A New Fuzzy Modelling Approach F-NoVaRM for Performance Evaluation of Mathematics Post-Graduate Students

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Abstract

For many years, Multi Criteria Decision Making (MCDM) has been a significant area of study, producing a large number of theoretical and applied articles and books. Numerous MCDM strategies work with unique alternatives that are determined by a set of parameters. Cardinal or ordinal data can be used to specify the values of the parameters. MCDM methods have been created to define the desired alternative, categorize alternatives into a small number of groups, or rank alternatives in a particular order of preference. It supports sound decision-making in areas where selecting the best option is very challenging.Mathematics was viewed as being an integral component of any curriculum from the beginning.There are also some renowned institutions conducting top-notch mathematical study in India.Although opinions varied, one of the most challenging classes is the MSc Mathematics programme. Evaluation of the impact of students' peers, teachers, and instructional styles on their mathematical achievement is a difficult task. These elements are regarded as being crucial to studying mathematics in order to guarantee success.This paper explores a new decision-making approach known as Fuzzy Normalized Value Based Ranking Method(F-NoVaRM) to evaluate the performance of post graduate students in Mathematics based on certain criteria.

1. Introduction

A performance evaluation system is a methodical means to examine how well someone is doing their job.This enables both businesses and educators to assess their students and staff. By recognizing their faults, this helps people inspire themselves to give their all for themselves. The performance evaluation process includes a standard assessment form, performance metrics, guidelines for giving feedback, and evaluation disciplines. Following the assessment process, encouraging good behavior and providing assistance to strengthen weaknesses are both necessary.A key component of raising students' academic success is performance evaluation of those students. The accomplishments of a student body at an educational institution largely decide its quality. The evaluation aids parents or instructors in identifying ways to raise performance in relation to the variables influencing performance.

Academic performance refers to how student deals with their studies and how they cope with or accomplish different tasks givento them by their teacher or instructors (extracted from wikki.answer.com).

The foundation of all scientific knowledge and technological advancement that is essential to the socioeconomic development of any particular nation or developing state has always been mathematics.Despite the crucial part that mathematics has played in our society, this field of study has not performed well across the country.Because Math is already regarded as a difficult topic, earning a master's degree in mathematics is always regarded as difficult.

Based on research literature, we find that issues in postgraduate research are not limited to developing countries but are also experienced in the developed world (Kearney, 2008), although developed countries are a little ahead in addressing these issues (Minnesota Measures, 2007)

Here in this paper performance evaluation of PG Mathematics students based on certain personal level criteria has been evaluated and a model based on F-NoVaRM has also been developed for this purpose.

2. PRELIMINARIES

We present here brief preliminaries on the theory of fuzzy sets which are used in our next section.

Definition 2.1.(Klir and Yuan, 2001)

Let X be the universal set. A fuzzy set in X is a set of ordered pairs,
$A = \{(x, \mu_A(x)); x \in X\}$, where $\mu_A: X \to [0,1]$ is called the membership function of A in X and [0,1] is called the membership set.

Definition 2.2. (Ban and Lucian, 2014)

A fuzzy number A is a fuzzy subset of the real line, $A: R \to [0, 1]$ satisfying the following properties:

(i) A is normal (i.e there exists $x_0 \in R$ such that $A(x_0) = 1$)

(ii) A is fuzzy convex.

(iii) A is upper semicontinuous on R.

(iv) The closure of the support, cl $\{x \in X: \mu_A(x) > 0\}$ is compact.

A graph of a fuzzy number is given below:

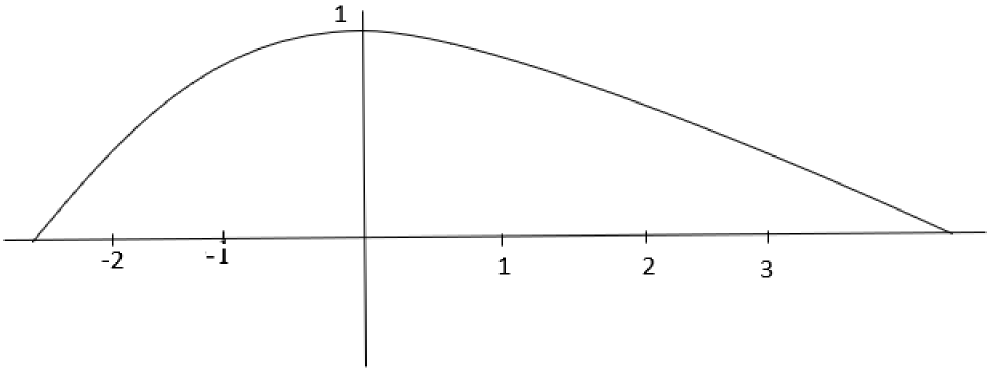

Figure 1. Graph of a fuzzy number

Definition 2.3.[Gani and Mohamed,2012]

A triangular fuzzy number (TFN) is defined as (a_1, a_2, a_3) where the membership function is given by

$$\mu_a(x) = \begin{cases} 0 & if \quad x \leq a_1 \\ \frac{x-a_1}{a_2-a_1} = \mu_1(x) & if \quad a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} = \mu_2(x) & if \quad a_2 \leq x \leq a_3 \\ 0 & if \quad x \geq a_3 \end{cases} \tag{1}$$

Definition 2.4. Fuzzy Conversion Scale: Conversion scales are usedto convert linguistic concepts into fuzzy numbers. Normally, the criteria and options are rated on a scale of 1 to 9. Here we make use of triangular fuzzy numbers to represent the five linguistic ratings that have a consistent representation from 1 to 9.

Table 1. Ratings of alternatives

Linguistic Variables	**TFN**
Very Poor	(1,1,3)
Poor	(1,3,5)
Fair	(3,5,7)
Good	(5,7,9)
Very Good	(7,9,9)

3. The pairwise comparison method [ODU G.O, 2019]

This method is used for analysing multiple populations in pairs to determine whether they are significantly different from one another. It can also be used where the decision-maker compares each criterion with others and determines the level of preference for each pair of such criteria. An ordinal scale (1 - 9) is adopted to help in determining the preference value of one criterion over the other. The number of comparisons can be determined by$C_p = \frac{n(n-1)}{2}$. Where C_p = the number of comparisons, and n = the number of criteria. This method has two main steps. The first step is to develop a matrix by comparing the criteria. Intensity values are used to fill the matrix, such as (1, 3, 5, 7, 9) representing equal importance, moderate importance of one over the other, strong importance, very strong importance, and extreme importance respectively. The diagonal in the matrix is always 1 and the lower left values are inverse values if activity i has one of the above numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i. To fill the lower triangular matrix, we use the reciprocal values of the upper diagonal. That completes the comparison matrix. The second step is to calculate the criteria weight, which is also known as the priority value or the principal eigenvector. In this step, we sum the values in each column, dividing each element by the column total, and dividing the sum of the normalized scores for each row by the number of criteria.

4. Fuzzy Normalized Value based Ranking Method[F-NoVaRM]

In this paper, we explore a new model to assess the student's academic performance based on certain criteria. The proposed model is as follows:

Step 1: Decide the ratings of alternativesas shown in table 1

Among the various criteria in decision-making, some might be a benefit criterion (a criterionthat provides a high potential in the study) and some others might be a cost criterion (a criterion providing a low potential in the study). In the present study, as per the suggestion obtained from the experts, we assumed $C_{1,}C_3,C_4$and C_6as the benefit criteria and $C_{2\,,}C_5$ as the cost criteria.

Step 2: Construct an initial fuzzy decision matrix as follows:

$$\widetilde{D}= \begin{matrix} & C_1\ C_2 \ldots C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} x_{11} & x_{12} \ldots & x_{1n} \\ x_{21} & x_{22} \ldots & x_{2n} \\ x_{31} & x_{32} \ldots & x_{3n} \end{pmatrix} \end{matrix} (2)$$

Step 3: Compute the criteria weights by Pairwise Comparison Method.

Step 4: Normalize the initial fuzzy decision matrix as follows: - [Neelima B.Kore, 2017]

For Benefit criteria, $C_j^* = {}^{max}{}_i c_{ij}$ (3)

Now, $\overline{r_{ij}} = \left({a_{ij}}/{C_j^*}, {b_{ij}}/{C_j^*}, {c_{ij}}/{C_j^*} \right)$(4)

For Cost criteria, $\overline{a_j} = {}^{min}{}_i a_{ij}$ (5)

Now, $\bar{r_{ij}} = \left(\bar{a_j}/c_{ij}\,, \bar{a_j}/b_{ij}\,, \bar{a_j}/a_{ij} \right)$(6)

Step 5:Determine the degree of satisfaction by using the following equation:

$d_{ij} = Val\,(a_{ij}, b_{ij}\;, c_{ij})$ (7)

Step 6: Calculate the score of each alternative by multiplying the degree by the corresponding weights of the criteria.

Step 7: Compute the total score for each alternative.

Step 8: Rank the alternatives in descending order.

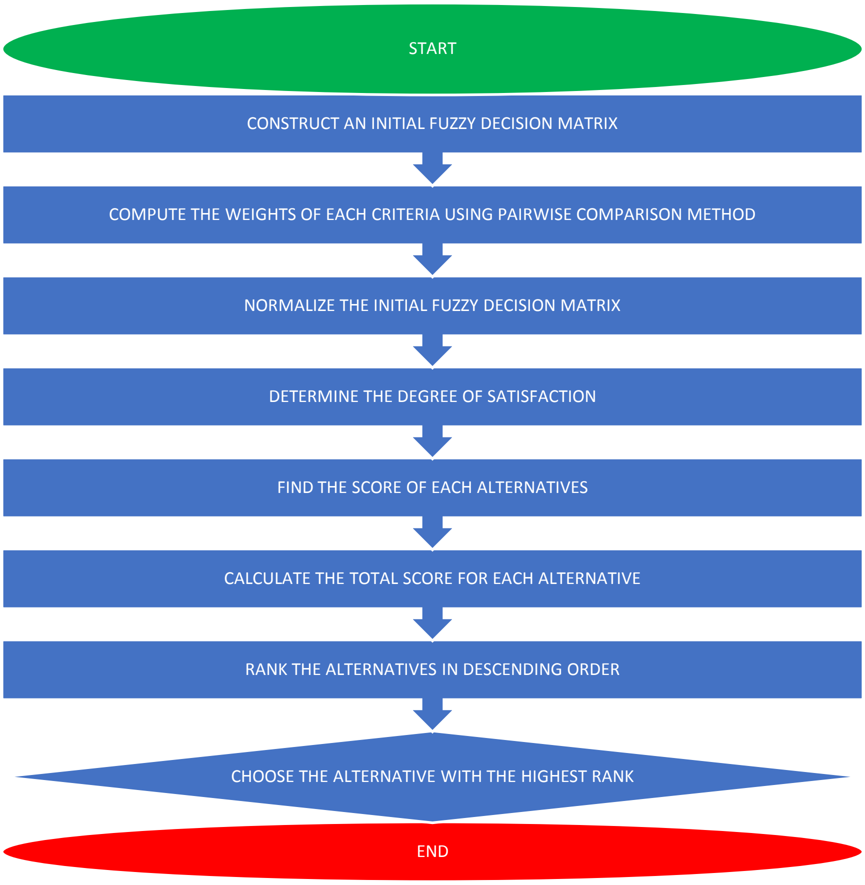

Figure 2. Framework of methodology

5. Performance Evaluation Model Based on F-NoVaRM- A Case Study

In this paper, we propose a new decision-making model for assessing students' performance based on certain factors. These factors are the input variable and performance level is the output variable.

The personal level factors such as Personal Caliber, Financial support, Motivation, Communication skill, Parental Background and Peer Influence are considered. Let it be the criteria C_1, C_2, C_3,C_4, C_5 andC_6respectively.

For a case study, 20 students of second year Post- Graduate (Mathematics)of Malankara Catholic College, Tamil Nadu were selected $(A_1, A_2, \ldots, A_{20})$. We need to rank these students based on the above-mentioned criteria. The first step is done by collectingjudgements from a decision maker (teacher) for these 20 alternatives (students) with respect to the above-mentioned criteria. Scaling of linguistic variables is done by Triangular fuzzy conversion scale specified in table 1. The weights of various criteria are computed by using Pair-wise comparison method.Surveymethod was adopted for data collection. A questionnaire comprising of 15 questions were prepared to compile expert's opinionand constructed pairwise comparison matrix. The questionnaire can be found here

https://docs.google.com/spreadsheets/d/1NzdJ8vwcC-7RZ4WAQ-BOVNkRzSmzMrYrvyhFGsNNfzw/edit?usp=share_link

Data collected from the expert were fuzzified based on the scalingmentioned in table 1 as follows:

Table 2. Initial Fuzzy Decision Matrix

Alternatives\Criteria	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(1,1,3)	(7,9,9)	(3,5,7)	(3,5,7)	(7,9,9)	(5,7,9)
A_2	(7,9,9)	(3,5,7)	(7,9,9)	(3,5,7)	(1,3,5)	(3,5,7)
A_3	(5,7,9)	(7,9,9)	(5,7,9)	(7,9,9)	(7,9,9)	(5,7,9)
A_4	(5,7,9)	(5,7,9)	(3,5,7)	(3,5,7)	(5,7,9)	(5,7,9)
A_5	(7,9,9)	(5,7,9)	(7,9,9)	(7,9,9)	(5,7,9)	(3,5,7)
A_6	(1,1,3)	(7,9,9)	(5,7,9)	(3,5,7)	(7,9,9)	(5,7,9)
A_7	(7.9,9)	(5,7,9)	(5,7,9)	(7,9,9)	(5,7,9)	(5,7,9)
A_8	(5,7,9)	(7,9,9)	(3,5,7)	(7,9,9)	(7,9,9)	(5,7,9)
A_9	(7,9,9)	(7,9,9)	(7,9,9)	(3,5,7)	(7,9,9)	(3,5,7)
A_{10}	(5,7,9)	(7,9,9)	(3,5,7)	(7,9,9)	(7,9,9)	(5,7,9)
A_{11}	(3,5,7)	(5,7,9)	(5,7,9)	(3,5,7)	(3,5,7)	(5,7,9)
A_{12}	(5,7,9)	(7,9,9)	(5,7,9)	(3,5,7)	(7,9,9)	(5,7,9)
A_{13}	(1,1,3)	(5,7,9)	(3,5,7)	(7,9,9)	(7,9,9)	(5,7,9)
A_{14}	(7,9,9)	(3,5,7)	(7,9,9)	(3,5,7)	(5,7,9)	(5,7,9)
A_{15}	(5,7,9)	(7,9,9)	(5,7,9)	(3,5,7)	(1,3,5)	(5,7,9)
A_{16}	(3,5,7)	(5,7,9)	(3,5,7)	(3,5,7)	(5,7,9)	(5,7,9)

A_{17}	(1,1,3)	(5,7,9)	(5,7,9)	(3,5,7)	(5,7,9)	(5,7,9)
A_{18}	(7,9,9)	(7,9,9)	(7,9,9)	(3,5,7)	(7,9,9)	(3,5,7)
A_{19}	(7,9,9)	(5,7,9)	(5,7,9)	(3,5,7)	(5,7,9)	(5,7,9)
A_{20}	(5,7,9)	(3,5,7)	(5,7,9)	(3,5,7)	(5,7,9)	(5,7,9)

By pairwise comparison method,the weights of various criteria obtained are: $W(C_1) = 0.43, W(C_2) = 0.21, W(C_3) = 0.15, W(C_4) = 0.12, W(C_5) = 0.06,\ W(C_6) = 0.03$.

Using equations (3), (4), (5) and (6), we got the normalized fuzzy decision matrix as follows:

Table 3. Normalized Fuzzy Decision Matrix

Alternatives/ Criteria	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.1,0.1,0.3)	(0.3,0.3,0.4)	(0.3,0.6,0.8)	((0.3,0.6,0.8)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_2	(0.78,1,1)	(0.4,0.6,1)	(0.8,1,1)	(0.3,0.6,0.8)	(0.2,0.3,1)	(0.3,0.6,0.8)
A_3	(0.6,0.8,1)	(0.3,0.3,0.4)	(0.6,0.8,1)	(0.8,1,1)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_4	(0.6,0.8,1)	(0.3,0.4,0.6)	(0.3,0.6,0.8)	(0.3,0.6,0.8)	(0.1,0.1,0.2)	(0.6,0.8,1)
A_5	(0.78,1,1)	(0.33,0.43,0.6)	(0.8,1,1)	(0.8,1,1)	(0.1,0.1,0.2)	(0.3,0.6,0.8)
A_6	(0.1,0.1,0.3)	(0.3,0.3,0.4)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_7	(0.8,1,1)	(0.3,0.4,0.6)	(0.6,0.8,1)	((0.8,1,1)	(0.1,0.1,0.2)	(0.6,0.8,1)
A_8	(0.56,0.78,1)	(0.3,0.3,0.4)	(0.3,0.6,0.8)	(0.8,1,1)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_9	(0.8,1,1)	(0.3,0.3,0.4)	(0.8,1,1)	(0.3,0.6,0.8)	(0.1,0.1,0.1)	(0.3,0.6,0.8)
A_{10}	(0.6,0.8,1)	(0.3,0.3,0.4)	(0.33,0.56,0.78)	(0.8,1,1)	(0.1,0.1,0.1)	(0.6,0.8,1)

A_{11}	(0.3,0.6,0.8)	(0.3,0.4,0.6)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.2,0.1)	(0.6,0.8,1)
A_{12}	(0.6,0.8,1)	(0.3,0.3,0.4)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_{13}	(0.1,0.1,0.3)	(0.3,0.4,0.6)	(0.3,0.6,0.8)	((0.8,1,1)	(0.1,0.1,0.1)	(0.6,0.8,1)
A_{14}	(0.8,1,1)	(0.4,0.6,1)	(0.8,1,1)	(0.3,0.6,0.8)	(0.11,0.143, 0.2)	(0.6,0.8,1)
A_{15}	(0.6,0.8,1)	(0.3,0.3,0.4)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.2,0.33,1)	(0.6,0.8,1)
A_{16}	(0.3,0.6,0.8)	(0.3,0.4,0.6)	(0.3,0.6,0.8)	(0.3,0.6,0.8)	(0.1,0.1,0.2)	(0.6,0.8,1)
A_{17}	(0.1,0.1,0.3)	(0.3,0.4,0.6)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.1,0.2)	(0.6,0.8,1)
A_{18}	(0.8,1,1)	(0.3,0.3,0.4)	(0.8,1,1)	(0.3,0.6,0.8)	(0.1,0.1,0.1)	(0.3,0.6,0.8)
A_{19}	(0.8,1,1)	(0.3,0.4,0.6)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.1,0.2)	(0.6,0.8,1)
A_{20}	(0.6,0.8,1)	(0.4,0.6,1)	(0.6,0.8,1)	(0.3,0.6,0.8)	(0.1,0.1,0.2)	(0.6,0.8,1)

Table 5. Degree of Satisfaction

d_{ij}	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.13	0.32	0.58	0.58	0.1	0.8
A_2	0.96	0.63	0.97	0.58	0.4	0.58
A_3	0.8	0.32	0.8	0.97	0.1	0.8
A_4	0.8	0.42	0.58	0.58	0.12	0.8
A_5	0.96	0.44	0.97	0.97	0.12	0.58
A_6	0.13	0.32	0.8	0.58	0.1	0.8
A_7	0.97	0.42	0.8	0.97	0.12	0.8
A_8	0.78	0.32	0.58	0.97	0.1	0.8
A_9	0.97	0.32	0.97	0.58	0.1	0.58

A_{10}	0.8	0.32	0.56	0.97	0.1	0.8
A_{11}	0.58	0.42	0.8	0.58	0.17	0.8
A_{12}	0.8	0.32	0.8	0.58	0.1	0.8
A_{13}	0.13	0.42	0.58	0.97	0.1	0.8
A_{14}	0.97	0.63	0.97	0.58	0.15	0.8
A_{15}	0.8	0.32	0.8	0.58	0.42	0.8
A_{16}	0.58	0.42	0.58	0.58	0.12	0.8
A_{17}	0.13	0.42	0.8	0.58	0.12	0.8
A_{18}	0.97	0.32	0.97	0.58	0.1	0.58
A_{19}	0.97	0.42	0.8	0.58	0.12	0.8
A_{20}	0.8	0.63	0.8	0.58	0.12	0.8

Table 6. Score Matrix

Score	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.06	0.07	0.09	0.07	0.006	0.02
A_2	0.42	0.13	0.15	0.07	0.02	0.02
A_3	0.34	0.07	0.12	0.12	0.006	0.02
A_4	0.34	0.09	0.09	0.07	0.01	0.02
A_5	0.42	0.09	0.15	0.12	0.01	0.02
A_6	0.06	0.07	0.12	0.07	0.01	0.02
A_7	0.42	0.09	0.12	0.12	0.01	0.02
A_8	0.33	0.07	0.09	0.12	0.01	0.02
A_9	0.42	0.07	0.15	0.07	0.01	0.02
A_{10}	0.34	0.07	0.08	0.12	0.01	0.02
A_{11}	0.25	0.09	0.12	0.07	0.01	0.02
A_{12}	0.34	0.07	0.12	0.07	0.01	0.02
A_{13}	0.06	0.09	0.09	0.12	0.01	0.02

A_{14}	0.42	0.13	0.15	0.07	0.01	0.02
A_{15}	0.34	0.07	0.12	0.07	0.03	0.02
A_{16}	0.25	0.09	0.09	0.07	0.01	0.02
A_{17}	0.06	0.09	0.12	0.07	0.01	0.02
A_{18}	0.42	0.07	0.15	0.07	0.01	0.02
A_{19}	0.42	0.09	0.12	0.07	0.01	0.02
A_{20}	0.34	0.13	0.12	0.07	0.01	0.02

On computing the total score, we obtained the following values:

Score (A_1) = 0.316, Score (A_2) = 0.804, Score (A_3) = 0.676, Score (A_4) = 0.620, Score (A_5) = 0.807, Score (A_6) = 0.350,Score (A_7) = 0.780, Score (A_8) = 0.645, Score (A_9) = 0.740, Score (A_{10}) = 0.643, Score (A_{11}) = 0.560, Score (A_{12}) = 0.630, Score (A_{13}) = 0.390, Score (A_{14}) = 0.799, Score (A_{15}) = 0.645, Score (A_{16}) = 0.530, Score (A_{17}) = 0.370, Score (A_{18}) = 0.740, Score (A_{19}) = 0.730, Score (A_{20}) = 0.690,

On ranking the alternatives based on the above total scores, we got

$$A_5 > A_2 > A_{14} > A_7 > A_9 \sim A_{18} > A_{19} > A_{20} > A_3 > A_8 \sim A_{15} > A_{10} > A_{12} > A_4 > A_{11} > A_{16} > A_{13} > A_{17} > A_6 > A_1$$

Based on the ranking,we can conclude that A_5 is the best student among all the 20 students selected. We can also provide remedial measures to those students who have optimum performance. A C++ program for the algorithm can be found here

https://www.onlinegdb.com/qxxrD8AmF

6. Conclusion

The newly developed Muti-Criteria Decision- Making Model can be applied to evaluate the performance of any number of students. This model also helps us to take remedial measures to improve their performance. This technique will help teachers to evaluate their students in the mid- semester itself. Here the case study is done by using a single decision maker. In future, researchers may use this technique for multiple decision makers. Further, instead of triangular fuzzy scaling one may go for higher dimensional scaling. Moreover, by using a software like MATLAB we may also create a model for the proposed approach. One advantage of this method is that it requires less computational effort as we compared with the existing MCDM techniques such as TOPSIS, AHP, F-MARCOS and so on.

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