspaceVisKS-semi-T_4.

Theorem1.31. AspaceUisKS-semi-T_4iffforanyKS-semi-opensetAcontainingKS-semi-closed setFthereexistsKS-semi-opensetGsuchthatF⊆ G⊆KS_{scl}(G)⊆ A.
Proof: AssumethatUisKS-semi-T_4space.SinceFandA^{c}aredisjointandKS-semi-closedsetsin U,thereexistsdisjointKS-semi-opensetsGandHsuchthatF⊆ GandA^c⊆ H.SinceGandH aredisjoint,G⊆H^c,wehaveKS_{scl}(G)⊆H^c⊆A. Thus,wehaveKS-semi-opensetGsuchthatF⊆G⊆KS_{scl}(G)⊆A.Conversely,assumethattheconditionholds.LetAand BbedisjointKS-semi-closedsetsinU.SinceB^cisKS-semi-openandcontainstheKS-semi-closedsetAbyassumption,thereisKS-semi-opensetVsuchthatA⊆V⊆KS_{scl}(V)⊆B^c.ThuswehaveKS-semi-opensetV⊇Aand[KS_{scl}(V)]^c⊇B.soUisKS-semi-T_4space.

2.KSα-T_i,(i=0,1,2,3,4)

Inthissection,wedefineanddiscusssomeofthepropertiesofKSα-T_0,KSα-T_1,KSα-T_2,KSα-T_3 andKSα-T4spacesinkasajtopologicalspaces.

Definition2.1.AKStopologicalspace$(U, \tau_R(X), KS_R(X))$issaidtobe

(i) KSα-T_0space(KSα-T_0space)ifapairofdistinctpointsx,y∈ UeitherthereexistsaKSα-opensetG∈$KS_R(X)$ suchthatx∈ G,y∉GorthereexistsaKSα-opensetH∈$KS_R(X)$suchthaty∈ H,x ∉H.

(ii) KSα-T_1space(KSα-T_1space)ifapairofdistinctpointsx,y∈ Uwith$x \neq y$,thereexistsaKSα-opensetG,H∈$KS_R(X)$suchthatx∈G,y∉G;y∈H,x∉H.

(iii) KSα-T_2space(KSα-T_2space)ifapairofdistinctpointsx,y∈ Uwith$x \neq y$,thereexistsaKSα-opensetG,H∈$KS_R(X)$suchthatx∈G,y∈H,G∩H=φ.

(iv) KSα-T_3space(KSα-T_3space)ifanelementx∈UandaKSα-closedsetF⊆Usuchthatx∉F,thereexistsdisjointKSα-opensetsG_1,G_2⊆Usuchthatx∈G_1,F⊆G_2.

(v)KS α-T_4space(KSα-T_4space) if a pair of disjoint KSα-closed sets C_1,C_2⊆U,

there exists disjointKSα-opensetsG_1,G_2⊆UsuchthatC_1⊆G_1,C_2⊆G_2.

Exemplar2.2. LetU={a,b,c,d,e}withU/R={{a,c},{b,d},{e}}andX={a,b,c}⊆U. Then$\tau_R(X)$={φ,U,{a,c},{b,d},{a,b,c,d}}.IfweconsiderS={c},S'={a,b,d,e}then$KS_R(X)$={φ,{a},{c},{a,c}{b,d},{a,b,d},{b,c,d},{a,b,c,d},{a,b,d,e}, U}andKSα-open={φ,{a},{c},{a,c},{b,d},{a,b,d},{b,c,d},{a,b,c,d},{a,b,d,e},{b,c,d,e},U}.

(i) Letc,d∈U,c ≠ d∃aKSα-openset={a,c}suchthatc∈{a,c}andd∉{a,c}.

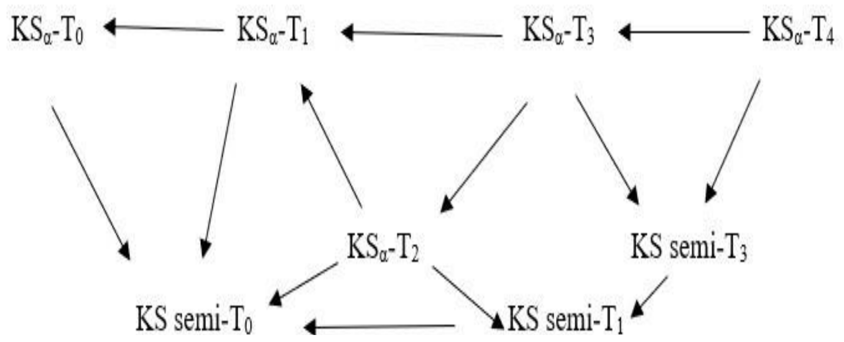
(ii) FromExemplar2.2(i)and∃aKSα-openset={b,d}suchthatd∈{b,d}andc∉{b,d}.

(iii) FromExemplar2.2((i)&(ii))∃{a,c}∩{b,d}=φ.

(iv) Lete∈U,{a,c}=KSα-closedsetsande∉{a,c}∃{e}and{a,c}=KSα-opensetssuchthate∈{e}and{a,c}⊆{a,c}.

(v) Let{a}and{c}=KSα-closedsetswhere {a}≠{c}∃{a}and{b,c,d}=KSα-open sets where {a}≠{b,c,d}suchthat{a}⊆{a}and{c}⊆{b,c,d}.

Remark2.3. Fromthedefinitionabove,wehavethefollowingdiagramrepresentstherelationbetween KS-spaces:



Theorem2.4. IfUisKSα-T_0spaceandVisasubspaceofUthenVisalsoKSα-T_0space.

Proof: LetUbeKSα-T_0spaceandVbeasubspaceofU.ToshowthatVisKSα-

T_0space,letx,y∈Vwithx≠y.SinceV⊆U,wehavex,y∈U.ButUisKSα-T_0space.SothereexistsaKSα-opensetGsuchthatGcontainsonlyonepointx∈Gandy∉GthenV∩GisaKSα-opensetinVsuchthat x ∈V∩Gandy∉V∩G.HenceVisKSα-T_0space.

Theorem2.5.AKStopologicalspace$(U, \tau_R(X), KS_R(X))$isaKSα-T_0spaceiffKSα-closureof distinct points are distinct.

Proof: Let x and y be distinct points of U. Since U is KSα-T_0space there exist a KSα-open set G such thatx∈Gandy∉G.ConsequentlyU-GisKS$_\alpha$-closedcontainingybutnotx.But$KS_{\alpha cl}(y)$istheintersectionofallKS$_\alpha$-closedsetcontainingy.Hencey∈$KS_{\alpha cl}(y)$.Butx∉$KS_{\alpha cl}(y)$ asx∉(U-G).Therefore $KS_{\alpha cl}(x)$ $KS_{\alpha cl}(y)$.Conversely, let $KS_{\alpha cl}(x) \neq KS_{\alpha cl}(y)$ for x≠ y.Then there exists at leastonepointz∈ Usuchthatz∈$KS_{\alpha cl}(x)$butz∉$KS_{\alpha cl}(y)$.Weclaimx∉$KS_{\alpha cl}(y)$becauseif x ∈$KS_{\alpha cl}(y)$, x ⊆$KS_{\alpha cl}(y)$ implies $KS_{\alpha cl}(x) \subseteq KS_{\alpha cl}(y)$.So, z ∈$KS_{\alpha cl}(y)$, which is a contradiction. Hencex∉$KS_{\alpha cl}(y)$,whichimpliesx∈U-$KS_{\alpha cl}(y)$,whichisaKS$_\alpha$-opensetcontainingxbutnoty.HenceUisaKSα-T_0space.

Theorem2.6.AKStopologicalspace$(U, \tau_R(X), KS_R(X))$isKSα-T_1spaceifandonlyifeachone-point set is KSα-closed set.

Proof: AssumethatUisKSα-T_1space.Letx∈U.Thenforeachy∈U-{x}thereexistsaKSα-openset Usuchthaty∈Uandx∉U.Sincex∉U,thesets{x}andUaredisjoint.Thatis,{x}∩U=φthatimpliesU⊆U-{x}.Thusy∈U⊆U-xthatimpliesU-{x}isaKSα-opensetthatimplies{x}isKSα-closedset.Conversely,assumethateachone-pointsetisKSα-closedset.Letx,y∈Uwith x≠y.So,U-{x}isaKSα-opensetcontainingyandbutnotx.Also,U-{y}isKSα-opencontaining xbutnoty.SoUisKSα-T_1space.

Theorem2.7. EverysubspaceofKSα-T_1spaceisKSα-T_1space.

Proof: Let$(U, \tau_R(X), KS_R(X))$beKSα-T_1space.Let$(V, \tau_R(Y), KS_R(Y))$beasubspaceofU.Letx,y∈Vwith x≠y.Since V⊆U,we have x,y ∈U. Since U is KSα-T_1space there exist KSα-open setsGandHsuchthatx∈G,y ∉G and y ∈H, x∉H.Let I =V∩G and J =V∩H.Then I and J are KSα-opensetsin V. Also, x∈I,y ∉Iandy ∈J,x∉J.So, V is KSα-T_1space.

Theorem2.8.AKStopologicalspaceUisKSα-T_1spaceiffeveryfinitesubsetofUisKSα-closedin U.

Proof: AssumethatUisKSα-T_1space.LetGbeafinitesubsetofU.LetG={x1,x2,...xn}.Then G={x1}∪{x2}∪...{xn} whichisKSα-closed,beginafiniteunionofKSα-closedsets.Conversely, leteachfinitesubsetofUisKSα-closedinU.Then{x}isKSα-closedsinceitisfinite.SinceeachsingletonisKSα-closed,UisKSα-T_1space.

Theorem2.9. LetU isKSα-T_1spaceifftheintersectionofallKSα-neighborhoodsof any point x in U is the singleton {x}.

Proof: AssumethatUisKSα-T_1space.Letx∈U.LetGbetheintersectionofallKSα-neighborhoodsof x. Let y be any point in U different from x. Since the space U is KSα-T_1space. There exists a KSα- neighborhoodNofxsuchthaty∉N.Sincey∉N,wehavey∉GsinceGistheintersectionofallKSα-neighborhoodsofx.Sincey∉G,nopointdifferentfromxisinG.HenceG={x}.Conversely,assume that the intersection of all KSα-neighborhoods of p in U is p.To prove that U is KSα-T_1space, letx,y∈Uwithx≠y.LetG = ∩{allKSα-neighborhoodsofxinU}.ThenG={x}.LetH= ∩{all KSα-neighborhoods of y in U}.Then G={y}.Since y≠ x, we have y ∉G that implies ∃a KSα-neighborhoodPof x with y ∉U.Since x ≠y, x ∉Hthat implies a KSα-neighborhood Q such that y withx ∉Q.HenceUisKSα-T_1space.

Theorem2.10. ForaKStopologicalspaceU,eachofthe followingstatementareequivalent:

(i) UisKSα-T_1space.

(ii) TheintersectionofallKSα-opensetscontainingthesetGisG.

(iii) TheintersectionofallKSα-opensetscontainingthepointx∈Uis{x}.

Proof:(i)⇒ (ii)SupposeUisKSα-T_1space.ByTheorem(2.13)eachsingletonsetisKSα-closedinX. LetG⊆U.Thenforeachx∈U-G,{x}isKSα-closedinUandhenceU-{x}isKSα-open.Clearly, G⊆ U-{x}foreachx∈ U-G.ThereforeG⊆ ∩{U-{x} :x∈ U-G}.Ontheotherhand,if y∉Gtheny ∈U-Gandy∉U-{y}.Thereforey∉∩{U-{x}:x∈U-G}⊆G.Therefore,theintersectionofallKSα-opensetscontainingthesetGisG.

(ii)⇒(iii) Suppose the intersection of all KSα-open sets containing the set G is G. Take G = {x}. Then G ={x}=∩{H: H is KSα-open and x ∈H}.Therefore, the intersection of all KSα-open sets containingthe point x∈Uis {x}.
(iii)⇒(i)Letx,y∈ Uandy≠ x.Theny∉{x} = ∩{H:HisKSα-openandx∈ H}.Hencethere existsaKSα-opensetHcontainingxbutnoty.Similarly,thereexistsaKSα-opensetIcontainingybutnotx.Thus,UisKSα-T_1space.

Theorem2.11. EachsingletonsetisKSα-T_2spaceisKSα-closed.

Proof: Let$(U, \tau_R(X), KS_R(X))$beKSα-T_2space.SinceUisKSα-T_2space.⇒UisKSα-T_1space.⇒{x}isKSα-closedforx∈U.Hence,eachsingletonsetinKSα-T_2spaceisKSα-closed.

Remark2.12. EachsingletonsetinKSα-T_2spaceisKS-semi-closed.

Theorem2.13. AsubspaceofKSα-T_2spaceisKSα-T_2space.

Proof: LetVbeasubspaceofKSα-T_2spaceU.Letp,q∈Vwithp≠q.Thenp,q∈U.SinceUis KSα-T_2space,thereexistKSα-opensetsGandHsuchthatp ∈G,q∈HandG∩H= φ.Thus,we haveG∩H,H∩VareKSα-openinV,(G∩V)∩(H∩V) = φ.p∈G∩Vandq∈H∩V.HenceV isKSα-T_2space.

Theorem2.14. InanyKStopologicalspace,thefollowingareequivalent:

(i) UisKSα-T_2space.

(ii) Foreachx≠y,thereexistsaKSα-opensetGsuchthatx∈G and y∉$KS_{\alpha cl}$(G).

(iii) Foreachx∈U,{x}=∩{$KS_{\alpha cl}$(U):UisaKSopensetinUandx∈U}

Proof:(i)⇒(ii)Assume(i)holds.Letx,y ∈Uandx≠y,thenthereexistsdisjointKSα-opensetsG andHsuchthatx∈Gandy∈H.ClearlyU-HisaKSα-closedset.SinceG ∩H=φ,G⊆U-H. ThereforeKS$_{\alpha cl}$(G)⊆KS$_{\alpha cl}$(U-H) = U-H.Nowy∉U-Hthatimpliesy∉KS$_{\alpha cl}$(G).
(ii)⇒(iii)Foreachx≠ythereexistsaKSα-opensetGsuchthatx∈Gand y∉KS$_{\alpha cl}$(G).So y∉∩{KS$_{\alpha cl}$(G):GisaKSα-opensetinUandx ∈ G}={x}.

(iii)⇒(i)Letx,y∈Uandx≠y.ByhypothesisthereexistsaKSα-opensetGsuchthatx∈Gand KS$_{\alpha cl}$(G).ThisimpliesthereexistsaKSα-closedsetHsuchthaty∉H.Thereforey∈U-Hand U-HisaKSα-openset.Thus,thereexiststwodisjointaKSα-opensetGandU-Hsuchthatx∈ G andy ∈ U-H.Therefore,UisaKSα-T_2space.

Theorem2.15. LettheKStopologicalspaceUisα-T_3spaceiffKStopologicalspaceUisKSα-

T_3space.

Proof: SupposeUisα-T_3space.Letx∈UandA⊆UisKSα-closedx∈U-A.Thereforex∈U andA⊆U.SinceUisα-T_3space,thereexistsdisjointαopensetsG,H∈U.x∈ GandA⊆ H. Thisimpliesthatx∈ GandA∈ H.SinceGandHaredisjointαopensets,wehaveG ∩H=φ. ThusG∩H =φ.Hence G and H are disjoint KS-semi-open sets.This implies that U is KSα-T_3space. Conversely,assumethatUisKSα-T_3space.Letx∈ UandAbeaαclosedsubsetofU.Therefore x ∈UandAisKSα-closedinU.SinceUisKSα-T_3thereexistsdisjointKSα-opensetsGandHsuch thatx∈GandA⊆H.Hencex∈GandA⊆H.ThisprovesthatUisKSα-T_3space.

Theorem2.16. AsubspaceofKSα-T_3spaceisKSα-T_3space.

Proof: LetUbeKSα-T_3spaceandVbeasubspaceofU.ToprovethatVisKSα-T_3space.LetP ∈V andFbeaKSα-closedsetinVsuchthatP∉F.SoF= V∩$KS_{\alpha cl}$(F).SinceP∉F.Weseethat P∉$KS_{\alpha cl}$(F).Since U is KSα-T_3space, there exist disjoint KSα-open sets G and H in U such that $KS_{\alpha cl}$(F)⊆G,P∈H.NowF⊆$KS_{\alpha cl}$(F) ⊆G.SinceF⊆V,F⊆V∩G.SinceP∈VandP∈H, P∈V∩H.Further(V∩G)∩(V∩H)=φ.since G∩H =φ.Thus V∩G, V∩Hare KSα-open sets inV,P∈V∩H,F⊆V∩Gand(V∩G)∩(V∩H) =φ.HenceVis KSα-T_3space.

Theorem2.17.AKStopologicalspaceUisKSα-T_3spaceiffforanyx∈UandKSα-neighborhood Nofx,thereisaKS_α-opensetGsuchthatx∈ G⊆ $KS_{\alpha cl}$(G)⊆N.

Proof: Assume that U is KSα-T_3space and N is KSα-neighborhood of x.Then $N^c$is a KSα-closed setandx∉N^c.Since U is KSα-T_3space, there exist disjoint KSα-open sets G and H such that x∈G and N^c⊆H.So,H^c⊆N.SinceG∩H=φ,G⊆H^c⇒$KS_{\alpha cl}$(G)⊆H^c.SinceH^cisaKS_α-closedset.Thusx∈G⊆$KS_{\alpha cl}$(G)⊆N.Conversely,assumethatthegivenconditionis satisfied.Let F be aKSα-closedset in U and x ∉F.Since $F^c$is KSα-neighborhood of x, by assumption there is a KSα-open setGsuchthatx∈G⊆$KS_{\alpha cl}$(G)⊆F^c.Thus,thedisjointKS_α-opensetsGand[$KS_{\alpha cl}$(G)]^{c}contains xandFrespectively.HenceUisKSα-T3space.

Theorem 2.18. The statements given below are equivalent:

(i) UisKSα-T_3space.
(ii)Forx∈UandeachKSα-openneighborhoodUthereexistKSα-neighborhoodofUsuchthat$KS_{\alpha cl}$(N)⊆U.

Proof:(i)⇒(ii) Let U be KSα-neighborhood of x, there exists G belong to KSα-open in U such thatx∈G⊆U.NowG^cbelongstoKSα-closedinUandx∉G^c.From(i)thereexistP,Qdisjointa KS_α-opensetsuchthatG^c⊆P, x ∈Q,P∩Q = φ.So,Q ⊆M^c.Now$KS_{\alpha cl}$(Q) ⊆ KS-αcl(P^c) = G^c andG^c⊆P.Thisimplies P^c⊆G ⊆U.Therefore $KS_{\alpha cl}$(Q) ⊆U.

(ii)⇒(i)LetKSα-closedFinUandx∉Forx∈F^candUisKSα-openandsoF^cisKSα-neighborhood ofx.Byhypothesis,thereexistKS_α-openneighborhoodNsuchthatx∈N, $KS_{\alpha cl}$(N)⊆F^c.Thisimplies F⊆ {U- $KS_{\alpha cl}$(N)}andN∩{(U- $KS_{\alpha cl}$(N)} =φ.ThusU isKS_α-T_3space.

Theorem2.19. LetUisKSα-T_3spaceiffforeveryGbelongsKSα-closedinUandpoint p∈(U-G)thenx∈U,G⊆NandKS$_{\alpha cl}$(N)∩$KS_{\alpha cl}$(U)=φwhereNandUareopensets.

Proof: Given that U is KSα-T_3space.Let G belongs to KSα-closed in U and U∉G.Then p ∈M andG⊆NandM∩N=φwhere M and N are open sets.This implies M∩$KS_{\alpha cl}$(N)=φ.Since U is KSα-T_3space,p∈Pand $KS_{\alpha cl}$(N) ⊆Q, P∩N= φ where P,Q are KS_α-open.Also $KS_{\alpha cl}(P)\cap Q = \varphi$. LetV= M ∩Pthen p ∈V, G ⊆Nand $KS_{\alpha cl}(N)\cap KS_{\alpha cl}(V) = \varphi$ where N, V are KS_α-open in U. Conversely,supposeforallGbelongstoKSα-closedinUandp∈(U-G),wehavep∈U,G⊆Nand $KS_{\alpha cl}(N)\cap KS_{\alpha cl}(V)=\varphi$whereN,VareKS$_\alpha$-opensets.Thisimpliesp∈U,G ⊆N,andV ∩N=φ.Therefore,UisKSα-T_3space.

Theorem2.20.AKSα-closedsubspaceofKSα-T_4spaceisKSα-T_4space.
Proof: LetVbeaKSα-closedsubspaceofKSα-T_4space.LetC_1andC_2aredisjointKSα-closed subsetsofV.SinceVisKSα-closedinU,C_1andC_2arealsoKSα-closedinU.Thereexistdisjoint KSα-opensetsGandHinUsuchthatC_1⊆GandC_2⊆H.SinceVcontainsbothC_1andC_2,wehave C_1⊆ V∩G,C_2⊆ V∩Hand(V∩G)∩(V∩H)=φ.SinceGandHareKSα-openinX,(V∩G)and(V∩H)areKSα-openinV.Thus,inthesubspaceV,wehavedisjointKSα-opensets(V∩G)containing$C_1$and(V∩H)containingC_2.HencethesubspaceVisKSα-T_4space.

Theorem2.21.AKStopologicalspaceUisKSα-T_4spaceiffforanyaKSα-opensetAcontainingaKSα-closedset FthereexistsaKSα-opensetGsuchthatF⊆ G⊆$KS_{\alpha cl}$(G)⊆ A.
Proof: Assume that U is KSα-T_4space. Since F and A^c are disjoint and KSα-closed sets in U, there existsdisjointKSα-open sets G and H such that F ⊆G and A^c⊆H.Since G and H are disjoint, G⊆H^c,wehave$KS_{\alpha cl}$(G)⊆H^c⊆A.Thus,wehaveaKS$_\alpha$-opensetGsuchthatF⊆G⊆$KS_{\alpha cl}$(G)⊆A.
Conversely,assumethattheconditionholds.LetAandBbedisjointKSα-closedsets inU.SinceB^cisKSα-openandcontainstheKSα-closedsetAbyassumption,thereisaKSα-opensetVsuchthat A⊆ V⊆ $KS_{\alpha cl}$(V)⊆B^c.Thus,wehaveaKS$_\alpha$-opensetV⊇ Aand $[KS_{\alpha cl}(V)]^c$⊇ B.soUisKSα-T_4space.

3.Conclusion

The class of KS-open and KS-closed sets has an important role to examine the separation axiom in kasajtopologicalspace.Inthisworkweintroducedandstudiednewtypesofseparationaxiomsnamely, KS-semi-T_i,(i=0,1,2,3,4) space and KSα-T_i,(i=0,1,2,3,4) space.Several characterizations and the relation betweenpropertiesoftheabovesetsofseparationaxiomsarediscussedandproved.Furthermore,useful results are investigated by comparing KS-semi-T_i,(i=0,1,2,3,4) space and KSα-T_i,(i=0,1,2,3,4) space inthe context of these new concepts.

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