Bipolar Pythagorean Neutrosophic Soft Extension of the MULTIMOORA Method for Solving Decision-Making Problems

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Abstract: Multi objective optimization by ratio analysis plus full multiplicative form (MULTIMOORA) is an efficient decision making method for solving multi-criteria decision making evaluation values. These strategies include ration system approach (RSA), reference point approach (RPA) and full multiplicative for (FMF). The novel multicriteria decision making approach MULTIMOORA – multi objective optimization by ratio analysis plus full multiplicative form under bipolar pythagorean neutrosophic soft (BPNS) is introduced in this paper. The main aim of the study is to extend the MULTIMOORA method using BPNS information to overcome the MCDM problem. The objective is to develop a proper method for selecting the best rank from the given alternatives using the extended MULTIMOORA method. An application for the proposed approach is given and also comparative of MOORA methods are provided.

Keywords: multi criteria decision making, MULTIMOORA methods, bipolar pythagorean neutrosophic soft set.

1 Introduction

The MULTIMOORA method was introduced by Brauers and Zavadskas[11] in 2010. The ordinary MULTIMOORA methods was proposed for usage with crisp numbers. Many extensions have been proposed to use it in solving a larger number of complex decision making problems. Fuzzy extension of the MULTIMOORA method was proposed by Brauers et al.[12] in 2011.Balezentis and Zeng [9] in 2013 introduced interval-valued fuzzy extension. In 2014, Balezentis et al.[10] proposed intuitionistic fuzzy extension and in 2015 Zavadskas et al. [27] proposed interval-valued intuitionistic extension of the MULTIMOORA method.To solve a wide range of problems this method has been applied.

In fuzzy set theory Zadeh [26] introducedaimportant approach to solve a complex decision-making problems by adapting multi criteria decision making methods for the purpose of using fuzzy numbers in 1965. Atanassov [7,8] in 1986 introduced the intuitionistic fuzzy sets by providing non-membership function that states non-membership to a set, thus having created the basis for solving a much larger number of decision making problems. Intuitionistic fuzzy sets are capable of operating with incomplete pieces of information since it is composed of membership and non-membership function in which both lies between closed interval 0,1 and sum of them lies between 0 and 1.Smarandache [21,22,23] in 1998 and 1999 extended the intuitionistic fuzzy sets to neutrosophic sets by introducing indeterminacy membership function. It is composed of three independent membership functions namely truth, falsity and indeterminacy membership functions.

Bipolar-valued fuzzy sets, an extension of fuzzy set and their operation was coined in 2000 by Lee[16]. Deli et al.[14] developed bipolar neutrosophic sets and study their application in decision making. The concept of soft set theory as a new mathematical tool was initiated by Molodtsov[19] in 1999 and presented the fundamental results of the soft sets. Bipolar soft sets and bipolar fuzzy soft sets are studied by Aslam et al.[5]. Pythagorean fuzzy set and subsets were discussed by Xindong[24] and Yagar[25]. The notion of bipolar neutrosophic soft set was introduced by Ali et al.[3] in 2017. The concept of pythagorean neutrosophic set was introduced by Jansi [15].

In this paper, we proposed a new ranking method using bipolar pythagorean neutrosophic soft set. In order to find the ranking alternatives of many decision making problems this method is more effective. Therefore the rest of this paper is organized as follows: In Section 2, some basic definitions are provided. In Section 3, the comparison between ordinary MOORA method and our proposed method is given in tabulated form. In Section 4, an example is considered with the aim to explain in detail the proposed methodology. In Section 5, the conclusion is presented.

2. Preliminaries

Definition 2.1[19] Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.2[16] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $x \in X$ to the set A. A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$, where $\langle \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ in $]^{-}0, 1^{+}$ are functions such that the condition: $\forall x \in X,\ 0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ is satisfied.

Definition: **2.3**[16] Let U be a universe. A bipolar fuzzy set A in U is defined as: $A = \{(u, T^{+}(u), T^{-}(u)) : u \epsilon U\}$ where $T^{+}: X \to [0,1]$ and $T^{-}: X \to [-1,0]$. The positive membership degree $T^{+}(u)$ denotes the truth membership corresponding to a bipolar fuzzy set A and the negative membership of an element $u \epsilon U$ to some implicit counter-property corresponding to a bipolar fuzzy set A.

Definition:2.4[5] Let U be a universe and E be a set of parameters that are describing the elements of U. A bipolar fuzzy soft set A in U is defined as; $A = \{(e, \{(u, T^{+}(u), T^{-}(u)) : u \epsilon U\}) : e \in E\}$ where $T^{+}: X \to [0,1]$ and $T^{-}: X \to [-1,0]$. The positive membership degree $T^{+}(u)$ denotes the truth membership corresponding to a bipolar fuzzy soft set A and the negative membership of an element $u \epsilon U$ to some implicit counter-property corresponding to a bipolar fuzzy soft set A.

Definition:2.5[13] Let U be a universe, N(U) be the set of all neutrosophic sets on U, E be a set of parameters that are describing the elements of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function $f_N: E \to N(U)$ where f_N is called an approximate function of the neutrosophic soft set N. For $x \in X$, the set $f_N(x)$ is called x-approximation of the neutrosophic soft set N which may be artitrary, some of them may be empty and some may have a nonempty intersection. In other words, the neutrosophic soft set is a parameterized family of some elements of the set N(U), and therefore it can be written a set of ordered pairs, N=$\left\{\langle x, \{< u, T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u)\rangle >: x \in X: x \in E\right\}$where $T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) \epsilon [0,1]$.

Definition: 2.6[14] Let U be the universe. A bipolar neutrosophic set A in U is defined as; $A = \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)): u\epsilon U\}$where $T^+: X \to [0,1]$, $I^+: X \to [0,1]$, $F^+: X \to [0,1]$, $T^-: X \to [-1,0]$, $I^-: X \to [-1,0], F^-: X \to [-1,0]$. The positive membership degree $T^+(u), I^+(u), F^+(u)$denotes the truth, indeterminate and false membership corresponding to a bipolar neutrosophic set A and the negative membership $T^-(u), I^-(u), F^-(u)$denotes the truth, indeterminate and false membership of an element $u\epsilon U$.

Definition:2.7[15] Let U be a universe. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components A on U is an object of the form $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}$ where $T_{A,}, I_{A,} F_A : U \to [0,1]$ and $0 \le \left((T_A(x))^2 + (I_A(x))^2 + (F_A(x))^2\right) \le 1$. Here $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

Definition: 2.8[18] Let X be a non-empty set. A bipolar pythagorean fuzzy set (BPFS) $A = \{(x, T_A^P, F_A^P, T_A^N, F_A^N): x \in X\}$where $T_A^P: X \to [0,1]$, $F_A^P: X \to [0,1]$, $T_A^N: X \to [0,1], F_A^N: X \to [0,1]$ are the mappings such that $0 \le \left(\left(T_A^P(x)\right)^2 + \left(F_A^P(x)\right)^2\right) \le 1$ and $-1 \le -\left(\left(T_A^N(x)\right)^2 + \left(F_A^N(x)\right)^2\right) \le 0$ and $T_A^P(x)$ denote the positive membership degree, $F_A^P(x)$ denote the positive non-membership degree, $T_A^N(x)$ denote the negative membership degree and $F_A^N(x)$ denote the negative non-membership degree.

Definition: 2.9[1]Let U be a universe and E be a set of parameters. A bipolar pythagorean neutrosophic soft set$\mathbb{A} = \{(e, \{(u, T^+(u), I^+(u), F^+(u), T^-(u), I^-(u), F^-(u)): u\epsilon U\}): e\epsilon E\}$ where $T^+: X \to [0,1]$, $I^+: X \to [0,1]$, $F^+: X \to [0,1]$, $T^-: X \to [-1,0]$, $I^-: X \to [-1,0], F^-: X \to [-1,0]$ are the mappings such that $0 \le \left((T^+(u))^2 + (I^+(u))^2 + (F^+(u))^2\right) \le 2$ and $-2 \le -\left((T^-(u))^2 + (I^-(u))^2 + (F^-(u))^2\right) \le 0$. The positive membership degree $T^+(u), I^+(u), F^+(u)$ denote the Truth, Indeterminacy and False membership of an element corresponding to' a bipolar pythagorean neutrosophic soft set $\mathbb{A}$ and the negative membership degree $T^-(u), I^-(u), F^-(u)$ denotes the Truth, Indeterminacy and False membership of an element $u\epsilon U$ to some implicit counter property corresponding to a bipolar pythagorean neutrosophic soft set.

3. MOORA Methods and Modified MOORA Methods Comparison

In this section, we give the comparison between the ordinary MOORAmethods and the proposed modified MULTIMOORA methods. Ratio System Approach (RSA), Reference Point Approach (RPA) and Full Multiplicative Form (FMF) are the three types of MOORA methods.

Depending on the dominance theory alternative rankings are calculated using MULTIMOORA based on bipolar pythagorean neutrosophic soft set (BPNSS).

Ratio System Approach (RSA) of MOORA–Algorithm

MOORA Method	Modified MOORA Method
1. Construct decision matrix	1. Construct decision matrix for the positive membership values of the Bipolar pythagorean neutrosophic soft set (BPNSS)
2. Compute normalized decision matrix using the equation $$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$	2. Compute normalized decision matrix for the BPNSS positive membership values using the equation $$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$
3. Calculate weighted normalized decision matrix using the equation $$u_{ij} = w_j x_{ij}^*$$ where w_j = the weight of j^{th} criterion	3. Calculate weighted normalized decision matrix for the BPNSS positive membership values using the equation $$u_{ij} = w_j x_{ij}^*$$ where w_j = the weight of j^{th} criterion
4. Obtain final preference p_i^* by using the equation $$p_i^* = \sum_{j=1}^{g} u_{ij} - \sum_{j=g+1}^{n} u_{ij}$$ here $j = 1$to g indicates the maximized criteria and $j = g + 1$ to n indicates minimized criteria	4. Obtain final preference $p_i^*$for the BPNSS positive membership values by using the equation $$p_i^* = \sum_{j=1}^{g} u_{ij} - \sum_{j=g+1}^{n} u_{ij}$$ here $j = 1$to g indicates the maximized criteria and $j = g + 1$ to n indicates minimized criteria
-	5. Repeat the first four steps for the BPNSS negative membership values. Denote its final preference values by p_i^{**}
-	6. Finally, the final score and ranking of alternatives is obtained by $p = p_i^* - p_i^{**}$.

The Reference Point Approach (RPA) of MOORA - Algorithm

MOORA Method	Modified MOORA Method
1. Construct decision matrix	1. Construct decision matrix for the positive membership values of the Bipolar pythagorean neutrosophic soft set (BPNSS)
2. Compute normalized decision matrix using the equation $$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$	2. Compute normalized decision matrix for the BPNSS positive membership values using the equation

	$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$
3. Calculate weighted normalized decision matrix using the equation $$u_{ij} = w_j x_{ij}^*$$ where w_j = the weight of j^{th} criterion	3. Calculate weighted normalized decision matrix for the BPNSS positive membership values using the equation $$u_{ij} = w_j x_{ij}^*$$ where w_j = the weight of j^{th} criterion
4. Find reference points (r_j^*) for each criteria and it can be determined by choosing maximum value for maximization criteria whereas minimum value for minimization criteria.	4. Find reference points (r_j^*) for each criteria and it can be determined by choosing maximum value for maximization criteria whereas minimum value for minimization criteria.
5. Calculate distance between the reference points and alternatives by subtracting reference point value from u_{ij} values for minimization criteria and for maximization criteria, subtracting u_{ij} values from the reference point values r_j^*. It can be denoted by d_i^*.	5. Calculate distance between the reference points and alternatives by subtracting reference point value from u_{ij} values for minimization criteria and for maximization criteria, subtractingu_{ij} values from the reference point values r_j^*.
6. Obtain the ranking of alternatives	6. Repeat the above five steps for the BPNSS negative membership values.
-	7. Finally, the final score and ranking of alternatives is obtained by $d = d_i^* - d_i^{**}$.

Full Multiplicative Form (FMF) of MOORA– Algorithm

MOORA Method	Modified MOORA Methods
1. Construct decision matrix	1. Construct decision matrix for the positive membership values of the Bipolar pythagorean neutrosophic soft set (BPNSS)
2. Find multiplicative ranking index z_i^* for each alternative by $z_i^* = \frac{P_i}{Q_i}$, where $$P_i = \prod_{j=1}^{g} x_{ij}{}^{w_j}$$ for maximization criteria $$Q_i = \prod_{j=g+1}^{n} x_{ij}{}^{w_j}$$ for minimization criteria.	2. Find multiplicative ranking index $z_i^*$for each alternative by $z_i^* = \frac{P_i}{Q_i}$, where $$P_i = \prod_{j=1}^{g} x_{ij}{}^{w_j}$$ for maximization criteria $$Q_i = \prod_{j=g+1}^{n} x_{ij}{}^{w_j}$$ for minimization criteria.
3. Obtain alternative rank.	3. Repeat the above two steps for the BPNSS negative membership values.

-	4. Finally, the final score and ranking of alternatives is obtained by $z = z_i^{*} - z_i^{**}$.

4. Application

An example has been taken from Stanujkic et al.(2015) to exhibit the efficiency and applicability of the given method. This example has been slightly altered to briefly establish the advantages of the proposed methodology. Consider that a mining and smelting company has to build a new flotation plant, for which an expert has been involved to assess the five Communication Circuit Designs (CCDs). Let$U = \{u_1, u_2, u_3, u_4, u_5\}$ be the CCDs alternatives based on the use of rod mills, based on the use of ball mills, based on the combined use of rod mills and ball mills, based on the use of autogenous mills and based on the use of semi-autogenousmills respectively. The parameters considered for the purpose of conducting an evaluation are $E = \{e_1, e_2, e_3, e_4, e_5\}$ where $e_1 =$ grinding efficiency, $e_2 =$ economic efficiency, $e_3 =$ technological reliability, $e_4 =$ capital investment costs and $e_5 =$ environmental impact.Out of these parameters, consider e_2, e_3, e_4 as the beneficial criteria and e_1 and e_5 as the non-beneficial criteria. The expert evaluates the five CCDs in relation to the selected evaluation criteria. Based on the three methods the alternatives are ranked and based on the theory of dominance the final ranking order and the final decision is made. That is on all ranking list the best ranked alternative is the alternative with the highest number of appearances in the first position. Let

$$(A) = \left\{ \begin{array}{c} \left\{ \begin{array}{c} e_1, \{(u_1, 0.4, 0.2, 0.3, -0.3, -0.3, -0.2), (u_2, 0.3,0.3,0.2, -0.2, -0.1, -0.5), \\ \{(u_3, 0.5, 0.1, 0.2, -0.1, -0.2, -0.1), (u_4, 0.3,0.3,0.1, -0.2, -0.1, -0.5), \\ (u_5, 0.3,0.3,0.2, -0.1, -0.1, -0.2)\} \end{array} \right\} \\ \left\{ \begin{array}{c} e_2, \{(u_1, 0.3, 0.5, 0.2, -0.6, -0.1, -0.4), (u_2, 0.2,0.8,0.6, -0.4, -0.9, -0.4), \\ \{(u_3, 0.3, 0.2, 0.6, -0.6, -0.4, -0.1), (u_4, 0.5,0.6,0.1, -0.2, -0.8, -0.6), \\ (u_5, 0.1,0.7,0.3, -0.6, -0.4, -0.7)\} \end{array} \right\} \\ \left\{ \begin{array}{c} e_3, \{(u_1, 0.5, 0.1, 0.3, -0.2, -0.6, -0.7), (u_2, 0.6,0.5,0.2, -0.3, -0.7, -0.7), \\ \{(u_3, 0.2, 0.3, 0.4, -0.7, -0.3, -0.2), (u_4, 0.6,0.2,0.4, -0.3, -0.7, -0.2), \\ (u_5, 0.8,0.9,0.5, -0.5, -0.9, -0.4)\} \end{array} \right\} \\ \left\{ \begin{array}{c} e_4, \{(u_1, 0.5, 0.8, 0.6, -0.4, -0.1, -0.3), (u_2, 0.8,0.5,0.6, -0.5, -0.4, -0.3), \\ \{(u_3, 0.1, 0.2, 0.6, -0.6, -0.8, -0.5), (u_4, 0.6,0.4,0.7, -0.5, -0.1, -0.4), \\ (u_5, 0.1,0.6,0.7, -0.6, -0.5, -0.4)\} \end{array} \right\} \\ \left\{ \begin{array}{c} e_5, \{(u_1, 0.5, 0.1, 0.3, -0.2, -0.6, -0.7), (u_2, 0.6,0.5,0.2, -0.3, -0.7, -0.7), \\ \{(u_3, 0.2, 0.3, 0.4, -0.7, -0.3, -0.2), (u_4, 0.6,0.2,0.4, -0.3, -0.7, -0.2), \\ (u_5, 0.8,0.9,0.5, -0.5, -0.9, -0.4)\} \end{array} \right\} \end{array} \right\}$$

Ratio System Approach (RSA) of MULTIMOORA:

Table 1: Decision Matrix for Positive Information System

Alternatives	min	max	max	max	min
	e_1	e_2	e_3	e_4	e_5

u_1	0.2	0.5	0.5	0.8	0.1
u_2	0.2	0.8	0.6	0.8	0.1
u_3	0.1	0.6	0.6	0.6	0.4
u_4	0.1	0.6	0.6	0.7	0.2
u_5	0.2	0.7	0.9	0.7	0.2

Table 2: Normalized Decision Matrix

Alternatives	e_1	e_2	e_3	e_4	e_5
u_1	0.1849	0.4622	0.4622	0.7396	0.0924
u_2	0.1538	0.6154	0.4615	0.6154	0.0769
u_3	0.0894	0.5367	0.3578	0.5367	0.3578
u_4	0.0891	0.5345	0.5345	0.6236	0.1782
u_5	0.1463	0.5119	0.6581	0.5119	0.1465

Table 3: Criteria Weights

Criteria	e_1	e_2	e_3	e_4	e_5
Weights	0.2	0.2	0.3	0.2	0.1

Table 4: Weighted Normalized Decision Matrix and Final Preference

Alternatives	e_1	e_2	e_3	e_4	e_5	p_i^*
u_1	0.0370	0.0924	0.1387	0.1479	0.0092	0.3328
u_2	0.0308	0.1231	0.1385	0.1231	0.0077	0.3462
u_3	0.0179	0.1073	0.1073	0.1073	0.0958	0.2682
u_4	0.0178	0.1069	0.1604	0.1247	0.0178	0.3564
u_5	0.0293	0.1024	0.1974	0.1024	0.0147	0.3582

Table 5: Decision Matrix for Negative Information System

Alternatives	min	max	max	max	min
	e_1	e_2	e_3	e_4	e_5
u_1	-0.3	-0.1	-0.2	-0.1	-0.7
u_2	-0.5	-0.4	-0.3	-0.3	-0.9
u_3	-0.2	-0.1	-0.2	-0.5	-0.6
u_4	-0.5	-0.5	-0.2	-0.1	-0.5
u_5	-0.2	-0.4	-0.4	-0.4	-0.5

Table 6: Normalized Decision Matrix

Alternatives	e_1	e_2	e_3	e_4	e_5
u_1	-0.3412	-0.1137	-0.2275	-0.1137	-0.7963

u_2	-0.3363	-0.2690	-0.2018	-0.2018	-0.6054
u_3	-0.2390	-0.1195	-0.2390	-0.5976	-0.7171
u_4	-0.6509	-0.2603	-0.2603	-0.1301	-0.6509
u_5	-0.1769	-0.3539	-0.3539	-0.3539	-0.4424

Table 7: Criteria Weights

Criteria	e_1	e_2	e_3	e_4	e_5
Weights	0.1	0.2	0.3	0.2	0.2

Table 8: Weighted Normalized Decision Matrix and Final Preference

Alternatives	e_1	e_2	e_3	e_4	e_5	p_i^{**}	p	Rank
u_1	-0.0341	-0.0227	-0.0682	-0.0227	-0.1592	0.0797	0.2531	5
u_2	-0.0363	-0.0538	-0.0605	-0.0403	-0.1210	0.0027	0.3435	2
u_3	-0.0239	-0.0239	-0.0717	-0.1195	-0.1434	-0.0478	0.3160	4
u_4	-0.0650	-0.0520	-0.0780	-0.0260	-0.1301	0.0391	0.3173	3
u_5	-0.0176	-0.0707	-0.1061	-0.0707	-0.0884	-0.1415	0.4997	1

The ranking of alternatives: $u_5 > u_2 > u_4 > u_3 > u_1$

Reference Point Approach (RPA) of MULTIMOORA:

Table 9: Positive information system -Weighted Normalized Decision Matrix and Final Preference

Alternatives	e_1	e_2	e_3	e_4	e_5	d_i^*
u_1	0.0192	0.0307	0.0587	0.0000	0.0015	0.0587
u_2	0.0130	0.0000	0.0589	0.1402	0.0000	0.1402
u_3	0.0001	0.0158	0.0901	0.1121	0.0281	0.1121
u_4	0.0000	0.0162	0.0370	0.1301	0.0101	0.1301
u_5	0.0115	0.0207	0.0000	0.1332	0.0070	0.1332
r_j^*	0.0178	0.1231	0.1974	0.1479	0.0077	

Table 10: Negative information system -Weighted Normalized Decision Matrix and Final Preference

Alternatives	e_1	e_2	e_3	e_4	e_5	d_i^{**}	d	Rank
u_1	0.0309	0.0000	0.0077	0.0000	0.0000	0.0000	0.0587	2
u_2	0.0287	0.0311	0.0000	0.0176	0.0382	0.0000	0.1402	4
u_3	0.0411	0.0012	0.0112	0.0968	0.0158	0.0012	0.1109	3
u_4	0.0000	0.0293	0.0175	0.0033	0.0291	0.0000	0.1301	5
u_5	0.0474	0.0480	0.0456	0.0480	0.0708	0.0456	0.0876	1
r_j^{**}	-0.0650	-0.0227	-0.0605	-0.0227	-0.1592			

The ranking of alternatives - $u_2 > u_4 > u_3 > u_5 > u_1$

Full Multiplicative Form (FMF) of MULTIMOORA:

Table 11: Positive information system – Multiplicative Ranking Index

Alternatives	e_1	e_2	e_3	e_4	e_5	z_i^*
u_1	0.7135	0.8570	0.7933	0.9415	0.7881	1.1383
u_2	0.6877	0.9075	0.7930	0.9075	0.7737	1.2284
u_3	0.6170	0.8830	0.7347	0.8830	0.9023	1.0288
u_4	0.6166	0.8822	0.8287	0.9099	0.8416	1.2819
u_5	0.6808	0.8747	0.8820	0.8747	0.8252	1.2012

Table 12: Negative information system – Multiplicative Ranking Index

Alternatives	e_1	e_2	e_3	e_4	e_5	z_i^{**}	z	Rank
u_1	-0.8981	-0.6474	-0.6413	-0.6474	-0.9555	-0.3132	1.4515	5
u_2	-0.8968	-0.7690	-0.6187	-0.7261	-0.9045	-.04259	1.6543	4
u_3	-0.8666	-0.6538	-0.6509	-0.9022	-0.9357	-0.4735	1.5023	3
u_4	-0.9580	-0.7640	-0.6678	-0.6651	-0.9177	-0.3860	1.6679	2
u_5	-0.8410	-0.8124	-0.7321	-0.8124	-0.8495	-0.6763	1.8775	1

The ranking of alternatives - $u_5 > u_4 > u_3 > u_2 > u_1$

Table 13: The final ranking order of the alternatives according to the MUTLIMOORA (MM)method

RSA Method	**RPA Method**	**FMF Method**	**MM Method**
5	2	5	5
2	4	4	4
4	3	3	3
3	5	2	2
1	1	1	1

The different ranks given by the respective parts of the MULTIMOORA method is summarized as the final ranking order of the alternatives. The final ranking orders is determined based on the dominance theory as all three approaches integrated in the MULTIMOORA have resulted in different ranking orders, as it can be seen in Table 13.

5. Conclusion

In solving different decision making problems the MULTIMOORA methods has proven the effective one. Numerous extensions have been proposed for the method in order to enable its application in solving large number of complex problems. Therefore, an extension of the MULTIMOORA method enabling the use of bipolar pythagorean neutrosophic soft is proposed in this paper. The usability and efficacy of the given extension is presented on the example of the communication circuit design selection. Using the introduced extension of the MULTIMOORA method, decision-makers are as well enabled to analyze different scenarios and make the most appropriate selection. Finally, it should be noted that the proposed extension of the MULTIMOORA method can be used for solving a much larger number of complex decision-making problems.

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