ANALYSIS OF MULTISERVER QUEUING MODEL WITH REST PERIOD AFTER EACH SERVICE

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ABSTRACT

This paper ,deals with many queuing system with reorientation time have been studied with more than one server. However, many times in our life , we face the problem of server failure and the need for standby arises .When the main server fails, we immediately switch to standby so that the work is not affected i.e .if the main power supply breaks the generator comes to our rescue .To gain the goodwill of customers ,standby may start operating whenever the queue exceeds a certain number. Multi-server queuing model is a queuing model consisting of two more service facilities that provide the same service in parallel. In the multi-server queuing model ,multiple stations can serve customers in the waiting line .Multi-server queuing system can describe real – world systems where a shared resource is divided into independent units providing service. For examples, call center, cashier and logical information transmission channels.MSQ system consists of more than one counter to provide the same service to different customers .For examples,large stores ,airports, and gas stations .A multi channel ,single step business has multiple servers and one step servicing process. For example ,an airline ticket counter with separate queues for business class and economy class passengers. In this paper we have discussed about the the expected number in system is a decreasing function of mean dependence rate when other parameter remain fixed. The mean number of customers in system is an increasing function of 'R' for given values of the parameters.

KEYWORDS: Multi-server, Main server, standby, server failure ,power supply .

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INTRODUCTION

Most of the queuing system has been studied in the past, in which the server serves the customers without any rest. Srinivas R. Chakravarthy (2014) has been studied about the multi-server queueing model with server consultations. This is a multi-server queueing model with a Markovian arrival process in which clients are served by the main server, which also serves as a consultant to other servers. During client service, regular servers might ask for consultations, which are given by the main server according to priority. Customer service is interrupted by the main server's preference for discussions above customer assistance. The model is evaluated in steady-state using matrix-analytic techniques, assuming exponential service and consultation times. The qualitative aspect of the model is illustrated with numerical examples.

The queuing system may describe as follows;

1) Arrival occurs singly with Poisson rate a

11) There are two server i.e. Main and Standby, Normally ,the main server works with rate ξ_1 but it can fall with probability ψ (independent from the time it is working).As soon as main fails standby start working with service rate ξ_2. Standby also start working as soon as the queue length reaches to a fixed number L. Repair time distribution of the main units is exponential with parameter θ .

SOLUTION OF THE MODEL-

Define

$Q_0(t)$ = Probability that at time 't' ,there are zero units in the system ,Main server failed and standby is operating.

$P_0(t)$ = Probability that at time 't' ,there are zero units in the system and main server is operating.

$P_{m\,n}(t)$ = Probability that at time 't' ,there is one customer into service ,n units in the waiting and main server is operating.

$P_{B\,n}(t)$ = Probability that time 't', there are n units in the system .Both the servers are working.

$P_{S\,,n}(t)$ = Probability that at time 't', there are n units in the queue and only standby is operating.

Elementary probability reasoning leads to the following set of differential equations :

$Q_0'(t) = -(\lambda+\theta)Q_0(t)+\phi P_0(t)+\xi_2 P_{S,0}(t)$
(1)

$P_0'(t) = -(\phi+\lambda)P_0(t)+\xi_1 P_{M,0}+\theta Q_0(t)$
(2)

$P_{M,0}'(t) = -(\lambda+\phi+\xi_1)P_{M,0}(t)+\xi_1 P_{M,1}(t)+\lambda P_{M,1}(t)+\lambda P_0(t)+\xi_2 P_{B,0}(t)+\theta P_{S,0}(t)$

(3)

$P_{M,N}'(t) = -(\lambda+\phi+\xi_1)P_{M,n}+\sum P_{M,n-1}(t)+\sum P_{M,n-1}(t)+\theta P_{S,n}(t)+\xi_2 P_{B,n}(t)$ (4)

$P_{M,L-1}'(t) = -(\lambda+\phi+\xi_1)P_{M,L-1}(t)+\lambda P_{M,L-2}(t)+\theta P_{S,L-1}(t)+\xi_2 P_{B,L-1}(t)$
(5)

$P_{B,0}'(t) = -(\lambda+\xi_1+\xi_2+\phi)P_{B,0}(t)+\xi_1 P_{B,1}(t)$
(6)

$P_{B,n}'(t) = -(\lambda+\xi_1+\xi_2+\phi)P_{B,n}(t)+\xi_1 P_{B,n+1}(t)+\lambda P_{B,n+1}(t)$
(7)

$P_{B,L-1}' = -(\lambda+\xi_1+\xi_2+\phi)P_{B,L-1}(t)+(\xi_1+\xi_2)P_{B,n+1}(t)+\lambda P_{B,L}(t)+\lambda P_{M,L-1}(t)$
(8)

$P_{B,n}'(t) = -(\lambda+\xi_1+\xi_2+\phi)P_{B,n}(t)+(\xi_1+\xi_2)P_{B,n+1}(t)+\lambda P_{B,n-1}(t))$ n≥ 1
(9)

$P_{S,0}'(t) = -(\lambda+\xi_2+\theta)P_{S,0}(t)+\phi P_{S,n-1}(t)+\lambda Q_0(t)$ (10)

$P_{S,n}'(t) = -(\lambda+\xi_2+\theta)P_{S,n}(t)+\phi P_{B,n-1}(t)+\xi_2 P_{S,n+1}(t)+\lambda P_{S,n-1}(t)$ n≥ 1
(11)

Define the Generating Function

$P_M(z,t) = \sum_{n=0}^{L-1} P_{M,n}(t)z^n$
(12)

$P_B(z,t) = \sum_{n=0}^{\infty} P_{B,n}(t)z^n$
(13)

$P_S(z,t) = \sum_{n=0}^{\infty} P_{S,n}(t)z^n$
(14)

Multiplying by appropriate power of z to (1) to (5) taking Laplace transforms and using (12),we get

$\bar{P}_M(z,t) = \frac{\lambda Z\bar{P}_0(s)-\xi_1\bar{P}_{M,0}(s)-\lambda\bar{P}_{M,L-1}(s)Z^{L+1}+\theta\sum_{n=0}^{L-1}\bar{P}_{B.n}(s)Z^{n+1}}{z(\lambda+\phi+\xi_1)-\xi_1-\lambda Z^2}$
(15)

Similarly multiplying (6.6) to (6.11)by appropriate power of using (6.13),(6.14),we get

$\bar{P}_B(z,t) = \frac{\lambda\bar{P}_{M,L-1}(s)Z^L-\xi_1\bar{P}_{B,0}(s)}{Z(\lambda+\xi_1+\xi_2+\phi)-\xi_1-\lambda^2}$
(16)

$\bar{P}_S(z,t) = \frac{z\xi_2 P_{S,1}(t)+\phi z^2 P_B(z,t)-\xi_2 P_O(z,t)+\lambda Q_0(t)}{Z(\lambda+\xi_2+\theta)-\xi_2}$
(17)

STEADY STATE SOLUTION

The probability generating function in steady state obtained from (15) to (17) ,we get

$P_M(z) = \frac{\lambda ZP_0-\xi_1 P_{M,0}-\lambda P_{M,L-1}Z^{L+1}+\theta\sum_{n=0}^{L-1}P_{S,n}Z^{n+1}+\xi_2\sum_{n=0}^{L-1}P_{B,n}z^{n+1}}{z(\lambda+\theta+\xi_1)-\xi_1-\lambda z^2}$
(18)

$P_B(z) = \frac{\lambda P_{M,L-1}Z^L - \xi_1 P_{B,O}}{Z(\lambda+\xi_1+\xi_2+\phi)-\xi_1-\lambda Z^2}$ (19)

$P_Z(z) = \frac{z\xi_2 P_{S,1}+\phi z^2 P_B(z)-\xi_2 P_O+\lambda Q_0}{(\lambda+\xi_2+\theta)-\xi_2}$ (20)

Equations (1),(2) and (11) gives

$P_{S,O} = \frac{\lambda+Q}{\xi_2} Q_0 - \frac{\phi}{\xi_2} P_O$ (21)

$P_{M,O} = \frac{\lambda+Q}{\xi_1} P_O - \frac{\theta Q_O}{\xi_1}$ (22)

$P_{B,1} = \frac{\lambda+\xi_1+\xi_2+\phi}{\xi_1} P_{B,O}$ (23)

$P_{B,n} = r^2 P_{B,0}$

$P_{S,n} = (\frac{\lambda+\xi_2+\theta}{\xi_2})^n P_{S,O} - \frac{\theta}{\mu_1}\sum_{j=0}^{n-1}(\frac{\lambda+\xi_2+\theta}{\xi_2})^{n-j-2} P_{B,j}$ (24)

Where

$r = \frac{(\lambda+\xi_1+\xi_2+\phi)-((\lambda+\xi_1+\xi_2+\phi)^2-4\lambda\xi_1)^{\frac{1}{2}}}{2\xi_1}$ (25)

If we put values of $P_{S,O}$,$P_{M,O}, P_{B,n}$ *and* $P_{S,n}$ (n=0,1,2,2,. . .,L-1) in (19) to (21),only four unknowns $P_O, Q_O, P_{B,O}$ *and* $P_{M,L-1}$ remained to be determined .

Since $P_M(z)$ is polynomial, hence the numerator must vanish for two zeroes r_1, r_2 of the denominators.

This gives,

$\lambda r_1 P_0 - \xi_1 P_{M,O}$-$\lambda P_{M,L-1} r_i^{L+1} + \theta\sum_{n=0}^{L-1} P_{S,n} r_i^{n+1} + \xi_2 \sum_{n=0}^{L-1} P_{B,n} r_i^{n+1}$ =0

(i=1, 2, 3 . . . n) (26)

$P_B(z)$ is convergent series in $|z|$< 1 and hence must vanish for the root of denominator which lies inside the unit circle (say r_3). This gives

$\lambda P_{M,L-1} r_3^L - \xi_1 P_{B,D}$ =0 (27)

Since

$P_B(1) + P_S(1) + P_M(1) = 1$, this gives

$$(\lambda P_0 - \xi_1 P_{M,0} - \lambda P_{M,L-1} + \theta \sum_{n=0}^{L-1} P_{S,n} + \xi_2 \sum_{n=0}^{L-1} P_{B,n})(\xi_2+\phi)$$

$(\lambda+Q)(\lambda P_{M,L-1} - \xi_1 P_{B,0})$ $(\lambda+\theta)(\theta) + (\xi_2 P_{S,1} + \phi P_B - \xi_2 P_0 + \lambda Q_0)$

$(\xi_2+\phi)(\theta) = \theta(\xi_2+\phi)$ $(\lambda+\theta)$ (28)

Solving (26) ,(27) and (28),we can obtain P_O ,$Q_O, P_{B,O}$ *and* $P_{M,L-1}$

OPERATIONAL CHARACTERISTICS

1) Average Number of Units in the System

Let L_1, L_2, L_3 be the average number of units in the system

$L_1 = P_B'(1) / P_B(1)$

$P_B'(1) = \frac{(\xi_2+\phi)\lambda P_{M,L-1}(\lambda P_{M,L-1}-\xi_1 P_{B,0})(\xi_1+\xi_2+\phi-\lambda)}{(\xi_2+\phi)^2}$ (29)

$L_2 = P_S'(1) / P_S(1)$

$P_S'(1) = \frac{(\lambda+\theta)(\xi_2 P_{S,1}+2\phi P_B(1)) - (\xi_2 P_{S,1}+\phi P_B(1)-\xi_1 P_0+\lambda Q_0)(\lambda+\xi_2+\theta)}{(\lambda\theta)^2}$ (30)

$L_3 = P_M'(1) / P_M(1)$

$P_M'(1) = \frac{\theta(\lambda P_0 - \lambda(L+1)P_{M,L-1} + \theta\sum_{n=0}^{L-1}(n+1)P_{S,n} + \xi_2\sum_{n=0}^{L-1}(n+1)P_{B,n} - (\lambda P_0 - \xi_1 P_{M,0} - \lambda P_{M,L-1} + \theta\sum_{n=0}^{L-1} P_{B,n})(\theta+\xi_1-\lambda)}{\theta^2}$ (31)

2) Proportion of Time the Server Remains in Idle State and Operative State

Let E_1, E_2, E_3 be the proportion of time the server remains and idle state respectively

$E_1 = P_M(1)$

$\frac{\lambda P_0 - \xi_1 P_{M,L-1} + \theta\sum_{n=0}^{L-1} P_{S,n} + \xi_2\sum_{n=0}^{L-1} P_{S,n}}{\theta}$ (32)

$E_2 = P_B(1)$

$\frac{\lambda P_{M,L-1} - \xi_1 P_{B,0}}{\xi_2 - \phi}$ (33)

$E_3 = P_S(1)$

$E_3 = \frac{\xi_2 P_{S,1} + \phi P_{B,1} - \xi_2 P_O}{\lambda+\theta}$ (34)

PARTICULAR CASES

When both the servers are identical i.e

$\xi_1 = \xi_2 = \xi$

Then the equation (29) to (34) give

$P_B'(1) = \frac{(\xi+\phi)\lambda P_{M,L-1} - (\lambda P_{M,L-1} - \xi P_{B,0})(2\xi+\phi-\lambda)}{(\xi+\phi)^2}$

$P_S'(1) = \frac{(\lambda+\theta)(\xi_2 P_{S,1} + 2\phi P_B'(1) - (\xi_2 P_{S,1} - P_0) + \phi P_B(1) + \lambda Q_0)(\lambda+\xi+\theta)}{(\xi+\theta)^2}$

$P_M'(1) = \frac{\theta(\lambda P_0 - \lambda(L+1) P_{M,L-1} + \theta\sum_{n=0}^{L-1}(n+1)P_{S,n} + \xi_2\sum_{n=0}^{L-1}(n+1)P_{B,n} - (\lambda P_0 \xi P_{M,0} - \lambda P_{M,L-1} + \theta\sum_{n=0}^{L-1} P_{S,n} + \xi\sum_{n=0}^{L-1} P_{B,n})(\theta+\xi-\lambda)}{\theta^2}$

E_1

$$= \frac{\lambda P_0 - \xi P_{M,0} - \lambda P_{M,L-1} + \theta \sum_{n=0}^{L-1} P_{S,n} + \xi \sum_{n=0}^{L-1} P_{S,n}}{\theta}$$

$$E_2 = \frac{\lambda P_{M,L-1} - \xi P_{B,0}}{\xi - \phi}$$

$$E_3 = \frac{(P_{N,0} - P_3) + \phi P_3(1)}{\lambda + \theta} \qquad (35)$$

Numerical Analysis

From table (1.1) it is observed that, the probability that the system is in state 0 is inversly proportional to the probability that the system is in state 1. The $P_n(1)$ values is increasing when the parameter r is increasing for fixwd values of the other parameters. P(o) is increasing function of M also. $P_n(1)$ is decreasing function of λ' when other parameter remain fixed .from the table (1.2) we observe that the expected number in system is a decreasing function of mean dependence rate when other parameter remain fixed. The mean number of customers in system is an increasing function of 'R' for given values of the parameters.

Table 1.1

R	M	μ	λ	λ'	e	L_s
0.06	0.2	0.1	0.5	0.2	0.125	4.574
0.06	0.2	0.1	0.5	0.3	0.125	4.573
0.06	0.2	0.1	0.5	0.4	0.125	4.573
0.06	0.2	0.1	0.5	0.5	0.125	4.572
0.06	0.2	0.1	0.5	0.6	0.125	4.572

r	M	μ	λ	λ'	e	L_s
0.06	0.3	0.1	0.4	0.2	0.125	4.577
0.06	0.3	0.1	0.4	0.3	0.125	4.576
0.06	0.3	0.1	0.4	0.4	0.125	4.576
0.06	0.3	0.1	0.4	0.5	0.125	4.575
0.06	0.3	0.1	0.4	0.6	0.125	4.575

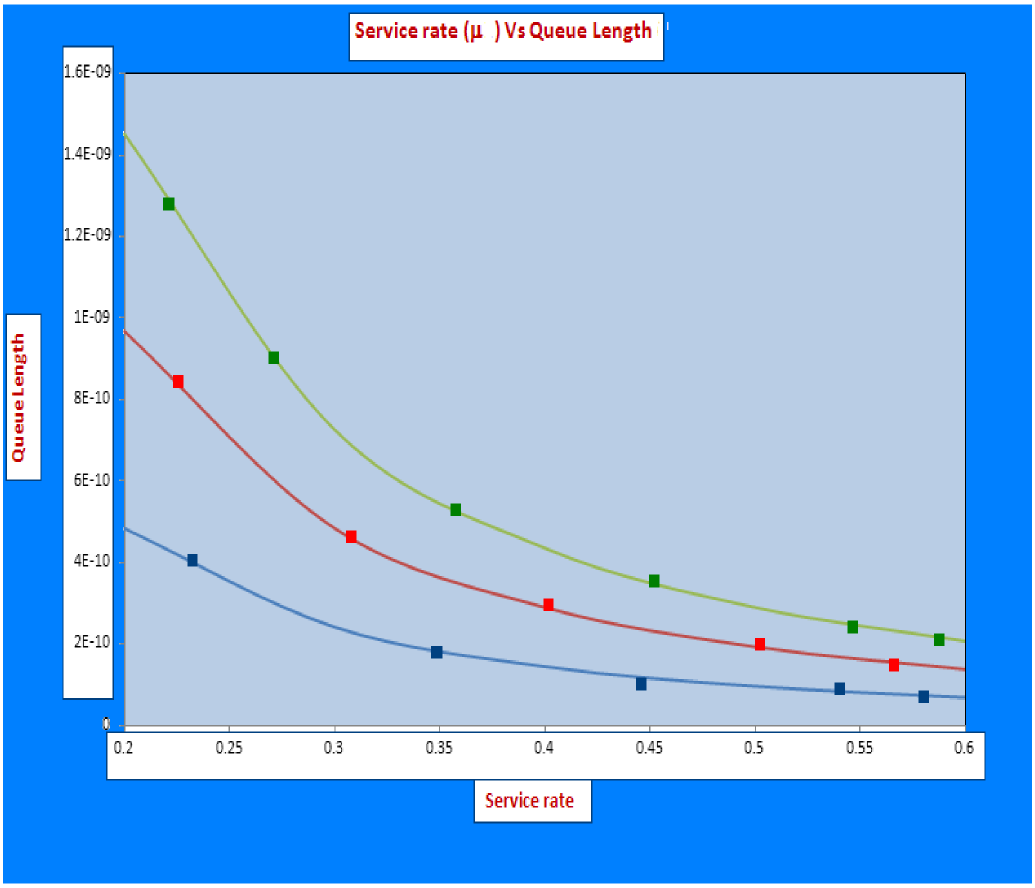
r	M	μ	λ	λ'	e	L_s
0.06	0.4	0.1	0.2	0.2	0.125	4.580
0.06	0.4	0.1	0.2	0.3	0.125	4.579
0.06	0.4	0.1	0.2	0.4	0.125	4.579
0.06	0.4	0.1	0.2	0.5	0.125	4.578
0.06	0.4	0.1	0.2	0.6	0.125	4.578

Different values are calculated by taking μ = 0.1, λ = 0.2, M = 0.4

Table – 1.2

B	R	M	μ	λ	λ'	e	$P_0(0)$
0.06	1	0.2	0.1	0.4	0.4	0.0625	4.843E-10
0.06	1	0.2	0.2	0.4	0.4	0.125	2.422E-10
0.06	1	0.2	0.3	0.4	0.4	0.1875	1.455E-10
B	R	M	μ	λ	λ'	e	P(0)
0.06	1	0.4	0.1	0.4	0.4	0.0625	9.68366E-10
0.06	1	0.4	0.2	0.4	0.4	0.125	4.84183E-10
0.06	1	0.4	0.3	0.4	0.4	0.1875	2.9051E-10

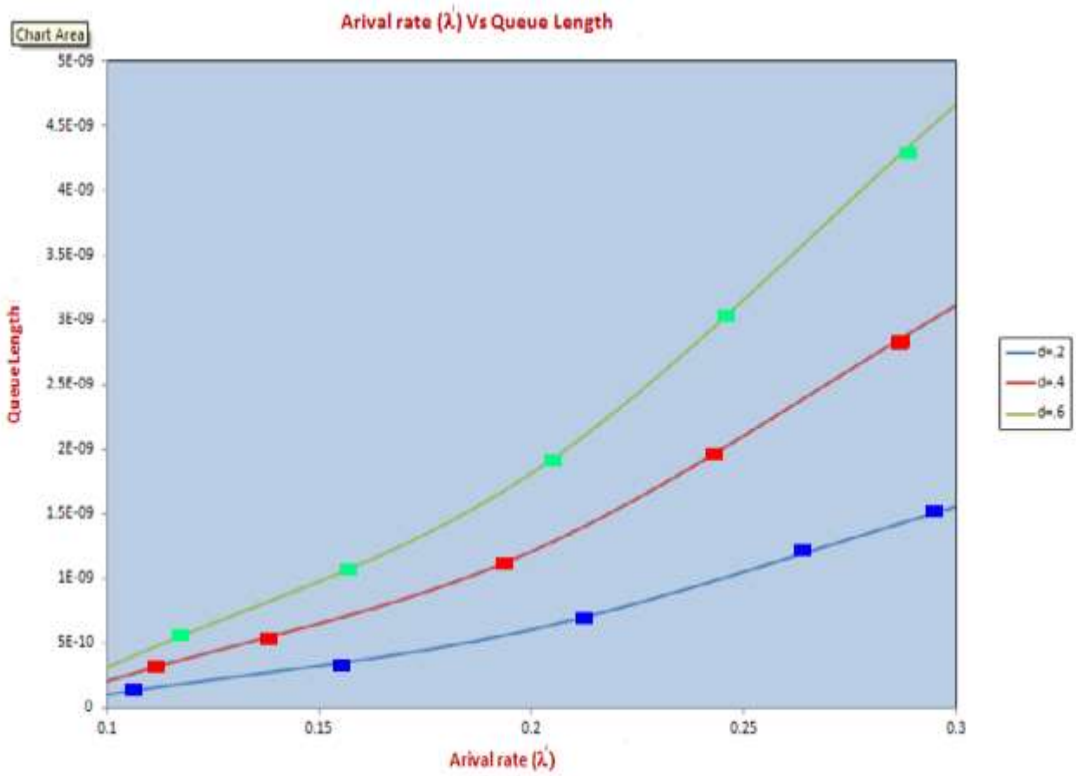
Different values are calculated by taking μ = 0.1, M = 0.



μ = 0.3 μ = 0.2 μ = 0.1

This graph shows that queue length decreases as service rate increases.

Graph 1.2



This graph shows that queue length increases as arrival rate increases.

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