



OnKSGp[KSgsp]-RegularandNormalSpacesinKasaj TopologicalSpaces

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ABSTRACT

The purpose of this article is to investigate the spaces namely $KSgp[KSgsp]$ -regular spaces, $KSgp[KSgsp]$ -normal spaces by utilizing $KSgp[KSgsp]$ -open sets in Kasaj Topological Spaces. Also we discuss their relationship with existing concepts in kasaj topological spaces.

Keywords: $KSg[KSgp, KSgsp]$ -regular spaces and $KSg[KSgp, KSgsp]$ -normal spaces.

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1. INTRODUCTION AND PRELIMINARIES

In 1970, Levine [3] introduced the concept of generalized closed sets in topological spaces. This was introduced as generalization of closed sets in topological space and interesting results were proved. Lellis Thivagaret al. [2] introduced nano topological spaces with respect to a subset X of a universal set U which is defined in terms of lower and upper approximation of X . In 2019, Chandrasekar [1] introduced a new topology namely, micro topology which is extension of nano topological space. A partial extension of micro topological space namely kasaj topological spaces was introduced by Kashyap G. Rachchhand Sajeed [4]. Further, the authors introduced the concept of kasaj generalized closed sets in kasaj topological spaces. Sathishmohan et al. [5] introduced and investigate the new type of closed sets known as $KSgp(KSgsp)$ -closed sets in Kasaj topological spaces. In this paper we shall define KS -regular spaces, KS -normal spaces, KS pre-regular spaces, KS pre-normal spaces, $KS\beta$ -regular spaces, $KS\beta$ -normal spaces, $KSgp[KSgsp]$ -regular spaces, $KSgp[KSgsp]$ -normal spaces in kasaj topological spaces and obtain some of their basic results.

Definition 1.1. A subset P of U in $(U, \tau_R(X), KSR(X))$ is called

- (1) KS -closed set [4] if $KS_{cl}(P) = P$. The complement of KS -closed set is KS -open set in U .
- (2) KS pre-closed set [4] if $KS_{cl}(KS_{int}(P)) \subseteq P$. The complement of KS -pre-closed set is KS -pre-open set in U .
- (3) $KS\alpha$ -closed set [4] if $KS_{cl}(KS_{int}(KS_{cl}(P))) \subseteq P$. The complement of $KS\alpha$ -closed set is $KS\alpha$ -open set in U .
- (4) $KS\beta$ -closed set [4] if $KS_{int}(KS_{cl}(KS_{int}(P))) \subseteq P$. The complement of $KS\beta$ -closed set is $KS\beta$ -open set in U .

Definition 1.2.[4] A subset P of U in $(U, \tau_R(X), KSR(X))$ is called a Kasaj-generalized-closed set (briefly KS_g -closed) if $KS_{cl}(P) \subseteq V$ whenever $P \subseteq V$ and V is KS -open set in U . The complement of KS_g -closed set is KS_g -open set in U .

Definition 1.3.[5] A subset P of U in $(U, \tau_R(X), KSR(X))$ is called a Kasaj-generalized-pre-closed set (briefly KS_{gp} -closed) if $KS_{pcl}(P) \subseteq V$ whenever $P \subseteq V$ and V is KS -open set in U . The complement of KS_{gp} -closed set is KS_{gp} -open set in U .

Definition 1.4.[5] A subset P of U in $(U, \tau_R(X), KSR(X))$ is called a Kasaj-generalized-semi-pre-closed set (briefly KS_{gsp} -closed) if $KS_{spcl}(P) \subseteq V$ whenever $P \subseteq V$ and V is KS -open set in U . The complement of KS_{gsp} -closed set is KS_{gsp} -open set in U .

2. Properties of $KS_{gp}[KS_{gsp}]$ -Regular Spaces

In this section, we introduce the new class of spaces namely $KS_{gp}[KS_{gsp}]$ -Regular Spaces in Kasaj Topological Spaces. By using KS_{gp} -open sets and studied some of their properties.

Definition 2.1. A Kasaj Topological Spaces U is called

- (i) KS - T_3 (briefly KS -regular) space iff for each KS -closed set F and a point $x \notin F$, there are disjoint KS -open sets G and H such that $x \in G$ and $F \subseteq H$.
- (ii) KS_{pre} - T_3 (briefly KS_{pre} -regular) space iff for each KS_{pre} -closed set F and a point $x \notin F$, there are disjoint KS_{pre} -open sets G and H such that $x \in G$ and $F \subseteq H$.
- (iii) KS_{β} - T_3 (briefly KS_{β} -regular) space iff for each KS_{β} -closed set F and a point $x \notin F$, there are disjoint KS_{β} -open sets G and H such that $x \in G$ and $F \subseteq H$.
- (iv) KS_{gp} - T_3 (briefly KS_{gp} -regular) space iff for each KS_{gp} -closed set F and a point $x \notin F$, there are disjoint KS_{gp} -open sets G and H such that $x \in G$ and $F \subseteq H$.
- (v) KS_{gsp} - T_3 (briefly KS_{gsp} -regular) space iff for each KS_{gsp} -closed set F and a point $x \notin F$, there are disjoint KS_{gsp} -open sets G and H such that $x \in G$ and $F \subseteq H$.

Theorem 2.2. In a Kasaj Topological Spaces $(U, \tau_R(X), KSR(X))$. Then

- (i) Every KS -regular space is KS_{gp} -regular.
- (ii) Every KS_{pre} -regular space is KS_{gp} -regular.
- (iii) Every KS_{α} -regular space is KS_{gp} -regular.
- (iv) Every KS_g -regular space is KS_{gp} -regular.

Proof:

i) Let U be a KS -regular and F be a KS -closed set not containing x implies F be a KS_{gp} -closed set not containing x . As U is KS_{gp} -regular, there exists KS_{gp} -open set G and H such that $x \in G$ and $F \subseteq H$. Therefore U is KS_{gp} -regular.

Proof of (ii)-(iv) are similar to (i).

Converses of the above results need not be true as shown in the following example.

Example 2.3. Let $U = \{a, b, c, d, e\}$, with $U \setminus R = \{\{a, d\}, \{b, c\}, \{e\}\}$ and $X = \{a, e\}$. Then the nanotology, $\tau R(X) = \{U, \emptyset, \{e\}, \{a, d, e\}, \{a, d\}\}$. Let $S = \{d\}$, $S' = \{a, b, c, e\}$. Then $x = \{d\}$, $F = \{a, d, e\}$ and $A = \{a, e\}$, $B = \{d\}$ it is KSgp-regular spaces but not in KS-regular spaces, KSpre-regular spaces, KS α -regular spaces, KSg-regular spaces.

Theorem 2.4. Every KSgp-regular space is KSgp-T2 space.

Proof: Let U be a KSgp-regular space and $x, y \in U$ with $x \neq y$. Since U is KSgp-regular, the subset $\{y\}$ is KSgp-closed. Since $x \notin \{y\}$ and U is KSgp-regular space, there exists disjoint KSgp-open sets G and H such that $x \in G, y \in H$ and $G \cap H = \emptyset$. Hence U is KSgp-T2 space.

Theorem 2.5. If U is a KSgp-regular space and V is a subspace of U , then V is also KSgp-regular space.

Proof: Let $(U, \tau R(X), KSR(X))$ be a KSgp-regular space and $(V, \tau R(Y), KSR(Y))$ be

a subspace of U . To prove that V is KSgp-regular, let $x \in V$ and F be a KSgp-closed set in V such that $x \notin F$. So $F = V \cap KS_{gpcl}(F)$.

Since $x \notin F$, we see that $x \notin KS_{gpcl}(F)$. Since U is KSgp-regular space, there exists disjoint KSgp-open sets G and H in U such that $KS_{gpcl}(F) \subseteq G, x \in H$. Now $F \subseteq KS_{gpcl}(F) \subseteq G$. Since $F \subseteq V, F \subseteq V \cap G$. Since $x \in V$ and $x \in H, x \in V \cap H$. Further $(V \cap G) \cap (V \cap H) = \emptyset$. Since $G \cap H = \emptyset$. Thus $V \cap G$ and $V \cap H$ are KSgp-open sets in $V, x \in V \cap H, F \subseteq V \cap G$ and $(V \cap G) \cap (V \cap H) = \emptyset$. Hence V is KSgp-regular space.

Theorem 2.6. A Kasaj Topological Spaces U is KSgp-regular space iff for any $x \in U$ and a KSgp-neighbourhood N of x , there is a KSgp-open set F such that $x \in F \subseteq KS_{gpcl}(F) \subseteq N$.

Proof: Assume that U is KSgp-regular space and N is a KSgp-neighbourhood of x . Then N^c is a KSgp-closed set and $x \notin N^c$. Since U is KSgp-regular, there exists disjoint KSgp-open sets F and G such that $x \in F$ and $N^c \subseteq G$. So $G^c \subseteq N$. Since $F \cap G = \emptyset, F \subseteq G^c$ this implies that $KS_{gpcl}(F) \subseteq G^c$. Since G^c is a KSgp-closed set. Thus $x \in F \subseteq KS_{gpcl}(F) \subseteq N$.

Conversely, assume that the given condition is satisfied. Let H be a KSgp-closed set in U and $x \notin H$. Since H^c is KSgp-neighbourhood of x , by assumption, there is a KSgp-open set L such that $x \in L \subseteq KS_{gpcl}(L) \subseteq H^c$. Thus the disjoint KSgp-open set L and $[KS_{gpcl}(L)]^c$ contain x and H respectively. Hence U is KSgp-regular space.

Theorem 2.7. In a Kasaj Topological Spaces $(U, \tau R(X), KSR(X))$. Then every KS-regular space (KSpre-regular space, KS α -regular space, KS β -regular space, KSg-regular space, KSgp-regular space) is KSgp-regular.

Proof:

It is obvious from Theorem 2.2.

Converses of the above results need not be true as shown in the following example.

Example 2.8. Let $U = \{a, b, c, d, e\}$, with $U \setminus R = \{\{a, b\}, \{c, d\}, \{e\}\}$ and $X = \{a, e\}$. Then the nanotology, $\tau R(X) = \{U, \emptyset, \{e\}, \{a, b, e\}, \{a, b\}\}$. Let $S = \{a\}$, $S' = \{b, c, d, e\}$. Then $x = \{e\}$, $F = \{d, e\}$ and $A = \{d\}$, $B = \{e\}$ it is KSgp-regular but not in KS-regular spaces, KSpre-regular spaces, KS α -regular spaces, KS β -regular spaces, KSg-regular spaces, KSgp-regular spaces.

Theorem 2.9. Every KS_{gsp} -regular space is KS_{gsp} - T_2 space.

Proof:

Let U be a KS_{gsp} -regular space and $x, y \in U$ with $x \neq y$. Since U is KS_{gsp} -regular, the subset $\{y\}$ is KS_{gsp} -closed. Since $x \neq \{y\}$ and U is KS_{gsp} -regular space, there exists disjoint KS_{gsp} -open sets G and H such that $x \in G, y \in H$ and $G \cap H = \emptyset$. Hence U is KS_{gsp} - T_2 space.

Theorem 2.10. If U is a KS_{gsp} -regular space and V is a subspace of U , then V is also KS_{gsp} -regular space.

Proof: Let $(U, \tau_R(X), KSR(X))$ be a KS_{gsp} -regular space and $(V, \tau_R(Y), KSR(Y))$ be a subspace of U . To prove that V is KS_{gsp} -regular, let $x \in V$ and F be a KS_{gsp} -closed set in V such that $x \notin F$. So $F = V \cap KS_{gspcl}(F)$. Since $x \notin F$, we see that $x \notin KS_{gspcl}(F)$. Since U is KS_{gsp} - T_3 space, there exists disjoint KS_{gsp} -open sets G and H in U such that $KS_{gspcl}(F) \subseteq G, x \in H$. Now $F \subseteq KS_{gspcl}(F) \subseteq G$. Since $F \subseteq V, F \subseteq V \cap G$. Since $x \in V$ and $x \in H, x \in V \cap H$. Further $(V \cap G) \cap (V \cap H) = \emptyset$. Since $G \cap H = \emptyset$. Thus $V \cap G$ and $V \cap H$ are KS_{gsp} -open sets in $V, x \in V \cap H, F \subseteq V \cap G$ and $(V \cap G) \cap (V \cap H) = \emptyset$. Hence V is KS_{gsp} -regular space.

Theorem 2.11. A Kasaj Topological Spaces U is KS_{gsp} -regular space iff for any $x \in U$ and a KS_{gsp} -neighbourhood N of x , there is an KS_{gsp} -open set F such that $x \in F \subseteq KS_{gspcl}(F) \subseteq N$.

Proof: Assume that U is KS_{gsp} -regular space and N is a KS_{gsp} -neighbourhood of x . Then N^c is a KS_{gsp} -closed set and $x \notin N^c$. Since U is KS_{gsp} - T_3 , there exists disjoint KS_{gsp} -open sets F and G such that $x \in F$ and $N^c \subseteq G$. So $G^c \subseteq N$. Since $F \cap G = \emptyset, F \subseteq G^c$ this implies that $KS_{gspcl}(F) \subseteq G^c$. Since G^c is a KS_{gsp} -closed set. Thus $x \in F \subseteq KS_{gspcl}(F) \subseteq N$. Conversely, assume that the given condition is satisfied. Let H be a KS_{gsp} -closed set in U and $x \notin H$. Since H^c is KS_{gsp} -neighbourhood of x , by assumption, there is an KS_{gsp} -open set L such that $x \in L \subseteq KS_{gspcl}(L) \subseteq H^c$. Thus the disjoint KS_{gsp} -open set L and $[KS_{gspcl}(L)]$ contain x and H respectively. Hence U is KS_{gsp} -regular space.

3. Properties of $KS_{gsp}[KS_{gsp}]$ -Normal Spaces

In this section, we defined and studied the concept of $KS_{gsp}[KS_{gsp}]$ -normal spaces in Kasaj Topological Spaces and investigated its properties with some of the existing results.

Definition 3.1. A Kasaj topological spaces U is called

- (i) KS - T_4 space (briefly KS -normal) if for each pair A and B of disjoint KS -closed sets in U , there are disjoint KS -open sets G and H such that $A \subseteq G$ and $B \subseteq H$.
- (ii) KS pre- T_4 (briefly KS pre-normal) space if for each pair A and B of disjoint KS pre-closed sets in U , there are disjoint KS pre-open sets G and H such that $A \subseteq G$ and $B \subseteq H$.
- (iii) $KS\beta$ - T_4 (briefly $KS\beta$ -normal) space iff for each pair A and B of disjoint $KS\beta$ -closed sets in U , there are disjoint $KS\beta$ -open sets G and H such that $A \subseteq G$ and $B \subseteq H$.
- (iv) KS_{gsp} - T_4 (briefly KS_{gsp} -normal) space iff for each pair A and B of disjoint KS_{gsp} -closed sets in U , there are disjoint KS_{gsp} -open sets G and H such that $A \subseteq G$ and $B \subseteq H$.
- (v) KS_{gsp} - T_4 (briefly KS_{gsp} -normal) space iff for each pair A and B of disjoint KS_{gsp} -closed sets in U , there are disjoint KS_{gsp} -open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

Theorem 3.2. In a Kasaj Topological Spaces $(U, \tau_R(X), KSR(X))$. Then

- (i) Every KS -normal space is KS_{gp} -normal.
- (ii) Every KS pre-normal space is KS_{gp} -normal.
- (iii) Every KS_{α} -normal space is KS_{gp} -normal.
- (iv) Every KS_g -normal space is KS_{gp} -normal.

Proof: (i) Let U be a KS -normal space and F and G be arbitrary pair of disjoint KS -closed sets. Since every KS -closed set is (KS pre-closed, KS_{α} -closed, KS_g -closed) KS_{gp} -closed set, F and G are (KS pre-closed, KS_{α} -closed, KS_g -closed) KS_{gp} -closed sets and U is KS_{gp} -normal, therefore there exists disjoint (KS pre-open, KS_{α} -open, KS_g -open) KS_{gp} -open sets L and M such that $F \subseteq L$ and $G \subseteq M$. Thus for every pair of disjoint KS -closed (KS pre-closed, KS_{α} -closed, KS_g -closed) KS_{gp} -closed sets F and G there exists disjoint (KS pre-open, KS_{α} -open, KS_g -open) KS_{gp} -open sets L and M such that $F \subseteq L$ and $G \subseteq M$. Hence U is (KS pre-normal, KS_{α} -normal, KS_g -normal) KS_{gp} -normal space. Proof of (ii)-(iv) are similar to (i).

Converses of the above results need not be true as shown in the following example.

Example 3.3. Let $U = \{a, b, c, d, e\}$, with $U \setminus R = \{\{a, d\}, \{b, c\}, \{e\}\}$ and $X = \{b, e\}$. Then the $\tau R(X) = \{U, \emptyset, \{e\}, \{b, c, e\}, \{b, c\}\}$. Let $S = \{b\}$, $S' = \{a, c, d, e\}$. Then $F = \{b\}$, $G = \{a, d\}$ and $A = \{b\}$, $B = \{a, c, d, e\}$ it is KS_{gp} -normal spaces, but not in KS -normal spaces, KS pre-normal spaces, KS_{α} -normal spaces, KS_g -normal spaces.

Theorem 3.4. Every KS_{gp} -normal space is KS_{gp} -regular space.

Proof: Let U be a KS_{gp} -normal space. Then U is KS_{gp} -normal space as well as KS_{gp} - T_1 space. To show that U is KS_{gp} -regular, it suffices to show that the space is KS_{gp} -regular. Let F be a KS_{gp} -closed subset of U and let x be a point of U such that $x \notin F$. Since U is KS_{gp} - T_1 space, $\{x\}$ is a KS_{gp} -closed subset of U such that $\{x\} \cap F = \emptyset$. Then there exists KS_{gp} -open sets G and H such that $\{x\} \subseteq G$, $F \subseteq H$ and $G \cap H = \emptyset$. Also $\{x\} \subseteq G$ this implies $x \in G$. Thus there exists KS_{gp} -open sets G , H such that $x \in G$, $F \subseteq H$ and $G \cap H = \emptyset$. Hence the U is KS_{gp} -regular space.

Theorem 3.5. A KS_{gp} -closed subspace for a KS_{gp} -normal space is KS_{gp} -normal.

Proof: Let V be a KS_{gp} -closed subspace for a KS_{gp} -normal space. Let A and B be disjoint KS_{gp} -closed subset of V . Since V is KS_{gp} -closed set in U , A and B are also KS_{gp} -closed set in U . Since U is KS_{gp} -normal, there exists disjoint KS_{gp} -open sets G and H in U such that $A \subseteq G$ and $B \subseteq H$. Since V contains both A and B , we have $A \subseteq V \cap G$, $B \subseteq V \cap H$ and $(V \cap G) \cap (V \cap H) = \emptyset$. Since G and H are KS_{gp} -open set in U , $(V \cap G)$ and $(V \cap H)$ are KS_{gp} -open set in V . Thus in the subspace V , we have disjoint KS_{gp} -open sets $(V \cap G)$ containing A and $(V \cap H)$ containing B . Hence V is KS_{gp} -normal.

Theorem 3.6. A Kasaj Topological Spaces U is KS_{gp} -normal space iff for any KS_{gp} -open set A containing a KS_{gp} -closed set F , there exists a KS_{gp} -open set G such that $F \subseteq G \subseteq KS_{gpcl}(G) \subseteq A$.

Proof: Assume that U is KS_{gp} -normal. Since F and A^c are disjoint and KS_{gp} -closed sets in U , there exists disjoint KS_{gp} -open sets G and H such that $F \subseteq G$ and $A^c \subseteq H$. Since G and H are disjoint, $G \subseteq H^c$, we have $KS_{gpcl}(G) \subseteq H^c \subseteq A$. Thus we have a KS_{gp} -open set G such that $F \subseteq G \subseteq KS_{gpcl}(G) \subseteq A$. Conversely, assume that the condition holds. Let A and B be disjoint KS_{gp} -closed set in U . Since B^c is KS_{gp} -open and contains the KS_{gp} -closed set A , by assumption, there is a KS_{gp} -open set V such that $A \subseteq V \subseteq KS_{gpcl}(V) \subseteq B^c$, thus we have KS_{gp} -open sets $A \subseteq V$ and $[KS_{gpcl}(V)]^c$. Hence U is KS_{gp} -normal space.

Theorem 3.7. In a Kasaj Topological Spaces $(U, \tau_R(X), KS_R(X))$. Then every KS -normal space (KS pre-normal space, KS_α -normal space, KS_β -normal space, KS_g -normal space, KS_{gsp} -normal space) is KS_{gsp} -normal.

Proof: It is obvious from Theorem 3.2.

Converses of the above results need not be true as shown in the following example.

Example 3.8. Let $U = \{a, b, c, d, e\}$, with $U \setminus R = \{\{a, b\}, \{c, d\}, \{e\}\}$ and $X = \{a, e\}$. Then the $\tau_R(X)$ topology, $\tau_R(X) = \{U, \emptyset, \{e\}, \{a, b, e\}, \{a, b\}\}$. Let $S = \{a\}$, $S' = \{b, c, d, e\}$. Then $F = \{c\}$, $G = \{e\}$ and $A = \{a, b, c, d\}$, $B = \{e\}$ is KS_{gsp} -normal spaces, but not in KS -normal spaces, KS pre-normal spaces, KS_α -normal spaces, KS_β -normal spaces, KS_g -normal spaces.

Theorem 3.9. Every KS_{gsp} -normal space is KS_{gsp} -regular space.

Proof: Let U be a KS_{gsp} -normal space. Then U is KS_{gsp} -normal space as well as KS_{gsp} - T_1 space. To show that U is KS_{gsp} -regular, it suffices to show that the space is KS_{gsp} -regular. Let F be a KS_{gsp} -closed subset of U and let x be a point of U such that $x \notin F$. Since U is KS_{gsp} - T_1 space, $\{x\}$ is a KS_{gsp} -closed subset of U such that $\{x\} \cap F = \emptyset$. Then there exists KS_{gsp} -open sets G and H such that $\{x\} \subseteq G$, $F \subseteq H$ and $G \cap H = \emptyset$. Also $\{x\} \subseteq G$ this implies $x \in G$. Thus there exists KS_{gsp} -open sets G, H such that $x \in G$, $F \subseteq H$ and $G \cap H = \emptyset$. Hence the U is KS_{gsp} -regular space.

Theorem 3.10. A Kasaj Topological Spaces U is KS_{gsp} -normal space iff for any KS_{gsp} -open set A containing a KS_{gsp} -closed set F , there exists an KS_{gsp} -open set G such that $F \subseteq G \subseteq KS_{gspcl}(G) \subseteq A$.

Proof: Assume that U is KS_{gsp} -normal. Since F and A^c are disjoint and KS_{gsp} -closed sets in U , there exists disjoint KS_{gsp} -open sets G and H such that $F \subseteq G$ and $A^c \subseteq H$. Since G and H are disjoint, $G \subseteq H^c$, we have $KS_{gspcl}(G) \subseteq H^c \subseteq A$. Thus we have an KS_{gsp} -open sets G such that $F \subseteq G \subseteq KS_{gspcl}(G) \subseteq A$.

Conversely, assume that the condition holds. Let A and B be disjoint KS_{gsp} -closed set in U . Since B^c is KS_{gsp} -open and contains the KS_{gsp} -closed set A , by assumption, there is an KS_{gsp} -open set V such that $A \subseteq V \subseteq KS_{gspcl}(V) \subseteq B^c$, thus we have KS_{gsp} -open sets $A \subseteq V$ and $[KS_{gspcl}(V)]^c$. Hence U is KS_{gsp} -normal space.

Theorem 3.11. A KS_{gsp} -closed subspace for a KS_{gsp} -normal space is KS_{gsp} -normal.

Proof: Let V be a KS_{gsp} -closed subspace for a KS_{gsp} -normal space. Let A and B be disjoint KS_{gsp} -closed subset of V . Since V is KS_{gsp} -closed set in U , A and B are also KS_{gsp} -closed set in U . Since U is KS_{gsp} -normal, there exists disjoint KS_{gsp} -open sets G and H in U such that $A \subseteq G$ and $B \subseteq H$. Since V contains both A and B , we have $A \subseteq V \cap G$, $B \subseteq V \cap H$ and $(V \cap G) \cap (V \cap H) = \emptyset$. Since G and H are KS_{gsp} -open set in U , $(V \cap G)$ and $(V \cap H)$ are KS_{gsp} -open set in V . Thus in the subspace V , we have disjoint KS_{gsp} -open sets $(V \cap G)$ containing A and $(V \cap H)$ containing B . Hence V is KS_{gsp} -normal.

CONCLUSION

In the existing paper, we had introduced and studied the concept of $KS_{gsp}[KS_{gsp}]$ -Regular space, $KS_{gsp}[KS_{gsp}]$ -Normal space using the concept of $KS_{gsp}[KS_{gsp}]$ -open sets in kasaj topological spaces. This shall be extended in the future research with some applications.

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