Dynamic Stiffness formulation of an orthotropic Plate for free vibration analysis by Classical Plate Theory

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Abstract*: Aircraft structures generally modeled as assemblies of thin-walled structural elements, in particular, the top and bottom skins, torsion box, ribs and webs of the wing idealized as plates. Thus, the free vibration analysis of such structures plays an important role in aircraft design. The purpose of this research is to develop the dynamic stiffness method for an accurate and efficient, free vibration analysis of an orthotropic plates and plate assemblies. Free vibration analysis of orthotropic plate investigated with the help of dynamic stiffness method using classical plate theory. In this study, firstly, the fundamental equation of the classical plate theory (CPT) for orthotropic plate briefly summarized. Secondly, the dynamic stiffness matrix based on the CPT formulated. Subsequent to this development, the assembly procedure and imposition of boundary conditions by suppressing appropriate degrees of freedom (penalty method) explained in Section. This followed by section, which highlights the application of the Wittrick–Williams (WW) logic for computation of natural frequencies of thick plates with various boundary conditions. The rectangular plates have two opposite edges simply supported, while all possible combinations of free (F),simply supported(SS) and clamped (C) boundary conditions are applied to the other two edges. Hamilton's principle used to derive the governing differential equations of motion and natural boundary conditions in free vibration. Experimental validation carried out using FFT Analyzer. Finally, the study closes with comparison of the results obtained by CPT, DSM and FFT analyzer with concluding remarks.

Keywords: *Dynamic Stiffness Matrix, Finite Element Method, Free Vibration, Penalty Method, Wittrick-Williams Algorithm.*

1. INTRODUCTION

Mechanical engineering design has undergone fundamental changes arising out of space age requirements. Design requirements of present space technology have led to emergence of many new technological disciplines. These requirements have also provided sufficient background material for transformation of already existing areas of technology. It is due to this fact that subjects of mechanical design as well as analysis, amongst many others have undergone fundamental changes. Mechanical design requirements of present day space technology are quite complex as well as they place demands of high orders of accuracy on various designed functional components of systems. Precise dynamic performance evaluation of designed mechanical systems has become a prerequisite before any action initiated for their manufacture. The subject of machine vibrations, which is concerned, with evaluation of dynamics of mechanical system needs methodologies which could successfully deal with emerging situations of present era.

Vibrations

Vibration is a mechanical occurrence in which case fluctuations transpire around an equipoise location. Initiation of word is Latin vibrationem ("shaking, displaying"). Vibration is an irregular or regular moving of elements of an elastic form or medium in then again inverse ways from condition of equilibrium when that steadiness is upset. Just like a thin cord generates pleasant sounding tones or molecules perceptible everywhere converses 2 sounds to ear). The oscillations might be steady, e.g., oscillations of a pendulum or unsteady e.g., bumps of a vehicle suspension or tire on a village, hilly or rocky street.

Vibrations are fluctuations in mechanical dynamic systems. Some of time these vibration sources insignificant or candid execution or safety issues in designed systems. As example, when an airplane wing exorbitantly vibrates, travelers in airplane grow to be awkward particularly at occurrences of oscillations related to NFs linked to humanoid as well as body part. Indeed, it is notable that thunderous frequency of humanoid abdominal lot (approximated. 4 to 8 Hz) must to be upheld a tactical place from no matter what when planning elite airplane as well as recyclable communication automobiles in light of fact that continued presentation could cause candid interior injury. On off chance that an airliner extension trembles ubiquitously amplitude of an all-encompassing timeframe, extension is going to face inevitable encounter with fatigue failure or similar and it would conceivably make airplane crash bringing about injuries as well as additionally fatalities. Wing vibrations of this sort are typically associated with widespread collection of fluster marvels welcomed on by fluid-structure associations. The paramount well-known conniving debacle ever was Tacoma Narrows Bridge catastrophe of 1940. It failed for the reason, which is similar kind of self-energized vibrational behavior that transpires in airplane wings.

Why Vibration exposure evaluation is important?

We could experience shaking as well as realize that individuals presented to it. Even though, we cannot decide whether what we feel will be destructive. For that, we must quantify these oscillations or vibrations. The investigation of wellbeing impact of vibrating parts want proportions of generally speaking "pressure waves" (vibration energy) produced by vibrating gear or structures. Vibrations arrive in organization from characteristic of body or organ in connection with parts causing vibrations or vibrating gear. For example, consider a laborer is utilizing a hand-held hardware, like a cutting apparatus, drill and the shaking affects hands as well as arms. Such a scenario named hand-arm vibrating arrangement. In such scenario, when the laborer seated or stations on a floor which vibrating or seating arrangement, shaking of machine upsets nearly whole body and named entire body vibrations.

The danger initiated by such shaking or of vibrations can cause an injury and it relies upon every day exposure of the person to such vibrations. An assessment of danger considers force as well as frequency of the vibrating body, span (long periods) of presentation as well as characteristic of body which is going through energy of vibrating body. Vibrating at Hand-arm makes harm for hands as well as fingers of person. It may cause great amount of harm to veins, bone joints and nerves in person’s fingers. A disorder follows subsequently which recognized usually as white finger infection or hand-arm vibrating disorders (HAVS). Single indications are that predisposed body part changes color to white, especially if that body part exposed to cold. Vibration-prompted issues noted above as white finger infection additionally affects lost grasp force as well as loss of affectability to contact.

After consideration of above case, problems of anxious, circulatory as well as stomach related body functions are not explicit to entire body. These problems could arise by a mix of various other working situations and way of life factors as opposed to by single physical factor alone. Analysis of vibrating parts and reducing these are important in such cases.

Scope of work

The finite element method is a mathematical method for investigating structures and continua. It models a structure as a gathering of little parts (elements). Every element is of straightforward math and along these lines is a lot simpler to examine than real structure. There are two types of removal based finite element method in dynamics. The initial, a rough method, interjects displacements utilizing piecewise polynomial shape capacities. The second, alluded to as explicit method, interjects displacements utilizing shape works that fulfill static equilibrium condition precisely, brings about a 24 continuum element. The finite element as well as continuum element is connected by methods for Simpson's speculation, which expresses that if an adequate amount of allowable finite elements is utilized, finite element modeling and continuum modeling are proportionate. Both expected shape as well as accurate static type of finite element method displays incorrectness since frequency-free shape capacities neglect to peak for candid Eigen capacities, which rely expressly upon relating natural frequency. DS Method could kill such error issues by utilizing frequency subordinate shape works that are definite arrangements of overseeing various conditions, consequently gives careful usual modes to a vibrating structure. It kills spatial discretization blunder and is fit for foreseeing infinite count of natural modes by methods for few degrees of freedom. Since shape capacities utilized are frequency-subordinate, they came about DS matrix$[D]$are intrinsically frequency-subordinate. The Eigen concern to free vibrating body or forced vibrating body by modular test portrayed by conditions are supported by

$$[\,D\,(\omega)\,]\{u\} = 0 \text{ and}$$

$$\text{and } [\,D\,(\omega)\,] = [\,K\,(\omega)\,] - \omega^2[\,M(\omega)]$$

Here,

$[D\,(\omega)\,]$ is DS matrix

Much of time, we can view DS Method as a definite method. In certain circumstances, just a single element is required to ascertain any ideal amount of frequencies to inside finite accuracy of software used in it. Discretization is required uniquely to isolate areas of unexpectedly contrasting material or mathematical properties, or to oblige concentric forces, upholds, and so forth. Subsequently, quantity of obscure nodal displacement or level of freedoms is significantly less in DS Method than in regular finite element method. DS Method is particularly valuable in investigation of bended structures, which could model a whole length of a part by only one element, with goal that it is conceivable to dodge mistakes brought about by ignoring shape of individuals during discretization of elements. Accordingly, technique of DS matrix in vibrating body investigation of structures has certain points of interest over regular finite element method, especially in case larger frequencies as well as improved exactness of results are necessary. This is on grounds that, in FEM, property of an individual element of the structure are from accepted shape works as are not 'exact', while property got from DS Method is dependent on shut structures analytical arrangement of various conditions of element as well as consequently which is reasonably named as 'exact'.

2. LITERATURE REVIEW

In 2019 A. Muc et al. [1] published a paper displaying the informative game plan of the waver concern if there ought to emerge an event of isotropic plates having restricted estimations (for instance a width as well as a length) distributed by Bolotin in 1961. The endorsement of the structure similarly investigated. The models with various stacking plan, fiber bearing, limit case as well as thermo-mechanic loads thought of. Some proposition for future appraisal intertwines, the amplification of the assessment to falter of structures, considering the impact of temperature dispersals on shiver characteristics. The testing has dissected in a logical way, the effects of breaking point conditions, in-plane mechanical as well as warm loads, orthotropic modulus extent and stacking courses of action on shiver ascribes. The assessment nuances the analysis coordinated for the standard moving toward stream.

A.Y.T. Leung et al. [2] in 2014 had an explanatory arrangements as a convoluted arrangement obtained by the strategy for partition of variable supporting the split face conditions, while redrafting the overseeing circumstance in Hamiltonian structure. Hypothetically, adequate amount of coefficient of the convoluted arrangement to fulfill any peripheral limit circumstance exists. Except if the limit is straightforward, the network relating the coefficient to the peripheral limit circumstance is not well adapted, e.g., round. An advancement of another two-level limited strategy (FEM) using the complex arrangement as global capacities while utilizing the traditionalist limited shape capacities as neighborhood capacities is done in this paper. As the diagnostic arrangement typified in the change, the exactness of the anticipated SIFs as well as their subordinates is high. FEDSM used effectively in the paper and the results achieved matched with the expected results. A mathematical model used effectively in the research and the result are analyzed. In the paper publishes in 2011 Abbas Assadi et al. [3] Nonlocal flexibility utilized to outline the scale consequences for the dependability concern of such structures. General clasping circumstance are inferred considering autonomous uprooting amplitudes for every graphene sheet which is conceivable by forcing a few limitations on burdens or relocations as controlling boundaries. Non-uniform in plane stacking through the cover thickness is viewed as which brings about various stacking on every graphene layer. Accordingly, various kinds of as single and non-harmony clasping modes are acquired which could not be acknowledged by demonstrating the entire structures as a traditional overlaid plate. The outcomes got for a triple layered sheet show the accompanying general ends, with the more straightforward cases confirmed by various references. For as single clasping methods of the model overlay, the impact of the nonlocal edges is impressively more than the non-harmony cases in this manner getting more huge. The arrangement of the overseeing circumstance requires a few suppositions identifying with the heap or dislodging relative proportions made (load/uprooting control issues). For a successful plan in designing applications, Alexey N. Fedorenkoa et al. [4] in 2019 assessed the disappointment conduct as well as mechanical reaction of covered composites is basic. As of late gigantic count of disappointment, models for composites were set up and looked at in relevance as well as precision. The numerical model for materials with reliance of versatile property on the kind of stacking was proposed. The constitutive connection to shear reaction with harm rate reliance supplements the created model. In the investigation, a phenomenal relationship among hypothetical and trial results was cultivated as well as mathematical reenactments for instance issues of composites under biaxial stacking and open-gap pressure plate test performed.

In 2017, Amirhadi Alesadi et al. [5] in their this paper, the iso-geometric approach (IGA) as well as Carrera's Unified Formulation (CUF) utilized with the expectation of complimentary vibrating body as well as linearized clasping inspection of covered composite plates. Cubic as well as quadratic NURBS works acceptably weaken the viability of shear securing meager and thick plates, which appeared with the assistance of the introduced mathematical outcomes. For arriving at the assemblies arrangement, if there should be an occurrence of slender as well as thick plates, 3rd request and fourth request model serves separately, which is thinking about the outcomes identified with various request of Taylor development. Published in 2017, Anjibabu Merneedi et al. [6] with the adjustment in area of patterns of various sizes along its askew as well as significant pivot line by thought of various viewpoint proportions of plates, an inspection made with the variety in the common recurrence of a rectangle shaped plate, which is essential for the investigation. Henceforth it cleared from the systematic calculations that there is an imperative impact of cutout's position, shape as well as size on the normal frequencies of the plate. Ashraf M. Zenkour et al. [7] in the paper published in 2006, for upheld practically reviewed rect. plate presented to a cross over uniform burden, the static reaction investigated in this examination. Non-dimensional anxieties relocations registered for plates with earthenware metal blend. It seen that the fundamental reaction of the plates that compare to property transitional to that of the metal as well as fired, is essentially lie in the middle of that of clay as well as metal. This conduct discovered to be the case independent of limit conditions. Along these lines, the inclinations in material property assume a significant part in deciding the reaction of the FGM plates.

For the investigation of shear deformable progressed composite pillars as well as plates, Atteshamuddin S. Sayyad et al. [8] in 2017, a dislodging based brought together shear distortion hypothesis is set up in the paper. To make up for the impact of cross over shear twisting, this hypothesis is created with the consideration of mathematical (TSDT), explanatory (PSDT), exponential (ESDT) and hyperbolic (HSDT) shape capacities in agreement of thickness organize. The cross over shear stresses fulfilling foothold free limit circumstance for the FG shafts as well as plates are gotten utilizing constitutive relations though for the covered and sandwich plates they are acquired utilizing circumstance of harmony of hypothesis of versatility ignoring body powers. All the hypotheses precisely foresee the impact of delicate adaptable center if there should be an occurrence of sandwich bar as well as plate twisting. In view of the Carrera Unified Formulation (CUF), B. Wua et al. [9] in 2019 a complete Lagrangian-type brought together definition of full mathematically nonlinear refined plate hypothesis examined in the examination. In particular, in the absolute Lagrangian portrayal, the Green–Lagrange strain vector as far as the dislodging segments composed as the total articulations of the digression and secant stiff networks of the bound together plate components have been inferred which is the principle versatile bit of leeway of CUF, in a bound together path regarding essential cores. To determine the nonlinear logarithmic cases for various mathematical instances of isotropic rectangular shaped plates, a Newton- Raphson linearization plot alongside a bend length and requirement connection examined and contemplated. In 2017, Bin He et al. [10] in paper, they have broadened the recently talked about FS-TMM. Two exchange methodologies created in this paper to break down the clasping issues of the stretched, open meager walled individuals under the limit state of essentially upheld stacked edges. To approve the strategy, a symmetric Iarea part, a deviated E-segment part and an X-segment count is included. Couple exchange strategies are created to handle the exchange issues at the intersection of the open expanded cross-part of slight walled individuals by utilizing semi-explanatory limited strip move framework strategy (FS-TMM) which is a joined utilization of semi-logical limited strip technique (SA-FSM) as well as move grid technique (TMM) for the clasping investigation. Contrasted with conventional SAFSM, this technique has a littler framework as well as higher computational productivity because of no global stiff network created.

3. PLATE THEORY AND DERIVATION OF GOVERNING EQUATION

At point when body limited by surfaces, level in math, whose parallel dim are enormous contrasted with partition amongst surfaces, known as Plate. Plate is basic part, which described by couple of key properties. Initially, its mathematical design is three-dim (3D) strong whose thickness is little when contrasted and various dimensions. Besides, impacts of loads that required exerting on it just create stresses whose result is, in handy terms, only typical to element's thickness.

Mathematically, plates are limited either by straight or bended limits. Loads types of which could be static or non-static. In case of plates are contradictory to plate faces.

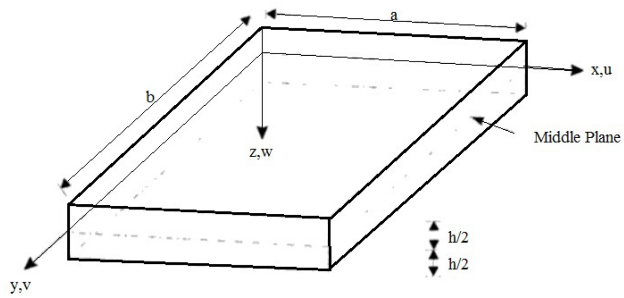

Fig.2 Co-ordinates and dimensions of rectangle shaped plate

The thickness of plate (h) estimated toward path typical to center plane of plate. Flexural property of plate to great extent relies upon its thickness as opposed to its other two-dim (2D) (length and width). Overall, plate issues classified into significant triple classifications - thin plate, tolerably thick plate as well as thick plate, contingent on thickness of plate. This additionally partitioned into few gatherings relying on idea of distortion as well as material property of plate.

Stress at point: Stress Tensor

Contemplate an elastics structural element of any ordinary profile exposed to peripheral stresses in evenness. Also, let us check a quantifiable three dimensional location somewhere in inner of structure. If we distribute Cartesian axes having co-ordinates x, y, and z, as revealed in fig, it is appropriate to allocate a miniscule constituent in the shape of a parallelepiped (dx; dy; dz), which have surfaces and axes planes parallel to each other.

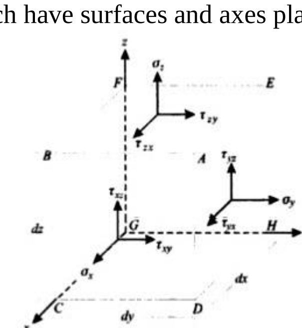

Fig.3 Stress components in Cartesian axes system

$$\sigma_x' = \left(\sigma_x + \frac{\partial \sigma_x}{\partial x}\, dx\right), \sigma_y' = \left(\sigma_y + \frac{\partial \sigma_y}{\partial y}\, dy\right)\ and$$

$$\sigma_z' = \left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\, dz\right)$$

$$\tau_{xy}' = \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\, dx\right)\ and\ \tau_{yx}' = \left(\tau_{yx} + \frac{\partial \tau_{yz}}{\partial y}\, dy\right)$$

$$\tau_{yz}' = \left(\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y}\, dy\right) and\ \ \tau_{xy}' = \left(\tau_{xy} + \frac{\partial \tau_{zy}}{\partial z}\, dz\right)$$

$$\tau_{zx}' = \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\, dz\right)\ and\ \ \tau_{xz}' = \left(\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\, dx\right)$$

Previously mentioned set of stresses constituents following, up on essences of element shapes stresses tensor, T_S, i.e.

$$T_s = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

Which, is symmetric concerning main askew due to correspondence law of shear stresses, for example,

$$\tau_{xy} \; matches \; with \; \tau_{yx}$$
$$\tau_{xz} \; matches \; with \; \tau_{zx} \; and$$
$$\tau_{yz} \; matches \; with \; \tau_{zy}$$

Along these lines, just6 stresses constituents out of 9 in stresses tensor are autonomous. Stress tensor, T_S, totally describes three-dim (3D) cases of stresses at focal point.

For elastics stresses investigation of plates, two-dim (2D) cases of stresses of unique significance. In this case

$$\sigma_z \; equals \; to \; \tau_{yz}$$
$$\tau_{xz} \text{ equals to } 0$$

Subsequently, two-dim (2D) stresses tensor has structure

$$T_s = \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$$

In above equation $\tau_{xy} \; matches \; with \; \tau_{yx}$

Equations of Equilibrium

The stresses constituents presented beforehand should fulfill accompanying differential articulation of equilibrium:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0$$
$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0$$
$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + F_z = 0$$

Here, $F_x, F_y \; and \; F_z$ are body forces (e.g., gravitational, attractive forces). In inferring these equations, correspondence of shear stresses utilized and the equations derived

Generalized Hooke's Law (Stress - Strain relations)

Preceding sections discusses theory of stresses dealing with static steadiness of body as well as theory of strains where strains related to constituents of displacements purely from geometrical consideration. Obviously constitutive law or stress-strains relation depends upon nature of material. In general, an elastics body classified as homogeneous or non-homogeneous. Homogeneous body is alone, in which structures as well as composition of material are similar at all points. Theory of elasticity extensively deals with homogeneous bodies. However, homogeneous elastics body could be either isotropic or anisotropic. Body said to be isotropic if point in it has identical property in every single direction, while substance said to be anisotropic if property varies in various directions. Materials such as wood, rock, reinforced concrete as well as glass-reinforced plastics are all anisotropic, while materials like steel, aluminum, and plastics are isotropic. Further, in theory of elasticity, relationship among stress-strains is self-determining of time or loading history. It also assumes that change in strains resulting from stresses is instantaneous, as well as that material is perfectly elastic, which means that all internal energy stored during loading is completely recovered after unloading. We will be concerned in this text with materials exhibiting linear elastics behavior wherein every stresses constituent linearly connected to strain. General form of strain-stress relation for an anisotropic body written as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_x \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix}$$

Or in condensed shape

$$\{\epsilon_i\} = [S_{ij}]\{\sigma_i\}(i, j = 1,2, \dots 6)$$

Here S_{ij} is a compliance matrix. Above solution indicates that for general anisotropic body, there are thirty-six independent elastics constants, which are functions of axes co-ordinates.

A material having above stress-strains connection is supposed to be transversely isotropic as well as quantity of autonomous elastics constants decreases to 5 for such material. In event that all triple mutually perpendicular planes are isotropic in nature, at that point stress-strains connection composed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_x \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix}$$

In addition, count of self-regulating elastics constants reduce to two.
For an orthotropic body compliance denoted with engineering elastics constants as

$$\left.\begin{matrix} S_{11} = \frac{1}{E_x}, S_{12} = -\frac{\nu_{xy}}{E_x}, S_{13} = -\frac{\nu_{xz}}{E_x} \\ S_{22} = \frac{1}{E_y}, S_{23} = -\frac{\nu_{yz}}{E_y}, S_{33} = \frac{1}{E_x} \\ S_{44} = \frac{1}{G_{yz}}, S_{55} = \frac{1}{G_{zx}}, S_{66} = \frac{1}{G_{xy}} \end{matrix}\right\}$$

Where E_x, E_y and E_z Are Young's modulii for x, y, and z directions$\nu_{yz}, \nu_{xz}, \nu_{xy}$ Are Poisson's ratios, and G_{yz}, G_{zx}, G_{xy} Are shear modulii in yz, zx and xy planes.
Due to symmetric of compliance matrix, following relationship happened among Young's modulus and Poisson's ratio.

$$E_x \nu_{\mathrm{Yx}} = E_y \nu_{xy},\ E_y \nu_{zy} = E_z \nu_{yz},\ E_z \nu_{xz} = E_x \nu_z$$

Hence, for an orthotropic material we have 9 independent engineering elastics constants.
The strain-stress relationship for an orthotropic body in terms of engineering elastics constants strains in three direction of the plane now transcribed as a matrix,

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{xz}}{E_x} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_y} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_v \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix}$$

By inverting above compliance matrix, classic Stiff matrix for an orthotropic body obtained. Here constants C_{11}, C_{12} ... in terms of engineering elastics constants revealed to be

$$C_{11}=\left(1-\nu_{yz}^2\frac{E_z}{E_Y}\right)\frac{E_x}{V}, C_{12}=\left(\nu_{xy}+\nu_{xz}\nu_{yz}\frac{E_z}{E_Y}\right)\frac{E_y}{V}$$
$$C_{13}=\left(\nu_{xz}+\nu_{xy}\nu_{yz}\right)\frac{E_z}{V}, C_{23}=\left(\nu_{yz}+\nu_{xy}\nu_{xz}\frac{E_y}{E_x}\right)\frac{E_z}{V}$$
$$C_{22}=\left(1-\nu_{xz}^2\frac{E_z}{E_x}\right)\frac{E_y}{V}, C_{33}=\left(1-\nu_{xy}^2\frac{E_y}{E_x}\right)\frac{E_z}{V}$$
$$C_{44}=G_{yz}, C_{55}=G_{zx}, C_{66}=G_{xy}$$

Where

$$V=\left[1-\nu_{xy}\left(\nu_{xy}\frac{E_y}{E_x}+2\nu_{yz}\nu_{xz}\frac{E_z}{E_x}\right)-\nu_{xz}^2\frac{E_z}{E_x}-\nu_{yz}^2\frac{E_x}{E_y}\right]$$

For transversely isotropic material analyzed or considered in above approach, in which yz plane is an isotropic plane, Stiff constants C_{11}, C_{12} ... in terms of engineering elastics constants will be,

$$C_{11}=\left(1-\nu_{yz}^2\right)\frac{E_x}{V}, C_{13}=C_{12}=\nu_{xy}\left(1+\nu_{yz}\right)\frac{E_y}{V}$$
$$C_{23}=\left(\nu_{yz}+\nu_{xy}^2\frac{E_y}{E_x}\right)\frac{E_y}{V}, C_{33}=C_{22}=\left(1-\nu_{xy}^2\frac{E_y}{E_x}\right)\frac{E_y}{V}$$
$$C_{44}=\frac{E_y}{2\left(1+\nu_{yz}\right)}, C_{55}=C_{66}=G_{xy}$$
$$and \quad V=\left[\left(1+\nu_{yz}\right)\left(1-\nu_{yz}-2\nu_{xy}^2\frac{E_y}{E_x}\right)\right]$$

For an isotropic material Stiff constants will be

$$C_{11}=C_{22}=C_{33}=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}=(\lambda+2G)$$
$$C_{12}=C_{13}=C_{23}=\frac{\nu E}{(1+\nu)(1-2\nu)}=\lambda$$
$$C_{44}=C_{55}=C_{66}=\frac{E}{2(1+\nu)}=G$$

Here λ and G are Lame's constants.

In certain practical engineering problems, the person is interested in state of stresses in thin body (measurement in z direction being very insignificant compared to measurement in y and z directions). In such forms, stresses in z direction being very insignificant, are neglected, and it is assumed that body is two-dim (2D) and in state of plane stress. Henceforward, we have for such type of case

$$\sigma_z=\tau_{yz}=\tau_{zx}=0$$

In addition, stress-strains relation for two-dim (2D) orthotropic substance below plane stresses case obtained as,

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & 0\\ C_{12} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

Here stiff constants C_{11}, C_{12} Etc. In terms of engineering elastics constants will be,

$$C_{11}=\frac{E_x}{1-\nu_{xy}\nu_{yx}}, C_{12}=\frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}}$$
$$C_{22}=\frac{E_y}{1-\nu_{xy}\nu_{yx}}, C_{66}=G_{xy}$$

For two-dim (2D) isotropic body beneath plane stresses form Stiff constants will be,

$$C_{11}=C_{22}=\frac{E}{1-\nu^2}, \quad C_{12}=\frac{\nu E}{1-\nu^2}, C_{66}=\frac{E}{2(1+\nu)}$$

Differential Equations

Displacement constituents viz. u, v *and* w considered as the basic unknowns. The problem reduction happens determining these triple unknowns satisfying prescribed case on exterior (boundary conditions). In dispensable differential solution as far as displacements would now be able to gotten by subbing stress-strains connection as well as strain-displacements relations in steadiness solution. It demonstrated that accompanying steadiness solution regarding displacements for an orthotropic body accomplished.

$$C_{11}\frac{\partial^2 u}{\partial x^2}+C_{66}\frac{\partial^2 u}{\partial y^2}+C_{55}\frac{\partial^2 u}{\partial z^2}+(C_{12}+C_{66})\frac{\partial^2 v}{\partial x\,\partial y}$$
$$+(C_{13}+C_{35})\frac{\partial^2 w}{\partial x\,\partial z}+X=0$$
$$(C_{12}+C_{66})\frac{\partial^2 u}{\partial x\,\partial y}+C_{66}\frac{\partial^2 v}{\partial x^2}+C_{22}\frac{\partial^2 v}{\partial y^2}+C_{44}\frac{\partial^2 v}{\partial z^2}$$
$$+(C_{23}+C_{44})\frac{\partial^2 w}{\partial y\,\partial z}+Y=0$$
$$(C_{13}+C_{55})\frac{\partial^2 u}{\partial x\,\partial z}+(C_{23}+C_{44})\frac{\partial^2 v}{\partial y\,\partial z}+C_{55}\frac{\partial^2 w}{\partial x^2}$$
$$+C_{44}\frac{\partial^2 w}{\partial y^2}+C_{33}\frac{\partial^2 w}{\partial z^2}+Z=0$$

The Stiff constants C_{11}, C_{12} ... regarding designing elastics constants for an orthotropic body considered given above solution. In event that Stiff constants regarding designing elastics constants for an isotropic body subbed, above solution for an isotropic body lessen to notable Lame's solution, which generally composed as

$$(\lambda+G)\frac{\partial e}{\partial x}+G\nabla^2 u+X=0$$
$$(\lambda+G)\frac{\partial e}{\partial y}+G\nabla^2 v+Y=0$$
$$(\lambda+G)\frac{\partial e}{\partial z}+G\nabla^2 w+Z=0$$

Here$\nabla^2=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$ is Laplacian operator and$e=\epsilon_x+\epsilon_y+\epsilon_z$isfirst invariant of the strain.

Then again we could keep stresses constituents as obscure and determine lot of6varioussolutions as far as 6 obscure stresses components. These solutions inferred for an isotropic body by consolidating steadiness solutions, stress-strains relations as well as strains similarity solution.

Henceforth governing differential solution as far as stresses constituents obtained as

$$\nabla^2\sigma_x+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial x^2}=-\left(\frac{\nu}{1-\nu}\right)\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}\right)-2\frac{\partial X}{\partial x}$$
$$\nabla^2\sigma_y+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial y^2}=-\left(\frac{\nu}{1-\nu}\right)\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}\right)-2\frac{\partial Y}{\partial y}$$
$$\nabla^2\sigma_z+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial z^2}=-\left(\frac{\nu}{1-\nu}\right)\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}\right)-2\frac{\partial Z}{\partial z}$$
$$\nabla^2\tau_{yz}+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial y\,\partial z}=-\left(\frac{\partial Y}{\partial z}+\frac{\partial Z}{\partial y}\right) \qquad (4.49)$$
$$\nabla^2\tau_{zx}+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial z\,\partial x}=-\left(\frac{\partial Z}{\partial x}+\frac{\partial X}{\partial z}\right)$$
$$\nabla^2\tau_{xy}+\left(\frac{1}{1+\nu}\right)\frac{\partial^2\theta}{\partial y\,\partial x}=-\left(\frac{\partial X}{\partial y}+\frac{\partial Y}{\partial x}\right)$$

Here $\theta=\left(\sigma_x+\sigma_y+\sigma_z\right)$ isfirst invariant of stress in three directions. Above solutions which gets derivation from the basic equations mentioned in the initial stages of the research also known as Beltrami-Michell equations of compatibility.

4. APPLICATION OF DYNAMIC STIFFNESS METHOD

The establishment of DS Method (DSM) was set somewhere near Kloušek, who presented regardless of precedent for mid 1940s frequency-subordinate DS coefficient for a Bernoulli–Euler shaft got from its free vibrating body reaction. Subsequently, coefficient used to known as Kloušek capacities in literature. Kloušek's prior research consequently remembered for a course book. DSM has gone through earth shattering changes since its origin &

there are presently elective structures as well as subordinates of approach known as Spectral element method (SEM) & Continuous Element Method (CEM). First idea created by Kloušek empowered researchers to build up a connection among amplitudes of forces & displacements at hubs of a freely vibrating auxiliary constituent by methods for its DS Matrix. Fundamental structures block of DSM is DS Matrix of an individual auxiliary element, which transformed from its neighborhood axes co-ordinates as well as amassed to shape common DS Matrix of last structures in a datum or global axes structure. Now, it ought to notice that there are numerous likenesses among DSM as well as customary FE approach (FEM) when tackling free-vibration issues of structures. In any case, there are additionally certain significant contrasts among couple of methods.

The equation of motion of a Levy-type plate

Figure shows a rectangle shaped Levy-type plate with couple of SS sides corresponding to x-co-ordinate& other couple of subjective B Cat contrary equal sides to y - co-ordinate.

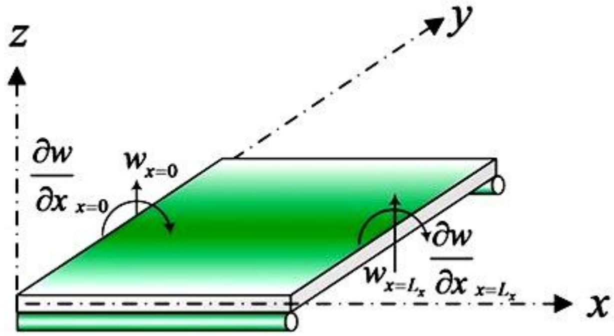

Fig.4 Levy-type rectangular plate in FEM scheme

Levy-type plating is dim of Lx& $Lx$for$x$ and y directions, separately. Homogeneous incomplete differential solution of free vibrating body with consideration deformation shear, transverse displacements of plate given by 2-variable RPT as:

$$D\left(\frac{\partial^4 w_k}{\partial x^4} + 2\frac{\partial^4 w_5}{\partial x^2\,\partial y^2} + \frac{\partial^4 w_b}{\partial y^4}\right) - \frac{\rho h^3}{12}\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right)$$

$$+\rho h\left(\ddot{w}_b + \ddot{w}_s\right) = 0$$

$$\frac{D}{84}(\frac{\partial^4 w_s}{\partial x^4} + 2\frac{\partial^4 w_s}{\partial x^2\,\partial y^2} + \frac{\partial^4 w_s}{\partial y^4}\Big) - \frac{5Eh}{12(1+\mu)}\left(\frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2}\right)$$

$$-\frac{\rho h^3}{1008}\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) + \rho h\left(\ddot{w}_b + \dot{w}_s\right) = 0$$

The governing solution of Bending Moments & Transverse shearing force of plate down course bound generally noted as:

$$M_{xxbn}(x,y;\omega_{nm}) = -D\left[\frac{\partial^2 w_b}{\partial x^2} + \mu\frac{\partial^2 w_b}{\partial y^2}\right]$$

$$Q_{xzbn}(x,y;\omega_{nm}) = -D\left[\frac{\partial^3 w_b}{\partial x^3} + (2-\mu)\frac{\partial^3 w_b}{\partial x\,\partial y^2}\right] + \frac{\rho h^3}{12}\frac{\partial \ddot{w}_b}{\partial x}$$

$$M_{xxsn}(x,y;\omega_{nm}) = -D\left[\frac{\partial^2 w_s}{\partial x^2} + \mu\frac{\partial^2 w_x}{\partial y^2}\right]$$

$$Q_{xzsn}(x,y;\omega_{nm}) = \frac{5Eh}{12(1+\mu)}\frac{\partial w_s}{\partial x}$$

$$-\frac{D}{84}\left[\frac{\partial^3 w_s}{\partial x^3} + (2-\mu)\frac{\partial^3 w_s}{\partial x\,\partial y^2}\right] + \frac{\rho h^3}{1008}\frac{\partial \omega}{\partial x}$$

Here, ρ– density of the mass, h- thickness and D- bend rig and this is articulated by

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

In above solutions, ν = Poisson's ratio $\&E$ = modulus of elasticity for plate. 1st & 2nd terms in solutions generally used for shear as well as twisting displacements in vertical direction of plate.

Levy type solution

In past conversation, we have determined governing differential solution what's more, BC from an infinitesimal steadiness approach for an orthotropic plate utilizing traditional plate theory. Presently in this section, we will see strategy to create DS Matrix.

The initial phase in building up DS Matrix is to unravel governing differential, which, inferred in past conversation. Arrangement looked for in customary Levi structure. A toll type arrangement, which fulfills BCs inferred above, looked for in accompanying structure:

$$w_0(x,y,t) = \sum_{m=1}^{\infty} W_m(x)e^{i\omega t}\sin(\alpha_m y)$$

Above ω is unidentifiedfrequencies and

$$\alpha_m = \frac{m\pi}{L}\ (\text{m} = 1, 2, \ldots\ldots\ldots\infty)$$

By replacement, subsequent 4th order ordinary differential solution obtained & it written in form:

$$D_x\frac{d^4W_m}{dx^4} - \alpha_m^2 D_{xy}\frac{d^2W_m}{dx^2} + (\alpha_m^4 D_y + \rho h\omega^2)W_m = 0$$

The arrangement of above solutions achieved by utilizing differential solutions method and it gives 4 roots. From these roots, 2 arrangements of differential equations are conceivable:

Case I. If

$$\alpha_m^2 \geq \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2} \rightarrow$$

Every single1roots are real

$$(r_{1_m}, -r_{1_m}, r_{2_m}, -r_{2_m})$$

$$r_{1_m} = \sqrt{\frac{\alpha_m^2 D_{xy} + \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2}}{2D_x}}$$

$$r_{2_m} = \sqrt{\frac{\alpha_m^2 D_{xy} - \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2}}{2D_x}}$$

The solution is:

$$W_m(x) = A_m\cosh\left(r_{1_m}x\right) + B_m sinh\left(r_{1_m}x\right) + C_m\cosh\left(r_{1_m}x\right) + D_m sinh\left(r_{1_m}x\right)$$

Case II. If

$$\alpha_m^2 \leq \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2} \rightarrow$$

Two real &2imaginary roots

$$(r_{1_m}, -r_{1_m}, ir_{2_m}, -ir_{2_m})$$

$$r_{1_m} = \sqrt{\frac{\alpha_m^2 D_{xy} + \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2}}{2D_x}}$$

$$r_{2_m} = \sqrt{\frac{-\alpha_m^2 D_{xy} + \sqrt{\alpha_m^4\left(D_{xy}^2 - 4D_xD_y\right) - 4D_x\rho h\omega^2}}{2D_x}}$$

The solution is:

$$W_m(x) = A_m \cosh(r_{1_m}x) + B_m sinh(r_{1_m}x) + C_m \cosh(r_{2_m}x) + D_m sinh(r_{2_m}x)$$

A. Procedure to obtain Dynamic Stiffness Matrix

The approach to get DS Matrix for main case mentioned underneath. Similar structure exerted to get DS Matrix for2ndcase. Presently from known displacementsw$_0$, rotation $\emptyset_y$, edge responses or net shear force V_x, &twisting2ndM$_{xx}$communicated in accompanying structure:

Turning:

$$\emptyset_{y_m}(x,y) = \emptyset_{y_m}(x)\sin(\alpha_m y)$$

$$\emptyset_{y_m}(x,y) = \frac{-\partial W_m(x)}{\partial x}\sin(\alpha_m y)$$

$$\emptyset_{y_m}(x,y) = -[\, A_m r_{1_m} \sinh(r_{1_m}x) + B_m r_{1_m} cosh(r_{1_m}x) + C_m r_{2_m} sinh(r_{2_m}x) + D_m r_{2_m} cosh(r_{2_m}x)\,]\ \sin(\alpha_m y)$$

Shear force, which is Net:

$$V_{x_m}(x,y) = v_{x_m}(x)\sin(\alpha_m y)$$

$$= \left[D_x \frac{\partial^3 w^0}{\partial x^3} + \left(\frac{D_{xy}}{2} - \frac{h^3}{6}Q_{66}\right)\frac{\partial^3 w^0}{\partial x \partial y^2}\right]\sin(\alpha_m y)$$

Assume,

$$H = \left(\frac{D_{xy}}{2} - \frac{h^3}{6}Q_{66}\right)$$

$$V_{x_m}(x,y) = [A_m \sinh(r_{1_m}x)(D_x r_{1m}^3 - H r_{1_m}\alpha_m^2) + B_m cosh(r_{1_m}x)(D_x r_{1m}^3 - H r_{1_m}\alpha_m^2) + C_m sinh(r_{2_m}x)(D_x r_{2m}^3 - H r_{2_m}\alpha_m^2) + D_m cosh(r_{2_m}x)(D_x r_{2m}^3 - H r_{2_m}\alpha_m^2)]\sin(\alpha_m y) \quad (5.70)$$

Bending moment (BM):

$$M_{xx_m}(x,y) = M_{xx_m}(x)\sin(\alpha_m y)$$

$$M_{xx_m}(x,y) = \left(D_x \frac{\partial^2 w^0}{\partial x^2} - \frac{h^3}{12}Q_{12}\frac{\partial^2 w^0}{\partial y^2}\right)\sin(\alpha_m y)$$

Assume

$$I = -\frac{h^3}{12}Q_{12}$$

$$M_{xx_m}(x,y) = [A_m \cosh(r_{1_m}x)(D_x r_{1m}^2 + \alpha_m^2) + B_m sinh(r_{1_m}x)(D_x r_{1m}^2 + \alpha_m^2) + C_m cosh(r_{2_m}x)(D_x r_{2m}^2 + \alpha_m^2) + D_m sinh(r_{2_m}x)(D_x r_{2m}^2 + \alpha_m^2)]\sin(\alpha_m y) \quad (5.71)$$

1) Levy type Plate - Boundary conditions

Figure below shows BC for Levy type plate & in this case, plate is SS on couple of reverse sides as well as complimentary on residual sides.

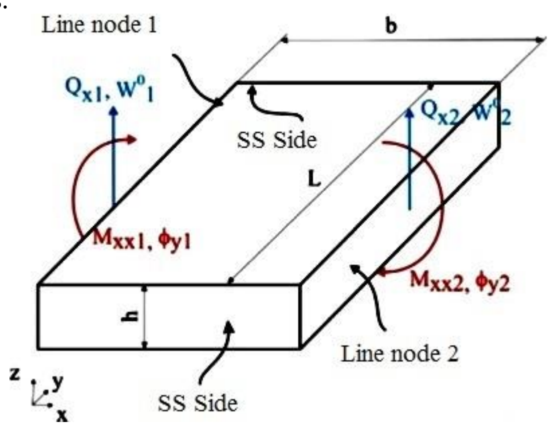

Fig.5 For a plate constituent BCs for displacements& forces

The BCs for displacement:

$$\text{at } x = 0 \rightarrow W_m = W_1 \; ; \; \emptyset_{y_m} = \emptyset_{y_1}$$
$$\text{at } x = b \rightarrow W_m = W_2 \; ; \; \emptyset_{y_m} = \emptyset_{y_2}$$

Correspondingly BC in case of forces is:

$$\text{at } x = 0 \rightarrow v_{x_m} = -V_1 \; ; \; M_{xx_m} = -M_1$$
$$\text{at } x = b \rightarrow v_{x_m} = V_2 \; ; \; M_{xx_m} = M_2$$

By relating these BCs for displacements& by replacement into preceding solution, we obtain subsequent solution:

$$W_1 = A_m + 0\,B_m + C_m + 0\,D_m$$
$$\emptyset_{y_1} = 0\,A_m - r_{1_m}\,B_m + 0\,C_m - r_{2_m}\,D_m$$
$$W_2 = Ch_1\,A_m + Sh_1 B_m + Ch_2\,C_m + Sh_2 D_m$$
$$\emptyset_{y_1} = (-r_{1_m}\,Sh_1)\,A_m - \left(r_{1_m} Ch_1\right) B_m - (r_{2_m} Sh_2)\,C_m - \left(r_{2_m} Ch_2\right) D_m$$

Here

$$Ch_1 = cosh(r_{1_m} b)\;, Ch_2 = \cosh\left(r_{2_m} b\right)$$
$$Sh_1 = sinh(r_{1_m} b)\;, Sh_2 = \sinh\left(r_{2_m} b\right)$$

This written in Matrix relationship as:

$$\begin{bmatrix} W_1 \\ \emptyset_{y_1} \\ W_2 \\ \emptyset_{y_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -r_{1_m} & 0 & -r_{2_m} \\ Ch_1 & Sh_1 & Ch_2 & Sh_2 \\ -r_{1_m} Sh_1 & -r_{1_m} Ch_1 & -r_{2_m} Sh_2 & -r_{2_m} Ch_2 \end{bmatrix} \begin{Bmatrix} A_m \\ B_m \\ C_m \\ D_m \end{Bmatrix}$$

Here,

$$[\delta] = [A]\{C\}$$

Consequently, by exerted BCs for F, succeeding matrix association formation attained:

$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 & R_1 & 0 & R_2 \\ L_1 & 0 & L_2 & 0 \\ -R_1 S_{h_1} & -R_1 C_{h_1} & -R_2 S_{h_2} & -R_2 C_{h_2} \\ -L_1 C_{h_1} & -L_1 S_{h_1} & -L_2 C_{h_2} & -L_2 C_{h_2} \end{bmatrix} \begin{Bmatrix} A_m \\ B_m \\ C_m \\ D_m \end{Bmatrix}$$

i.e.

$$[F] = [R]\{C\}$$

Here

$$R_i = D_x r_{i_m}^3 - H\, r_{i_m} \alpha_m^2\,, L_i = -D_x r_{i_m}^2 - I\,\alpha_m^2\; with\; i = 1, 2$$

By means of mentioned solution, DS Matrix K, in case of plate constituents having basis on CPT easily obtained by getting rid of constant vector C to present:

$$[F] = [K]\{\delta\}$$

Here

$$[K] = [R][A]^{-1}$$

Thus, we get 4×4 DS Matrix with6independent terms as

$$s_{vv}\,, s_{vm}\,, s_{mm}\,, f_{vv}\,, f_{vm}\,, f_{mm}$$

It illustrates consequence on shear as well as moment owing to displacements, which is unit. Thus, *K* described as:

$$[K] = \begin{bmatrix} s_{vv} & s_{vm} & f_{vv} & f_{vm} \\ s_{vm} & s_{mm} & -s_{vm} & f_{mm} \\ f_{vv} & -f_{vm} & s_{vv} & -s_{vm} \\ f_{vm} & f_{mm} & -s_{vm} & s_{mm} \end{bmatrix}$$

Unambiguous solution of constituent derivative of a widespread arithmetical exploitation utilizing MATLAB:

$$S_{vv} = (R_2 S_{h_2} r_1 r_2 (C_{h_1} - C_{h_2}))/\Delta - (C_{h_2} R_2 r_1 (S_{h_1} r_1 - S_{h_2} r_2))/\Delta - (R_1 S_{h_1} r_1 r_2 (C_{h_1} - C_{h_2}))/\Delta - (C_{h_1} R_1 (S_{h_2} r_2{}^2 - S_{h_1} r_1 r_2))/\Delta$$

$$S_{vm} = (C_{h_2} L_2 r_1 r_2 (C_{h_1} - C_{h_2}))/\Delta - (L_2 S_{h_2} r_1 (S_{h_1} r_1 - S_{h_2} r_2))/\Delta - (C_{h_1} L_1 r_1 r_2 (C_{h_1} - C_{h_2}))/\Delta - (L_1 S_{h_1} (S_{h_2} r_2{}^2 - S_{h_1} r_1 r_2))/\Delta$$

$$F_{vv} = (R_2 r_1 (C_{h_1} - C_{h_2}))/\Delta - (R_1 r_2 (C_{h_1} - C_{h_2}))/\Delta$$

$$F_{vv} = (L_2(S_{h_2}r_1 - S_{h_1}r_2))/\Delta - (L_1(S_{h_2}r_1 - S_{h_1}r_2))/\Delta$$

$$S_{vm} = (L_1(C_{h_1}S_{h_2}r_1 - C_{h_2}S_{h_1}r_2))/\Delta - (L_2(C_{h_1}S_{h_2}r_1 - C_{h_2}S_{h_1}r_2))/\Delta$$
$$F_{mm} = (L_2(S_{h_2}r_1 - S_{h_1}r_2))/\Delta - (L_1(S_{h_2}r_1 - S_{h_1}r_2))/\Delta$$

Here

$$R_1 = D_x r_{1_m}^3 - H\, r_{1_m}\alpha_m^2\,, R_2 = D_x r_{2_m}^3 - H\, r_{2_m}\alpha_m^2\,,$$
$$L_1 = -D_x r_{1_m}^2 - I\,\alpha_m^2\,, L_2 = -D_x r_{2_m}^2 - I\,\alpha_m^2$$
$$\Delta = ({C_{h_1}}^2 r_1 r_2 - 2\,C_{h_1}C_{h_2}r_1 r_2 + {C_{h_2}}^2 r_1 r_2 - {S_{h_2}}^2 r_1 r_2 + S_{h_1}S_{h_2}{r_1}^2 + S_{h_1}S_{h_2}{r_2}^2 - {S_{h_2}}^2 r_1 r_2)$$

5. CONCLUSION

Conventional methods for smearing BC in finite element exploration want work to adjust to calculation limits. This thusly necessitates multifaceted matrix logic applied to mechanized work creation from CAD calculation, especially while utilizing quadrangular as well as hexahedral elements. Penalty boundary method (PBM) is obtainable as a technique that fundamentally decreases stretch essential for producing FE modulus in light of fact that work not needed to adjust to CAD calculation. PBM likewise dispenses with discretization mistake as BC exerted utilizing CAD calculation legitimately as opposed to a guess of math. PBM utilizes penalty approach to apply BC on a basic, ordinary work. Boundary condition could be applied similarly as we apply in FE method. Penalty approach commonly used to apply BC to stifle explicit DOF. In this method, an enormous estimation of stiff is included to proper term main inclining of DS Matrix structure for applying BC summed up and shown as follows:

- Free (F): No penalization
- Simply Supported (SS): Penalization is done for W_i
- Fastened (C): Penalization is done for $W_i \& U_{y_i}$

Here, i is constrained node.

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