Some Results On Eccentric Coloring In Graphs

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Abstract

Foragraph$G$= (V,E),anEccentriccoloringisacolorfunction:$V \to N$sothat

(i) $(color(u)=color(v)) \Rightarrow d(u,v) > color(u), \forall u,v \in V$

(ii) $color(v) \leq e(v), \forall v \in V$.

IsEccentriccoloringofagraph.Inthispaper,wehaveattainedtheEccentriccoloring forsomestandardgraphsandwehaveobservedthatonlyforsomespecialclassesofg raphseccentriccoloringissatisfied.TheEccentriccoloringisrestrictedtosomeclas sesofgraphs.

Keywords:Eccentriccoloring,Eccentricvertex,EccentricChromaticnumber, Eccentricityofavertex,Eccentricity,Firecrackergraph, Bistar graph.

1 Introduction

Graph Coloring is a simple way of labeling graph components such as edges, vertices, and regions under some constraints.In graph 'G'no two adjacent vertices, adjacent edges, or adjacent regions are colored with a minimum number of colors. This number is called the chromatic number and the graph is called a properly colored graph.

The objective of the graph coloring problem aims to give particular graph nodes colors within specific bounds.Vertex coloring is the most prevalent problem with graph coloring. The task is to figure out how to use 'm' distinct colors to color each vertex of a graph so that no two vertex neighbors have the same color. Edge coloring is one of the other problems with graph color.

Graphcoloringproblemhasa widerangeofapplications. MakingaScheduleorTimeTable, MobileRadio Frequency Assignment, Clustering, Data mining, Image Capturing,Image segmentation, Networking,Sudoku, RegisterAllocation, Resource Allocation, Processes Scheduling,BipartiteGraphs, andMapColoring.

In this current work, we concentrate on simple, finite graphs. Inourpreviouspaper,someresultson the EccentricColoringofsomeclassesofgraphswereobtained. Continuingthesameworkinthispaperforsomeotherclassesofgraphs we have obtained some results. The result is restricted only to some classes of graphs.

DEFINITIONS

Definition1.1
Distance:The length of the shortest path between u and v in a graph *G* is represented by the distance symbol, *d(u, v).*

Definition1.2
Diameter and Radius: The diameter *diam(G)* denotes the vertices with the highest eccentricity, while the radius *rad(G)* denotes the vertices with the least eccentricity.

Definition1.3
Eccentricity:The distance *e(u)* between vertex *u* and the farthest vertex is what is referred to as eccentricity. *v* is the eccentric vertex of *u* if *d(u, v) = e. (u).*

Definition1.4
EccentricColoring:ForagraphG=(V,E)isafunctioncolor:$V \rightarrow N$suchthat

(i) $(color(u)=color(v)) \Rightarrow d(u,v)>color(u), \forall u,v \in V.$
(ii) $v \in V, color(v) \leq e(v), \forall v \in V.$
IsEccentriccoloringofagraph .

Definition1.5
EccentricChromatic number:$\chi_e \in N$isan Eccentric Chromatic number.GivenagraphG,thesmallestnumberofcolorsthatmaybeeccentricallycolore dbycolors$V \rightarrow \{1,2,....\chi_e\}$.

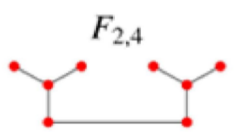
EccentricColoringofFirecrackergraph

Definition 2.1
Star Graph:

The unique type of graph known as a star graph consists of one vertex with a degree of n-1 and *n-1* vertices with a degree of 1. The central vertex of a star graph is connected to *n-1* of the total n vertices. *Sn* stands for a star graph with nvertices.

Definition 2.2
Firecracker graph:Concatenation of mn-stars is formed by connecting one leaf rom each N-star to a star with n pendant vertices. This results in a firecracker graph.

Example:



FIRECRACKERGRAPHOF $F_{2,4}$

Theorem1: EccentricColoringofFirecrackerGraph$f_{m,n}$

$$f_{m,n} = \begin{cases} m\leq 2, n\leq 2 \text{ noteccentriccolorable} \\ \text{otherwise 3-colorable} \end{cases}$$

Proof: Firecrackergraph$f_{m,n}$isagraphobtainedbytheconcatenationofmn-stars by linkingone leaf fromeach N-staris a star with$n$pendantvertices.Firecrackergraph$f_{m,1}$ and$f_{m,2}$ is1.
Weassigncolors
Color(C_1)=Red,
Color(C_2)=Blue,*Color*(C_3)=Green
.

Case(i):(a)For$f_{m,n}$wherem=2,n=2.
$f_{2,2}$isnotEccentriccolorable
Fromtheproofofthetheoremfromthearticle[11],If$G$is agraphwith
$diam(G)\leq 2$,then$G$isnoteccentriccolorable.

(b)For$f_{m,n}$wherem=2,n=3.Form
=2,n=m+1=2+1
$E(u_1)=E(u_k)=m+1$,

WeassigncolorC_1forthefirstleafandcolorC_2forthesecondleaflinkedtothe firstleaf.
$E(v_1)=E(v_k)=m+2$,

WeassigncolorC_3forthefirstleafandsecondleafofvertex$v$connectedto$n$star.

$E(w_1)=E(w_k)=m+3$,
We assign color $C_2$for the first leaf and thesecond leaf of vertex$w$connected to the$n$ star.
The sameprocedurecontinuesform=2,n=4, andm=2,n=5.

Case(ii):(a)For$f_{m,n}$wherem=3,n=2

Form=3,n=(m-1)=3-1=2
$E(u_1)=E(u_k)=m$

WeassigncolorC_1forthefirstleafandthe lastleafcolorC_2forthesecondleaflinked to thefirstleaf.
$E(v_1)=E(v_k)=m+1$

WeassigncolorC_2forthefirstleafandlastleafandcolorC_3forthesecondleafofvertex$v$connectedto the$n$star.
$f_{3,2}$isEccentricallycolorable

(b)For$f_{m,n}$wherem=3,n=3.
Form=3,n=m=3
$E(u_1)=E(u_k)=m+1$,
$E(u_2)=m$

WeassigncolorC_1forthefirstand lastleaf andcolorC_2forthesecondleaflinkedto the firstleaf.

$E(v_1)=E(v_k)=m+2$,

WeassigncolorC_2forthefirstleafand thelastleafofvertex$v$connectedto$n$ star.
$E(w_1)=E(w_k)=m+3$,

WeassigncolorC_3forthefirstleafand thelastleafofvertex$w$connectedto$n$ star.
The sameprocedurecontinuesform=3,n=4,andm=3,n=5.

Case(iii):(a)For$f_{m,n}$wherem=4,n=2.Form=4,n =(m-2)=4-2=2
$E(u_1)=E(u_k)=m$

WeassigncolorC_1forthefirstleafand thelastleafcolorC_2forthesecondleaflinkedto thefirstleaf and colorC_3forthethirdleaf.
$E(v_1)=E(v_k)=m+1$

WeassigncolorC_2forthefirstleafandlastleafandcolorC_3forthesecondleafofvertex$v$connectedto the$n$star.
$f_{4,2}$isEccentricallycolorable

(b)For$f_{m,n}$wherem=4,n=3.
Form=4,n=m-1=3
$E(u_1)=E(u_k)=m+1$,
$E(u_2)=m$
$E(u_3)=m$

WeassigncolorC_1forthefirstleafandlastleaf,colorC_2forthesecondleaf linked to thefirstleaf.
$E(v_1)=E(v_k)=m+2$,

WeassigncolorC_2forthefirstleafand thelastleafofvertex$v$connectedto$n$ star.
$E(w_1)=E(w_k)=m+3$,

WeassigncolorC_3forthefirstleafand thelastleafofvertex$w$connectedto$n$ star.
The sameprocedurecontinuesform=4,n=4,andm=4,n=5.

So,wefindtheEccentricColoringfor thenextsetofgraphs,weobservethatEccentricColoringfor theFirecrackergraphis3-colorablefor$m>2,n>2$.

3 EccentricColoringofCycle$C_k$having ChordWithDifferentDistances

Definition3.1
Cycle: A graph without a direction is called a cycle. A cycle *Ck* is a path that starts at one vertex and ends at the same vertex.

Definition3.2
Cycle: A graph without a direction is called a cycle. A cycle *Ck* is a path that starts at one vertex and ends at the same vertex.

Definition3.3

ChordalGraph: A chord, or edge connecting two of a cycle's vertices but not making up a part of the cycle, is required for every cycle in a graph with four or more vertices.

Eccentric Coloring is found for a cycle $C_k, K \geq 9,$ having a chord between two vertices at distances two and three [11]. We have continued the work, to find Eccentric Coloring for a Cycle having Chord at distances four and five.

Theorem2: Achordbetweentwoverticesatdistancefourfromeachotheriseccentriccolorableforacycle$C_k, k\geq 9$.

Proof:acycle$C_k, k\geq 9$theverticeslabeledas$\{v_1, v_2, v_3, ...v_k\}$.So$'e'$bethechordbetween$v_{k-2}andv_2$.FollowingaretheEccentricColoringforacycle:

Case(1):$k=4p+5, p\geq 1$.

Thiscasegivestheeccentriccoloringfor asetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\}$,

$Y=\{v_{4m-1}/1\leq m\leq \frac{k-5}{4}\}\cup\{v_k\}$and

$Z=\{v_{k-1}\}$aregivencolorc_1, c_3, c_2 andc_4 respectively.

Case(2):$k=4p+6, p\geq 1$.

Thiscasegivestheeccentriccoloringfora setofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-2}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k-6}{4}\}\cup\{v_k\}$and

$Z=\{v_{k-2}\}$aregivencolorc_1, c_3, c_2 andc_4 respectively.

Case(3):$k=4p+7, p\geq 1$.

Thiscasegivestheeccentriccoloringforsetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-3}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k-3}{4}\}\cup\{v_k\}$and

$Z=\{v_{k-1}\}$aregivencolorc_1,c_3,$c_2$and$c_4$respectively.**Case(4):**k=4p+8

,$p\geq 1$.

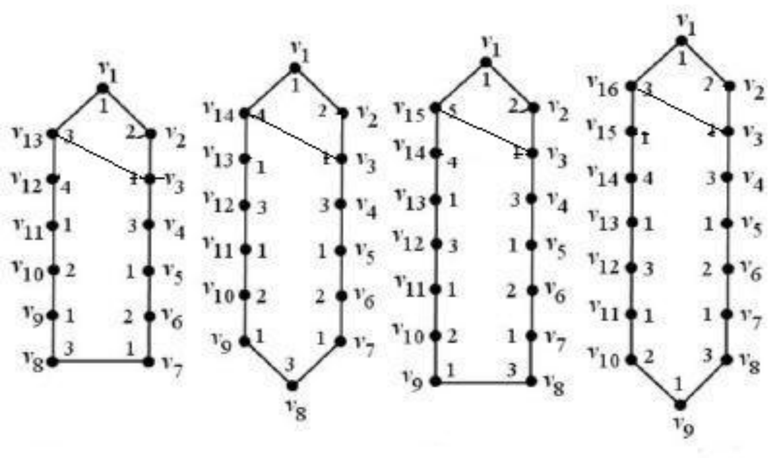
Thiscase gives the eccentriccoloring for a setof vertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k-5}{4}\}\cup\{v_k\}$and

$Z=\{v_{k-2}\}$aregivencolorc_1,c_3,$c_2$and$c_4$respectively.Thegraphinthefigureis anExample



A Cycle$C_k$of chord with the Distance of Four between two vertices

Wehaveconsidered$p$=3throughoutthisexample.k =4p+5=17,Inthefiguregraphisshownanditseccentriccoloringisas followsthesetof verticesobtainedare

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,=$\{v_1,v_3,v_5.....,v_{15}\}$

$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\}$,=$\{v_2,v_6,v_{10},v_{14}\}$

$Y=\{v_{4m-1}/1\leq m\leq \frac{k-5}{4}\}\cup\{v_k\}$and=$\{v_4,v_8\}\cup\{v_{13}\}$

$Z=\{v_{k-1}\}=\{v_{12}\}$
aregivencolorc_1,c_3,c_2 andc_4 respectively.

Similarly,othercaseswiththeirrespectivecolorsareshownrespectively.

Theorem3: Achordbetweentwoverticesat adistance offivefromeachotheriseccentriccolorableforacycle$C_k,k\geq 9$.

Proof:acycle$C_k,k\geq 9$theverticeslabeledas$\{v_1,v_2,v_3,.....v_k\}$.So$'e'$bethechordbetweenv_{k-1}andv_2.FollowingaretheEccentricColoringforacycle:

Case(1):$k=4p+5,p\geq 1$.

Thiscasegivestheeccentriccoloringfor asetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k-5}{4}\}$and

$Z=\{v_{k-1}\}$aregivencolorc_1,c_3,c_2 andc_4 respectively.

Case(2):$k=4p+6,p\geq 1$.

Thiscasegivestheeccentriccoloringfor asetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-2}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k-6}{4}\}$and

$Z=\{v_k\}$aregivencolorc_1,c_3,c_2 andc_4 respectively.

Case(3):$k=4p+7,p\geq 1$.

Thiscasegivestheeccentriccoloringfor asetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$

$X=\{v_{4m-2}/1\leq m\leq \frac{k-3}{4}\}$

$Y=\{v_{4m}/1\leq m\leq \frac{k-3}{4}\}$

$Z=\{v_{k-1}\}$and$Q=\{v_k/$aregivencolorc_1,c_3,c_2 andc_4 respectively.

Case(4):$k=4p+8,p\geq 1$.

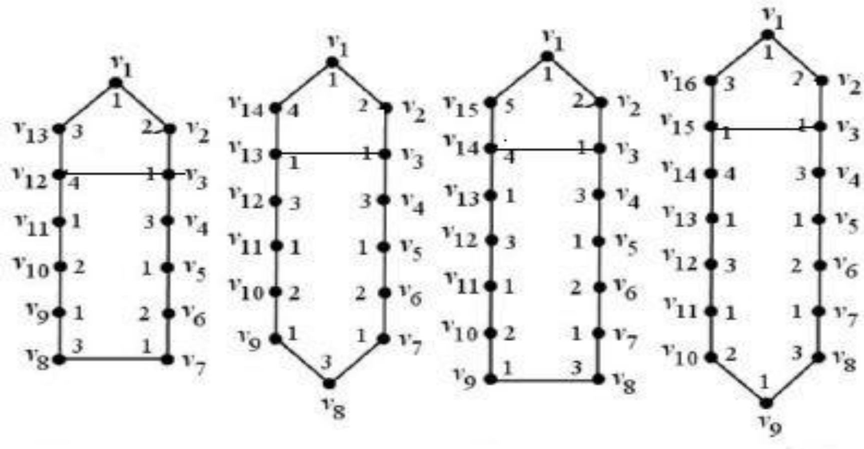
Thiscasegivestheeccentriccoloringfor asetofvertices.

$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\}$,

$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\}$,

$Y=\{v_{4m}/1\leq m\leq \frac{k}{4}\}$

and
$Z=\{v_{k-2}\}$aregivencolorc_1,c_3,$c_2$and$c_4$respectively.Thegraphi nthefigureisanExample



A Cycle$C_k$of chord with the Distance Five between two vertices

Wehaveconsidered$p$=3throughoutthisexample.k =4p+5=17,Inthefiguregraphisshownanditseccentriccoloringisas followsthesetof verticesobtainedare

$$W=\{v_{2m-1}/1\leq m\leq \frac{k-1}{2}\},=\{v_1,v_3,v_5.....,v_{15}\}$$

$$X=\{v_{4m-2}/1\leq m\leq \frac{k-1}{4}\},=\{v_2,v_6,v_{10},v_{14}\}$$

$$Y=\{v_{4m-1}/1\leq m\leq \frac{k-5}{4}\}\text{and}=\{v_4,v_8\}$$

$$Z=\{v_{k-1}\}=\{v_{12}\}$$

aregiven colorc_1,c_3,$c_2$and$c_4$respectively.

Similarly,othercaseswiththeirrespectivecolorsareshownrespectively.

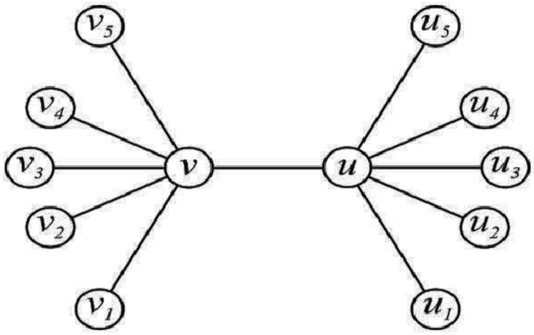
4 EccentricColoringofBistarGraph

Definition

Bistar graph

A Bistar graph, also known as $B_{n,n}$, is created by joining the center vertex of two copies of $K_{1,n}$together by an edge.

Example:



BistarGraph$B_{n,n}$

Theorem4: Eccentric Coloring of a Bistar graph $B_{n,n}$ for n ≥ 2 is always 3-colorable.

Proof : Let($B_{n,n}$)=$v_1,v_2,......v_n,v,u,u_1,u_2,u_3,...u_n$Bistargraph$B_{n,n}$ contains2n+2verticesand2n+1edges.

firsttwovertices(u,v)areconnectedtoeachotherbyanedgeandeachvertexisconnectedtootherverticesu$u_1,u_2,u_3...u_n,$and$v_1,v_2,v_3,...v_n$respectively.wheretheeccentricity′e′oftheverticesuandvarealwaysequalandtheeccentricityoftheverticesconnectedto(u,v)isalso$e$+1foralltheverticesofbothvertices(u,v).

the Bistargraph, the eccentricityis the sameforverticesconnectedto eachotherbyabridgeforallthegraphs,thentheeccentricityofthetwocopiesofgraph$k_{1,n}$connectedto each vertex(u,v) byabridgeisalsothe sameforallthegraphs

TheBistargraphsatisfiestheconditionofeccentriccolorfromthedefinition $c(u)=c(v)\Rightarrow d(u,v)>color(u)$forall($u,v$)∈vand forall$v$∈v,c(v)≤e(v).

Let us assignthree colorsC_B,C_G, and$C_R$where
ColorC_B= *Blue*
ColorC_G=*Green*ColorC_R=*Red.*

WeassignColorC_B tothefirstvertex(v)andColorC_G tothesecondvertex(u)whichareconnectedtoeachother.ThenweassignColorC_R, and ColorC_B alternativelyto therestoftheverticesconnectedtovertex(u).Sincevertex(u)isgivenColorC_G,othervertices connectedto′u′aregivenColor$C_R$andColorC_BSincetheseverticesareadjacenttothevertex′u′andColorC_R,Color$C_G$respectivelytothe restoftheverticesconnectedtovertex′v′.

Thisprocesscontinuesas thenumberofverticesincreasesonbothsidesofthevertex′u′and′v′

$Ecc(v_i)=Ecc(v)+1$wherei=1, 2,3......n

$Ecc(u_i)=Ecc(u)+1$wherei=1 ,2,3......n

Hereweconsidertwocases

Case(i): WhennisevenForn=2,4,6,8..........
The Colorassignedtotheverticesv$v_i$wherei=1,3,5,7......isalwaysColor(C_R).

AndtheColorassignedtotheverticesv$v_i$iswherei=2,4,6,8......isalwaysColor(C_G).

SothetwoColors,Color(C_R)andColor(C_G)areassignedtotheverticesofoneofthecopiesof B_n.

The same process continues for the vertices$u_i$on theother side of the copy.T h e Colorassignedtothevrtices$u_i$wherei=1,3,5,7....isalwaysColor(C_R).

Butthe color assigned to the vertices$u_i$where i=2,4,6,8....isalways Color(C_B).
Soforanothercopyof$B_n$thetwocolorsassignedareColor(C_R)andColor(C_B).

Case(ii): WhennisoddForn=3,5,7,9..........
The Colorassignedtothevertices$u_i$wherei=1,3,5,7......isalwaysColor(C_R).

AndtheColorassignedtothevertices$u_i$iswherei=2,4,6,8......isalwaysColor(C_B).

SothetwoColors,assignedtothevertices$u_i$ofoneofthecopiesof$B_n$arealwaysColor(C_R)and Color(C_B).

The sameprocesscontinuesforthevertices $v_i$ontheothersideofthecopy wherei=1,3,5,.....isalwaysColor(C_R)

AndtheColorassignedtothevertices$u_i$wherei=2,4,6,....isalwaysColor(C_G).
Soforanothercopyof$B_{n,}$ thetwocolorsassignedareColor(C_R)andColor(C_G).

Hencefor theBistargraphfor$n \geq 2 B_{n,n}$is3-colorable.

5 Conclusion

Inthisarticle,TheEccentricColoringoftheFirecrackerGraph,BistarGraph,andCycleofchordswithdifferentdistanceshavebeendetermined.Sincethererearerestrictionstoonlysomeclassesofgraphsforeccentriccolorable,wehaveworkedonsomeclassesofgraphs.

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