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BONDAGE SET OF STRONG ARCS IN COMPLETE INTUITIONISTIC FUZZY GRAPHS

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Abstract

One way to amplify the concept of connectivity in IFGs applied Strong arcs. The N-Bondage Set and M-Non Bondage Set concepts were presented in the intuitionistic fuzzy Graph. In intuitionistic fuzzy graph associate with complete graph G used to bondage set of Graph. The properties of bondage and non-bondage IFGs were analysed.

Keywords –Complete graph , bondage ($\alpha(G)$) , Non-bondage ($\alpha_k(G)$).

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I. INTRODUCTION

Graph deals with different domain problems of optimisation concepts of the network field. An interaction flow of intuitionistic Fuzzy Graph methods is much better at dealing with the natural existence of the biochemical field. Initially, we need basic techniques of Strong arc associated with bondage and Non-bondage set. In 1965 Zadeh [1] developed fuzzy set relation. In 1975, Rosenfeld [6] first introduced analogues of set-theoretic concepts. Zadeh deals with various applications of various fields, including chemical engineering and telecommunication one network field to another. He continued ot work on fuzzy graphs in various fields.

Krassimir T. Atanassov[2] contributed to the progress of fuzzy graphs on discrete set theory. J.A. Bondy [3] first used the term "fuzzy relation" in 1976. Rosenfeld contributed to paths and cycles for connectivity. In 2006, concrete intuitionistic fuzzy graphs were developed and used to find the shortest network distance using a dynamic programming approach. M.G. Karunambigai, R. Parvathi, and R. Bhubaneswar[7] introduced regularity for IG of Graph using strong arc weights to minimize this parameter further..

The system of components depends on N-bondage connectedness between two vertices, and the authors extended to the concepts of N- bondage and M- non-bondage with suitable illustrations. The necessary and sufficient conditions for their equivalence are studied here. The paper is organised as follows:

Section 2 includes preliminary material, and Section 3 introduces the content of N-bondage and M-non-bondage. In this section, vertices cannot form the connectivity of vertices; we also examine the relationship between an N-bondage and an M-non-bondage in an IF graph. Here briefly, a vertex ($v_{i's}$, $v_{j's}$) of G is an IF graph bridge if and only if it is dominated. Also examines vertex connectivity in regular IFGs.

II PRELIMINARIES

Here are a few preliminary concepts and properties of strong arc, bondage and non-bondage sets below.

Definition 2.1: A fuzzy graph G*(σ^*, μ^*) is defined by two functions that map sets to the interval (0,1). The function μ^* maps pairs of elements in Vn to (0,1) such that for all pairs of elements u and v in Vn , the value of $\mu^*(u, v)$ is less than or equal to the minimum of the values of $\sigma^*(u)$ and $\sigma^*(v)$.

Definition 2.2: A minimax IF Graph set Ĝ(ђ, ϐ) contains a graph (Vn, En) where Vn is a set of vertices and En is a set of edges. The functions ђ1 and ϐ1 map elements in Vn to (0,1), representing their degrees of membership and non-membership respectively. The sum of these values for any element vi in Vn must be between 0 and 1. The functions ђ2 and ϐ2 map pairs of elements in Vn to (0,1), representing their degrees of membership and non-membership as edges. The value of ђ2(vi,vj) must be less than or equal to the minimum value between ђ1(vi) and ђ1(vj), while the value of ϐ2(vi,vj) must be less than or equal to the maximum value between ϐ1(vi) and ϐ1(vj). Additionally, for any pair (vi,vj), the sum of their membership and non-membership values must be between 0 and 1.

Definition 2.3: An IF graph Ĝ is considered robust if for all pairs i,j in Vn ,the value ђ2ij equals min(ђ1i ,ђ1j)and the value ϐ2ij is less than max(ϐ1i ,ϐ1j).

Definition 2.4: An IFG Ĝ is considered a complete strong IF graph if for all pairs i,j in Vn ,the value ђ2ij is less than min(ђ1i ,ђ1j)and the value ϐ2ij equals max(ϐ1i ,ϐ)

Definition 2.5: An IF graph is complete if its second type edge function and second type vertex function are defined as the minimum and maximum of its first type vertex functions respectively.

Definition 2.6: A fuzzy graph is solid if its edge function is defined as the minimum of its vertex functions. It is complete if its edge function is defined as the minimum of its vertex functions for all vertices in $\sigma*$. Two vertices are neighbors if their edge value is greater than 0.

Definition 2.7: In an IF graph, an arc between two vertices is called a strong arc if it has Nă minimum elements in the least effective arc dominate set. The closed neighborhood of a vertex includes all vertices connected to it by a strong arc.

Definition 2.8: An IF graph is semi-strong if its second type edge function is defined as the minimum of its first type edge functions for all pairs of vertices.

Definition 2.9: An IF graph is semi-strong if its second type vertex function is defined as the maximum of its first type vertex functions for all pairs of vertices.

Definition 2.11: An IF graph is strong if both its second type edge and vertex functions are defined as the minimum and maximum respectively of their first type counterparts for all edges.

III PROPERTIES OF STRONG ARC WITH BONDAGE AND NON-BONDAGE

3.1 Bondage of Set

In IFG,H is a dominant set of complete graphs, the contribution of an exists subset of B⊆ M such that $\eta_i(\hat{G} - B) > \eta_i(G)$, then B is called an N-Bondage set of S of $\hat{G}$, where S is collection of arc in IF set of all Strong Arc in IF of $\hat{G}$ and also the strong arc of $\alpha_i(\hat{G})$. N-Bondage is denoted by BS. When compared to all BS'sIF g has the lowest set of the cardinality of $\hat{G}$.

3.2 Non-Bondage set Let B is domination of $\hat{G}$. Then set of Strong Arc in all vertices of B ⊆ M is form of an M-Non Bondage set (NBS) if satisfy this condition of $\eta_i(\hat{G} - B) = \eta_i(\hat{G})$, Here S is the collection of set of all strong arc of G and also $\alpha_k(\hat{G})$ formed by maximum cardinality of B in all the set of strong arc in IF Graph of $\eta_i(\hat{G} - B) = \eta_i(\hat{G})$.

IV SOME IMPORTANT CONCEPTS OF BONDAGE SET OF STRONG ARC IN COMPLETE GRAPH of G.

Theorem: 1

Consider $\hat{G}$ is a fuzzy graph and M be a count of minimum 2−bondage set G of a bondage set of strong arc if $\eta_{2i}(\hat{G} - k_i) = \eta(\hat{G} - k_i)$. k_i is denoted by strong arc in $\hat{G}$.

Proof : Let $\hat{G}$ be a fuzzy graph and M be the strong arc of a minimum 2−bondage set of $\hat{G}$ with $\eta_i(\hat{G} - k_i) = \eta(\hat{G} - k_i)$. Therefore $\eta_i(G) \geq \eta_i(G)$ and given $\eta_i(\hat{G} - k_i) = \eta_i(\hat{G} - k_i)$.

Since X is a minimum 2−bondage set of G then we have $\gamma_i(\hat{G} - k_i) > \gamma_i(\hat{G})$.

Thus $\eta_i(\hat{G} - k_i) > \eta_i(\hat{G}) \geq \eta(\hat{G})$.

$$\eta_i(\hat{G} - k_i) > \eta_i(\hat{G}).$$

$$\eta(\hat{G} - k_i) > \eta(\hat{G}).$$

Therefore, the N-bondage set of G with a strong arc. Hence the proof.

Theorem 3.2 Let the value of ň$_2$ (G) represents the maximum set of a number of arcs in a stable set of all collection of arcs in the fuzzy graph G, while given value of ꞵ$_2$ (G) of the 2−bondage set represents the minimum set of number of arcs are deleted from G into order to respectively all the arc of its Graph G. It is possible for ꞵ$_2$ (G) to be greater than 0 even if ň$_2$ (G) = n, meaning that the removal of some nodes is still necessary to turn G into a graph G.

Proof This means that removing the arc e causes at least one node in the minimum 2-dominating set to no longer be 2-dominated, implying that this node must have had only one neighbour in the 2-dominating set before the deletion, in this case which is not possible. Thus, the deletion of any arc *e* from G does not increase ň₂ (G). Therefore, G does not have a 2−bondage set. Hence ꞵ₂ (G) = 0.

Note: ꞵ₂ (G)-2−bondage set.

Theorem 3.3 states that the presence of an isolated arc in a graph does not necessarily mean that the independence number is 1. The proof explains that according to IFG, in the strong arc of an isolated arc pk, it is associated with all dominated arcs in a minimum collection of independent sets. Both ui and vj are directed to be members of the respective dominant set of G form (G-pk). Formally (G-pk) > ηi(G) and {pk} forms a Bondage Set of given graph G. Hence cardinality of dominating αk(G) = 1.

Theorem 3.4 states that the maximum coordinate size of an independent set in a Graph is associated with the cardinality number of an independent set or i(G) set. The value of αi(G) can be larger than 1 even if G is an independent Graph (IFG). The actual value depends on the specific structure of the Graph. Given two vertices not in a set independently connected by strong arcs.

The proof explains that η(G)=1 indicates that counting from the directed Graph form G, a single vertex dominates all other vertices. This is consistent with the statement that the center vertex in G* dominates all other vertices. By eliminating any one vertex p from G we obtain (G-p)=2 because each vertex will create a BS and bondage number will be G=1.

V BONDAGE AND NON-BONDAGE EXAMPLE

Let G be a IFG. Calculating all the BS and NBS of the given Graph and dominating cardinality in the complete Graph.

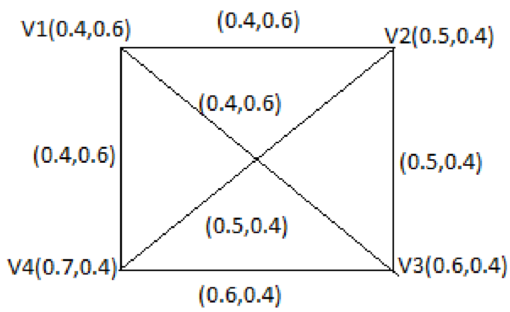

Figure 1.

From the above Graph, by the definition of domination set of G is associated with a cardinality of η(G) calculated for all the vertices by adjacent of G.

D_1={V_1,V_2,V_4}, V-D_1={V_3} so D_1 is dominance vector. This passage describes the calculation of the strong arc of all arcs dominantly in a graph G. The order of G is denoted by |V| with a cardinality of dominating set with all vertices. The maximum strength in the Graph for every arc connectedness of G is collected.

Two arcs Vi and Vj in V are considered to be the μ1-connectedness strength of G. Another arc μ2-strength of the arc connectedness is CON(G)(Vi,Vj) = maximum of { ημ1 } and CON(G)(Vi,Vj) = minimum of { ημ2 }. This includes all paths in G including Vi and Vj.

Now calculate $CON_{(G)}(V_1, V_2) = CON_{(G)}(p_1)$. In this concept we have two cardinal paths from V_1 to V_2. Contribute to one by one vertex in graph G form the form of V_1 to V_2 containing e_1 arc, but the second path of V_1 to V_4 to V_3 to V_2 contains e_4, e_3, and e_2 arcs. According to the definition, by the step of cardinality on the strength of all paths.

$\varphi_{1ij} \geq CON(G)_{\varphi 1}(G)(v_i, v_j)$ and $\varphi_{2ij} \leq CON(G)_{\varphi 2}(G)(v_i, v_j)$.

$CON_{(G)}(v_1, v_2) = CON_{(G)}(e_1) = (0.4 \vee 0.4, 0.6 \wedge 0.6)$

$= (0.4, 0.6)$,

$CON_{(G)}(v_2, v_3) = CON_{(G)}(e_2) = (0.5 \vee 0.4, 0.4 \wedge 0.6)$

$= (0.5, 0.4)$,

$CON_{(G)}(v_3, v_4) = CON_{(G)}(e_3) = (0.6 \vee 0.4, 0.4 \wedge 0.6)$

$= (0.6, 0.4)$,

$CON_{(G)}(v_1, v_4) = CON_{(G)}(e_4) = (0.4 \vee 0.4, 0.6 \wedge 0.6)$

$= (0.4, 0.6)$,

$CON_{(G)}(v_1, v_3) = CON_{(G)}(e_5) = (0.4 \vee 0.4, 0.6 \wedge 0.6)$

$= (0.4, 0.6)$,

$CON_{(G)}(v_2, v_4) = CON_{(G)}(e_6) = (0.5 \vee 0.5, 0.4 \wedge 0.4)$

$= (0.5, 0.4)$,

Since all e's apply the condition of strong arc connectedness, $\varphi_{1ij} \geq CON(G)_{\varphi_1}(G)(p_i, p_j)$ and

$\varphi n_{2ij} \leq CO(G)_{\varphi_2}(G)(p_i, p_j)$.

So G= $\{v_1, v_4\}$. So that form of $\eta_r(G)$ is equal to,

$$\eta_r(G) = \frac{1+0.4-0.6}{2} + \frac{1+0.7-0.4}{2}$$

$$\eta_r(G) = 1.05.$$

The Bondage Set of G is the subset of Strong arcs from IFG by eliminating arcs in that set. When G produces G, which is a more significant (G) of G, we shall determine the Bondage Set of G.

The concept of minimal cardinality H = $\{e_1\}$ is a IF Graph of G fit for all set of Strong Arc. Finding of all values should calculate of G − $\{e_1\}$ by giving the IF graph then the amplification of Graph follows all the conditions $\varphi_{1ij} \geq CON(G)_{\varphi_1}(G)(p_i, p_j)$ and $\varphi n_{2ij} \leq CON(G)_{\varphi_2}(G)(p_i, p_j)$ as, Hence the given G of set will be, $\{e_2, e_3, e_4, e_5, e_6\}$. Secondly then the strong arc of G − $\{e_1\}$ with the lowest value of the cardinal of arc is $\{v_3, v_4\}$, then, its $\eta_1(G)$ will be,

$$(\eta_1(G)-\{e_1\}) = \frac{1+0.7-0.4}{2} + \frac{1+0.6-0.4}{2}$$

$$\eta_1(G - \{e_1\} = 1.25 > 1.05.$$

Hence H = $\{e_2\}$ as a Bondage Set.

Similarly we form H = $\{e_3\}$ is a subset of the given Strong Arc of G then the set of G − $\{e_3\}$.

As per the condition of the connectedness of strong arc in $\hat{G}$ of $\varphi_{1ij} \geq CON(\hat{G})_{\varphi_{1i}}(p_i, p_j)$ and $\varphi_{2ij} \leq CON(\hat{G})_{\varphi_{2i}}(p_i, p_j)$

The Strong Arc will be formulated as follows$\{e_1, e_2, e_4, e_5, e_6\}$.The $\{v_1, v_2\}$ is G − $\{e_3\}$ IF Graph's form and the dominating set with the lowest value cardinality $\eta_2(G)$ will be

$$(\eta_2(G)-\{e_3\}) = \frac{1+0.4-0.6}{2} + \frac{1+0.5-0.4}{2}$$

$(\eta_2(G - \{e_3\}) = 0.95 \ngtr 1.05$.

So that H = $\{e_3\}$ is a NBS bondage.

Formatting the same process, consider H = $\{e_4\}$ consider as a subset G. Manipulate strong Arc of G − $\{e_4\}$.

Then the satisfied the value of $\varphi_{1ij} \geq CON(G)_{\varphi i1}(G)(p_i, p_j)$ and

$\varphi_{2ij} \leq CON(G)_{\varphi_2}(G)(p_i, p_j)$.

Further Strong arc formed by,$\{e_1, e_2, e_3, e_5, e_6\}$.

The $\{v_2, v_3\}$ is G − $\{e_4\}$ included lowest cardinality of dominating value of $\eta_3(G)$ will be,

$$(\eta_3(G)-\{e_4\}) = \frac{1+0.5-0.4}{2} + \frac{1+0.6-0.4}{2}$$

$(\eta_3(G - \{e_4\}) = 1.15 \ngtr 1.15$.

Therefore, H = $\{e_4\}$ is a NBS bondage.

To be continued for H = $\{e_5\}$the lowest cardinality of Strong Arc of H . We find the

Strong Arc of H of G − {e_5}.

Same process to be followed any strong arc of dominating set to be formed by $\varphi_{1ij} \geq CON(G)_{\varphi_1}(G)(p_i, p_j)$ and $\varphi_{2ij} \leq CON(G)_{\varphi_2}(G)(p_i, p_j)$ as follows the same,{e_1, e_2, e_3, e_4, e_6}.

The {v_2, v_4} is G − {e_5} and its η_4(G) will be,

$$(\eta_4(G)\text{-}\{e_5\}) = \frac{1+0.5-0.4}{2} + \frac{1+0.7-0.4}{2}$$

$$(\eta_4(G - \{e_5\}) = 1.25 > 1.05.$$

H = {e_5} is a Bondage of Set bondage.

So for H = {e_6} a subset of the form Strong of all collections arc of given set of IF G − {e_6}.

$\varphi_{1ij} \geq CON(G)_{\varphi_{1i}}(p_i, p_j)$ and $\varphi_{2ij} \leq CON(G)_{\varphi_{2i}}(p_i, p_j)$ of the formatted of{e_1, e_2, e_3, e_4, e_5}.

The {v_1, v_3} is G − {e_6} cardinality value and its η_5(G) will be,

$$(\eta_5(G)\text{-}\{e_6\}) = \frac{1+0.4-0.6}{2} + \frac{1+0.6-0.4}{2}$$

$$(\eta_5(G - \{e_6\}) = 1 \ngtr 1.05.$$

For the above discussion, we collect the bondage set of cardinality is H = {e_1,e_5}, then the cardinality of domination set is $\alpha(\hat{G})$ of given $\hat{G}$.

$$\text{CON of } \alpha(\hat{G}) = \frac{1+0.4-0.6}{2}$$

$$\text{CON of } \alpha(\hat{G}) = 0.4.$$

Final collection of the cardinal value of G is a complete graph of IFG in Bondage and Non-bondage set with a strong Arc.

VI CONCLUSION

Intuitionistic fuzzy graphs (IFGs) are a fuzzy Graph where each edge is assigned not only an ordering point, reflecting the uncertainty or ambiguity inherent in many real-world situations. The concept of complete IFG refers to a special case where the connection of one vertex to another and the edge weights reflect the degree of compatibility or conflict between the vertices. Studying the properties of IFGs, exceptionally complete IFGs, can have collaborative Network applications in various fields. For example, IFGs performed in clustering, classification, and pattern recognition in computer science. In engineering, IFGs can be used in modelling and control systems design. In operational research, IFGs can be used in decision-making and optimisation problems. Therefore, the study of IFGs is an important and promising research area with numerous potential applications.

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