THE CONVEXITY NUMBER OF A LINE GRAPH OF A GRAPH AND JUMP GRAPH OF A GRAPH AND APPLICATIONS OF CONVEX SETS IN MICRO CARDIAC NETWORK GRAPH

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Abstract
The convexity number $C(G)$ of G is defined as the maximum cardinality of a proper convex set of G, that is $C(G) = max\{|S|: S$ is a convex set of G and $S \neq V(G)\}$. In this paper convexity number of a Line graph of a graph, jump graph of a graph are determined and application of convex sets in micro cardiac network graph is given.

Keywords: convex, convexity number, line graph, jump graph, micro cardiac network graph.
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1.Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology, we refer to [2]. A vertex v is adjacent to another vertex u if and only if there exists an edge $e = uv \in E(G)$. If $uv \in E(G)$, we say that u is a *neighbor* of v and denote by $N_G(v)$, the set of neighbors of v. A vertex v is said to be *universal vertex* if $\deg_G(v) = p - 1$. A vertex v is called an *complete vertex* if the subgraph induced by v is complete.

A shell graph is a cycle C_p with $(p - 3)$ chords sharing a common end vertex called the apex. Bistar is the graph obtained by joining the p pendent edges to both the ends of K_2. The *length* of a path is the number of its edges. Let u and v be

vertices of a connected graph G. A shortest u-v path is also called a u-v *geodesic*. The (shortest path) *distance* is defined as the length of a u-v geodesic in G and is denoted by $d_G(u,v)$ or $d(u,v)$ for short if the graph is clear from the context. For a set S of vertices, let $I[S] = \bigcup_{x,y\in S} I[x,y]$. A set $S \subset V$ is called a *convex set* of G if $I[S] = S$

These concepts were studied in[2,3].

The convexity number $C(G)$ of G is defined as the maximum cardinality of a proper convex set of G, that is $C(G) = max\{|S|: S$ is a convex set of G and $S \neq V(G)\}$. A convex set S in G with $|S| = C(G)$ is called a maximum convex set or C-set. . The line graph $L(G)$ of a graph Gwhose vertices are edges of G and where two vertices of $L(G)$ are adjacent if and only if the corresponding edges are adjacent in G. We call the complement of line graph $L(G)$ as the jump graph $J(G)$ of G.. The jump graph $J(G)$ of a graph G is a graph whose vertices are edges of G and where two vertices of $J(G)$ are adjacent if and only if the corresponding edges are not adjacent in G.

The following Theorems are used in sequel.

Theorem 1.1. Let G be a connected graph of order $p \geq 3$. Then $2 \leq C(G) \leq p-1$.

Theorem 1.2. Let G be a connected graph of order $p \geq 3$. Then $C(G) = p-1$ if and only if G contains a complete vertex.

Theorem 1.3. Let G be a cycle of order $p \geq 3$. Then $C(G)$= $\left\lceil \frac{p}{2} \right\rceil$.

2. The Convexity Number of a Line graph of a graph

Theorem 2.1. For the star graph $G = K_{1,p-1}$, $C(L(G))$ = $p-2$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_{p-1}\}$. Then $L(G)$ has $p-1$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_{p-1}\}$. Since $L(G) \cong K_{p-1}$, By Theorem 1.2, $C(L(G))$ = $p-2$. ■

Theorem 2.2. For the fan graph = F_p $(p \geq 4)$, $C(L(G))$ = $p-1$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_{p-1}\}$. Then $L(G)$ has $2p-3$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_{p-2}\} \cup \{f_1, f_2, \dots, f_{p-1}\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-2)$ and $f_i = xv_i$ $(1 \leq i \leq p-1)$. Since $f_1, f_2, \dots, f_{p-1}$ are adjacent in G. $\langle f_1, f_2, \dots, f_{p-1} \rangle$ is the complete graph K_{p-1}. Therefore $S = \{f_1, f_2, \dots, f_{p-1}\}$ is a convex set in $L(G)$ and so $C(L(G)) \geq p-1$. We prove that $C(L(G)) = p-1$. On the contrary. Suppose that $C(L(G)) \geq p$. Then there exists a convex set S' in $L(G)$ such that $|S'| \geq p$. Then $I_{L(G)}[S'] \neq S'$, which is a contradiction. Therefore $C(L(G))$ = $p-1$. ■

Theorem 2.3. For the wheel graph = W_p $(p \geq 4)$, $C(L(G))$ = $p-1$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_{p-1}\}$. Then $L(G)$ has $2p+2$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_{p-2}\} \cup \{f_1, f_2, \dots, f_{p-1}\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-2), e_{p-1} = v_{p-1} v_1$ and $f_i = xv_i$ $(1 \leq i \leq p-1)$. Since $f_1, f_2, \dots, f_{p-1}$ are adjacent in G. $\langle f_1, f_2, \dots, f_{p-1} \rangle$ is the complete graph K_{p-1}. Therefore $S = \{f_1, f_2, \dots, f_{p-1}\}$ is a

convex set in $L(G)$ and so $C(L(G)) \geq p-1$. We prove that $C(L(G)) = p-1$. On the contrary. Suppose that $C(L(G)) \geq p$. Then there exists a convex set S' in $L(G)$ such that $|S'| \geq p$. Then $I_{L(G)}[S'] \neq S'$, which is a contradiction. Therefore $C(L(G)) = p-1$.

■

Theorem 2.4. For the cycle $= C_p$ $(p \geq 3)$, $C(L(G)) = \left\lceil \frac{p}{2} \right\rceil$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_p\}$. Then $L(G)$ has p vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_p\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-1)$ and $e_p = v_p v_1$. Since $L(G) \cong C_p$, by Theorem 1.3, $C(L(G)) = \left\lceil \frac{p}{2} \right\rceil$. ■

Theorem 2.5. For the path $= P_p$ $(p \geq 3)$, $C(L(G)) = p-2$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_p\}$. Then $L(G)$ has $p-1$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_{p-1}\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-1)$. Since $L(G) \cong P_{p-1}$, by Theorem 1.2, $C(L(G)) = p-2$. ■

Theorem 2.6. For the shell graph G of order $(p \geq 5)$, $C(L(G)) = p-1$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\}$. Then $L(G)$ has $2p-3$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_p\} \cup \{f_1, f_2, \dots, f_{p-3}\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-1)$, $e_{p-1} = v_{p-1} v_1$ and $f_i = v_1 v_i$ $(3 \leq i \leq p-1)$. Since $f_1, f_2, \dots, f_{p-1}$ are adjacent, $< e_1, e_p, f_1, f_2, \dots, f_{p-3} >$ is the complete graph K_{p-3}. Therefore $S = \{e_1, e_p, f_1, f_2, \dots, f_{p-1}\}$ is a convex set in $L(G)$ and so $C(L(G)) \geq p-1$. We prove that $C(L(G)) = p-1$. On the contrary. Suppose that $C(L(G)) \geq p$. Then there exists a convex set S' in $L(G)$ such that $|S'| \geq p$. Then $I_{L(G)}[S'] \neq S'$, which is a contradiction. Therefore $C(L(G)) = p-1$. ■

Theorem 2.7. For the sun let graph G of order $2p$ $(p \geq 3)$, $C(L(G)) = 2p-1$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\} \cup \{u_1, u_2, \dots, u_p\}$. Then $L(G)$ has $2p$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_p\} \cup \{f_1, f_2, \dots, f_p\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-1)$, $e_p = v_p v_1$ and $f_i = v_i v_i$ $(1 \leq i \leq p)$. Since $L(G)$ contains complete vertex and by Theorem 1.2, $C(L(G)) = 2p-1$. ■

Theorem 2.8. For the comb graph $= P_p \odot K_1$ $(p \geq 3)$, $C(L(G)) = 2p-2$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\} \cup \{u_1, u_2, \dots, u_p\}$. Then $L(G)$ has $2p-1$ vertices. Let $V(L(G)) = \{e_1, e_2, \dots, e_{p-1}\} \cup \{f_1, f_2, \dots, f_p\}$ where $e_i = v_i v_{i+1} (1 \leq i \leq p-1)$, and $f_i = v_i u_i$ $(1 \leq i \leq p)$. Since $L(G)$ contains complete vertex and by Theorem 1.2, $C(L(G)) = 2p-2$. ■

Theorem 2.9. For the complete graph $G = K_{r,s} (r < s$ and $r, s \geq 3)$, $C(L(G)) = s$.

Proof. Let $V = \{v_1, v_2, \dots, v_r\}$ and $U = \{u_1, u_2, \dots, u_s\}$ be the two bipartite sets of G. Then $L(V(G))$ has rs vertices. Let $V(L(G)) = \{e_{ij}\}$ $(1 \leq i \leq r, 1 \leq j \leq s)$, where

$e_{ij} = v_i u_j$. For each $v_i \in V$ $(1 \leq i \leq r)$, there are s edges incident at $v_i (1 \leq i \leq r)$. Hence it follows that for each $v_i \in V$ $(1 \leq i \leq r)$, there is a complete graph K_s in $L(G)$. Hence it follows that $C(L(G)) \geq s$. Since $V \cap U = \emptyset$, there does not exists any complete graph of order at least $s+1$ in $L(G)$. Therefore $C(L(G)) = s$. ■

Theorem 2.10. For the Ladder graph $G = L_p$ of order $2p$, $C(L(G)) = 3p - 4$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\} \cup \{u_1, u_2, \dots, u_p\}$. Then $V(L(G))$ has $3p-2$ vertices. Let $V(G) = \{e_1, e_2, \dots, e_{p-1}\} \cup \{f_1, f_2, \dots, f_{p-1}\} \cup \{g_1, g_2, \dots, g_p\}$ where $e_i = v_i v_{i+1}$ $(1 \leq i \leq p-1)$, $f_i = u_i u_{i+1}$ $(1 \leq i \leq p-1)$ and $g_i = u_i v_i$ $(1 \leq i \leq p)$. Then $S = V(L(G)) - \{f_{p-1}, g_p\}$ is a convex set of $L(G)$ such that $C(L(G)) \geq 3p-4$. To prove $C(L(G)) = 3p-4$. Suppose $C(L(G)) \geq 3p-3$. Then there exists a convex set S' of $L(G)$ such that $|S'| \geq 3p-3$. Then $I_{L(G)}[S'] \neq V(L(G))$, which is a contradiction. Therefore $C(L(G)) = 3p-4$. ■

3 The Convexity number of a Jump graph of a graph

Theorem 3.1. For the path $= P_p$ $(p \geq 5)$, $C(J(G)) = p-3$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\}$. Then $J(G)$ has $p-1$ vertices. Let $V(J(G)) = \{e_1, e_2, \dots, e_{p-1}\}$ where $e_i = v_i v_{i+1}$ $(1 \leq i \leq p-1)$. Let $S = \{e_1, e_3, e_4, \dots, e_{p-3}, e_{p-1}\}$ is a convex set of $J(G)$ such that $C(J(G)) \geq p-3$. We prove that $C(J(G)) = p-3$. On the contrary, suppose that $C(J(G)) \geq p-2$. Then there exists a convex set S' in $J(G)$ such that $|S'| \geq p-2$. Then $I_{J(G)}[S'] \neq V(J(G))$, which is a contradiction. Therefore $C(J(G)) = p-3$. ■

Theorem 3.2. For the cycle $= C_p$ $(p \geq 5)$, $C(J(G)) = \left\lceil \frac{p}{2} \right\rceil$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_p\}$. Then $J(G)$ has p vertices. Let $V(J(G)) = \{e_1, e_2, \dots, e_p\}$ where $e_i = v_i v_{i+1}$ and $e_p = v_p v_1$. Since $J(G) \cong C_p$ and by Theorem 1.3, $C(J(G)) = \left\lceil \frac{p}{2} \right\rceil$. ■

Theorem 3.3. For the fan graph $= F_p$ $(p \geq 5)$, $C(J(G)) = p-4$.

Proof. Let $V(G) = \{x, v_1, v_2, \dots, v_{p-1}\}$. Then $J(G)$ has $2p-3$ vertices. Let $V(J(G)) = \{e_1, e_2, \dots, e_{p-2}\} \cup \{f_1, f_2, \dots, f_{p-1}\}$ where $e_i = v_i v_{i+1}$ $(1 \leq i \leq p-2)$ and $f_i = x v_i$ $(1 \leq i \leq p-1)$. Since $< e_1, e_{p-2}, f_3, f_4, \dots, f_{p-4} >$ is the complete graph K_{p-4}. Therefore $S = \{e_1, e_{p-2}, f_3, f_4, \dots, f_{p-4}\}$ is a convex set of $J(G)$ and so $C(J(G)) \geq p-4$. We prove that $C(J(G)) = p-3$. On the contrary, suppose that $C(J(G)) \geq p-3$. Then there exists a convex set S' in $J(G)$ such that $|S'| \geq p-3$. Then $I_{J(G)}[S'] \neq V(J(G))$, which is a contradiction. Therefore $C(J(G)) = p-4$. ■

4. Application of convex sets in micro cardiac network graph

The human heart is a muscular organ that is about the size of a closed fist and is in charge of pumping blood throughout the body. Deoxygenated blood enters the body

through the veins, where it is oxygenated in the lungs before being pushed into the many arteries, which transport the blood throughout the body and supply nutrients and oxygen to the body's tissues. In the thoracic cavity, the heart is positioned medial to the lungs and posterior to the sternum. The base of the heart's better end is where the aorta, vena cava, and pulmonary arteries are all joined. The apex, or bottom point of the heart, is right above the diaphragm.. The midline of the body is where the peak of the heart, which faces the left, is situated. The left side of the body houses the heart, therefore approximately two thirds of the mass of the heart starts on the left side, with the remaining third on the right. Figure 4.1 in [1] illustrates the many parts of the human heart.. The maximum cardinality of a convex set S shows the region in which the blood circulation is more in the valves connecting the heart.

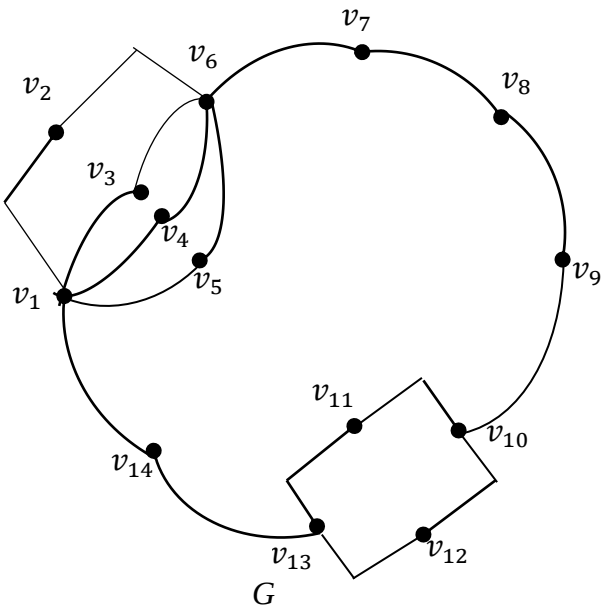

Figure 4.1
Micro Cardiac Network Graph

For the graph G given in Figure 4.1, $S = \{v_1, v_2, v_3, v_4, v_5, v_{11}, v_{12}, v_{13}, v_{14}\}$ is a maximum convex set.

Conclusion

In this article, we studied the convexity number of line graph of a graph and jump graph of a graph. Finally, we give an application of convex set in micro cardiac Network Graph.

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