



## STUDY ON UNDERWATER TECHNIQUE FOR MULTI-SOURCE DATA CORRECTION AND COOPERATIVE POSITIONING ERROR COMPENSATION

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**Article History:** Received: 12.12.2022

Revised: 29.01.2023

Accepted: 15.03.2023

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### Abstract:

The duration and distance of information transmission are the primary factors in underwater formation cooperative driving errors of navigation and placement. Time-related errors are connected to transmission lag, clock skew, and other elements. It is challenging to measure how different variables like water temperature, ocean current, and sea depth effect transmission distance inaccuracy. The Kalman filter technique is employed in this study to simulate the trajectory of the micro-platform, examine the mechanisms causing navigation and positioning mistakes, sort the errors into systematic and random errors, and categorize the errors. The positioning error characteristics of the micro-platform in the coordinated movement of underwater formation are investigated in conjunction with the multiple linear regression model and the approach based on Bayesian particle filter. To achieve the compensation and estimation of the positioning error, the experimental settings are altered, and the corresponding connection between the dependent variables and independent variables in the model is examined.

**Keywords:** Kalman filter, multiple linear regression model, particle filter, and error estimate for undersea formations

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**DOI: 10.31838/ecb/2023.12.s3.114**

## 1. Introduction

Whether in the military's underwater mission execution of special combatants or the civil area of marine engineering underwater maintenance operations, underwater multi-person / aircraft formation (Ou et al., 2021) cooperative cooperation is an essential method of functioning. In a wide variety of complex water environments, it is necessary to integrate multi-source data generated by various sensors in order to achieve formation maintenance and an accurate formation route. This will inevitably result in navigation and positioning errors brought on by systematic errors and random errors. The inaccuracy created by the cumulative calculations of sensors like inertial navigation and Doppler ergometer is referred to as the systematic error. Random error refers to the error generated by environmental conditions or specific factors according to particular laws, which is unpredictable. Its impact on navigation and placement frequently follows certain principles. This research establishes the motion and positioning models of the micro-platform based on the Kalman filtering technique and analyses the relative positioning error mechanism under cooperative formation working conditions. Both the Bayesian particle filter algorithm and the multiple linear regression model approach are used. The estimation of navigation and positioning error is finished, and the corresponding connection between the dependent variable and the independent variable in the model is explored by altering the experimental circumstances.

## 2. Creation of a motion model for a micro platform

The master and slave micro-motion platforms first synchronise their clocks before beginning the underwater formation's cooperative navigation. The position data and corresponding variance of the master micro-motion platform, as well as the separation between the master and slave micro-motion platforms, can be analysed under the assumption that the underwater acoustic signal of the autonomous micro-motion platform is received from the micro-motion platform at a specific time. The necessary data of the primary micro motion platform may then be examined from the micro motion platform when the underwater acoustic signal packets originating from the autonomous micro motion platform are received again from the micro motion platform at  $(\tau + 1)$ . The position estimate information at the time of the micro motion platform may be solved using the position information collected from these two times and the dead reckoning system from the micro motion platform.

It is assumed that the motion state of the micro platform at  $K$  time is changed from the state at  $(K-1)$  time in combination with the Kalman filter algorithm(He et al., 2022), and the transformation formula is as follows:

$$x_k = F_k x_{k-1} + B_k u_k + w_k \quad (1)$$

$B_k$  is the input control model operating on the controller  $u_k$ ;  $F_k$  is a state transformation model acting on state  $x_{k-1}$ ;  $Q_k$  is the covariance matrix, which is a multivariate standard normal distribution; and  $w_k$  is the process noise, with a mean of zero.

$$w_k \sim N(0, Q_k) \quad (2)$$

The measurement variable  $Z_k$  of state  $x_k$  at time  $k$  has the following model formula:

$$z_k = H_k x_k + v_k$$

$H_k$  is the observation model,  $v_k$  is the observation noise, and is the function that transforms the state quantity into the observation quantity are some of them. The covariance matrix,  $R_k$ , is like the preceding  $w_k$  and follows the multivariate standard normal distribution:

$$v_k \sim N(0, R_k)$$

It is believed that the starting state and the noise at each time,  $\{x_0, w_1, \dots, w_k, v_1, \dots, v_k\}$ , are independent of one another.

The value of the motion state  $x$  of the micro platform with time may be obtained when combined with the Kalman filtering technique. Specifically, the error  $P_{k-1|k-1}$  between the last state value and the measured value is used to anticipate the error  $P_{k|k-1}$  between the current state value and the measured value. This is to predict the current state optimum estimation  $x_{k-1}$  by the last state optimal estimation  $x_{k|k-1}$ . The motion state's prediction update equation is:

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1} + B_k u_k \quad (\text{Estimating the predictive state})$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + u_k \quad (\text{Matrix of Prediction, Estimation, and Covariance})$$

The estimator  $\hat{x}_{k-1}$  derived by comparing the most recent state value  $x_{k-1}$  with  $Z_{k-1}$  is denoted by the formula  $\hat{x}_{k|k-1}$ . The anticipated value of the current estimator,  $\hat{x}_k$ , which is derived from the prior estimator,  $\hat{x}_k$ , is represented by the expression identical  $\hat{x}_{k|k-1}$

By deducting the system state equation from the state prediction equation:

$$x_k - \hat{x}_{k|k-1} = F_k (x_{k-1} - \hat{x}_{k-1|k-1}) + w_k$$

$$P_{k|k-1} = E \left[ \left( F_k (x_{k-1} - \hat{x}_{k-1|k-1}) + w_k \right) \times \left( F_k (x_{k-1} - \hat{x}_{k-1|k-1}) + w_k \right)^T \right]$$

The state estimate error may be transformed into: since it is not connected with the system noise.

$$\begin{aligned} & E \left[ (x_{k-1} - \hat{x}_{k-1|k-1}) w_k^T \right] \\ &= E \left[ w_k^T (x_{k-1} - \hat{x}_{k-1|k-1})^T \right] \\ &= 0 \end{aligned}$$

$$\rightarrow P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$$

The formula can alternatively be revised to read:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (Z_k - H_k \hat{x}_{k|k-1})$$

$$P_{k|k} = P_{k|k-1} + K_k H_k P_{k|k-1}$$

$K_k$  is a Kalman gain, which decides how to turn the predicted value into the updated value to satisfy:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

Thus, the following may be established for the micro-platform motion measurement model:

$$Z_{k+1} = g(X_{k+1}^S, X_{k+1}^M, X_k^M, DX_{k+1}^M)$$

### 3. Navigation positioning system error correction using multiple linear regression equations

System error is the error that occurs from a specific or specific factor changing in accordance with a specific law under a specific set of experimental circumstances, and its impact on the determination results frequently complies with a specific rule. It is challenging to create a closed-loop motion for the placement and navigation of micro-platforms across vast distances. The closed-loop path of short-range eight-character navigation (shown in Figures 1 and 2) is suggested to replace the cooperative working state of underwater formation in long distance, so as to carry out error correction analysis. This is done in order to find the corresponding relationship between navigation errors.



Figure 1: Diagram of a closed-loop node setup

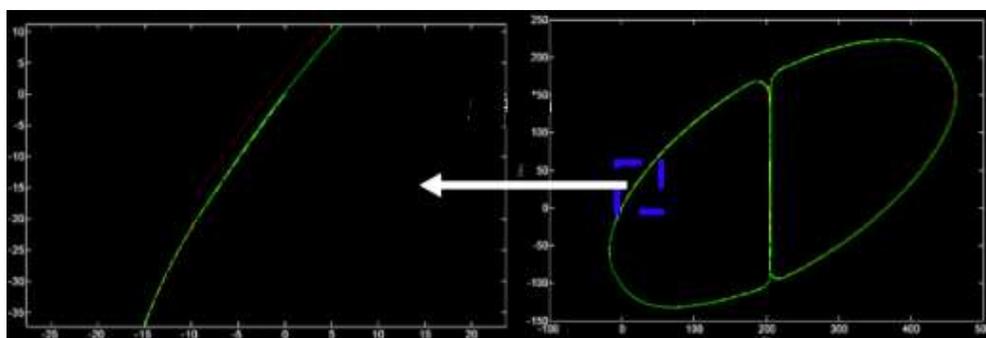


Figure 2: Driving trajectory comparison technique at nodes

Figure 3 depicts three instances of a micro-platform traversing an 8-shaped closed-loop map:

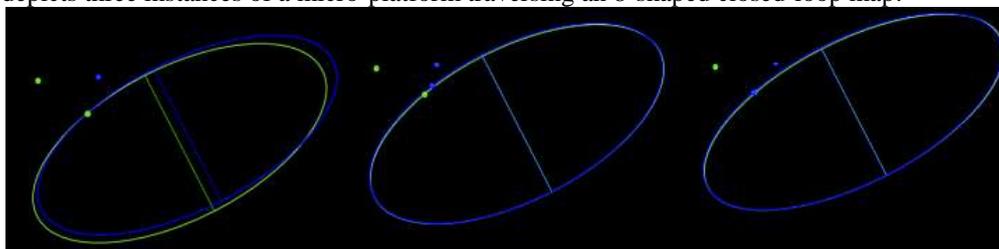


Figure 3: Node-level closed-loop correction diagram

A multivariate linear regression equation between the system error of the micro-platform and multiple influencing factors is established by combining with the environmental characteristics around the underwater micro-platform and the parameters of the dead reckoning information in light of the

mentioned situation. Feature extraction and matching are performed at the starting and ending points of the closed-loop route. In order to fully correct the navigation and positioning error, the dead reckoning position of the micro-platform is

corrected by combining the parameter matching and optimization methods, so that the calculated position value closely matches the actual position value of each point in the driving area (Figure 3(c)).

Here is how the system error is stated:

$$\mu(y) = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p$$

It is possible to write it down as a matrix by looking at group N data:

$$Y = X\beta + \varepsilon$$

X stands for observation quantity, Y for observation error, for observation matrix, and for observation error. The multivariate linear regression equation (Michailidis et al., 2020) can be represented as the following matrix form since the origin of system error is frequently certain deterministic factors, not random variables. Assuming the regression equation fulfills G-M requirements and observation error obeys normal distribution.

$$Y = X\beta + \varepsilon$$

$$\varepsilon \sim N(0, \sigma^2 I_n)$$

The factor parameters affecting the system error are determined using the least squares estimation and maximum likelihood estimation methods. The regression equation is then reintroduced into the driving process of the micro platform prior to the node for positioning correction comparison in order to identify the source and magnitude of the system error and complete the compensation and correction of the system error (Gao & Guo, 2018).

#### 4. Bayesian filtering-based random error correction for location and navigation

Random mistake frequently results from something unforeseen and unintentional that happens during the measurement process; these events have no regularity and cannot be predicted, while repeated measurements have statistical regularity. By following the 8-word path on the closed map, it is possible to extract the random error sample function of the state space propagation of the micro-platform, which can roughly reflect the posterior probability density of the positioning error of the micro platform. The mapping link between small distance and big distance is constructed in conjunction with the particle filter based on Bayesian filtering (Zheng et al., 2005), after which the positioning inaccuracy of the large-scale micro platform is fixed (Hightower et al., 2000).

The system state follows a first-order Markov process, and the system observation  $e_k$  is independent if  $(k - 1)$  time and  $P_{(\tau_{k-1}|e_{1:k-1})}$  are known.

$$p(\tau_k | e_{1:k-1}) = \int p(\tau_k | \tau_{k-1}) p(\tau_{k-1} | e_{1:k-1}) d\tau_{k-1}$$

Where  $P_{(\tau_k|e_{1:k-1})}$  is the system state's transition probability density.

The research object for this work is  $\tau$ . The process equation of the system may be built in accordance with the content, where  $\tau$  can be derived by using input. (Rebello & Nascimento, 2021) The observation is the environmental information error, where the difference between the real-time environmental detection and the projected value, and it also reflects the positioning error of the system's inertial navigation.

Based on this, the system's observation equation may be formed as follows:

The process equation:

$$\tau_k = g(\tau_{k-1}, U_k)$$

The observation equation:

$$e_k = h(\tau_k)$$

The undersea micro platform's navigation and positioning state variable  $\tau_k$  is iteratively evaluated using Bayesian and Markov hypothesis:

Let  $\tau_{0:k}$  represent the state variable series  $\tau_{0:k} = (\tau_i, i = 0, 1, \dots, k)$  and  $\tau_{1:k}$  represent the variables used to make observations. The Bayesian formula demonstrates that:

$$p(\tau_{0:k} | \tau_{1:k}) = \frac{p(e_{1:k} | \tau_{0:k}) p(\tau_{0:k})}{p(\tau_{1:k} | \tau_{0:k}) d(\tau_{0:k})}$$

The likelihood probability density when the observation is  $e_{1:k}$  is  $P_{(\tau_{0:k}|e_{0:k})}$  and the posterior probability density is  $P_{(\tau_{0:k}|e_{0:k})}$ . (Lomax et al., 2000) The edge density of  $P_{(e_{0}|e_{1:k})}$ , or the posterior probability density  $P_{(\tau_k|e_{1:k})}$ , is the subject of the filtering issue.

$$p(\tau_k | e_{1:k}) = \iint \dots \int p(\tau_{0:k} | e_{1:k}) d\tau_0 d\tau_1 \dots d\tau_{k-1}$$

The posterior probability density  $P_{(\tau_k|e_{1:k})}$  will need to be computed once for the formula above everytime a new observation data set is received, which is quite unpleasant. As a result, the posterior probability density is produced using the recursive updating approach described below:

The system model is utilized to forecast the probability density of time, and the prior probability density  $P_{(\tau_k|e_{1:k-1})}$  of time is acquired. Beginning with the posterior probability density  $P_{(\tau_k|e_{1:k-1})}$  obtained at  $k - 1$  time.

Update: When the observed value  $e_k$  at  $k$  arrives, it is used to correct the above prior probability density to obtain the posterior probability density  $P_{(\tau_k|e_{1:k})}$  at  $k$ ;

Update: The posterior probability density  $P_{(\tau_k|e_{1:k})}$  at is obtained by correcting the prior probability density above with the observed value  $e_k$  at  $k$ .

Initially, the prediction step yields the prior probability density  $P_{(\tau_k|e_{1:k-1})}$  of a  $(k - 1)$  time system state without  $k$ -time data.

Then, by updating step with the observation value at k-time points, the posterior probability density of the system state at K time points is obtained:

$$p(\tau_k | e_{1:k}) = \frac{p(e_{1:k} | \tau_k) p(\tau_k)}{p(e_{1:k})} = \frac{p(\tau_k, \tau_{1:k-1} | \tau_k) p(\tau_k)}{p(e_k, e_{1:k-1})}$$

Defined by conditional probabilities:

$$p(e_k, e_{1:k-1}) = p(e_k | e_{1:k-1}) p(e_{1:k-1})$$

From the joint probability formula:

$$p(e_k, e_{1:k-1} | \tau_k) = p(e_k | e_{1:k-1}, \tau_k) p(e_{1:k-1} | \tau_k)$$

From Bayesian formula:

$$p(e_{1:k-1} | \tau_k) = \frac{p(\tau_k | e_{1:k-1}) p(e_{1:k-1})}{p(\tau_k)}$$

$$p(\tau_k | e_{1:k}) = \frac{p(e_k | e_{1:k-1}, \tau_k) p(\tau_k | e_{1:k-1}) p(e_{1:k-1})}{p(e_k | e_{1:k-1}) p(e_{1:k-1})}$$

Assuming that each system observation is independent of each other, we can get:

$$p(e_k | e_{1:k-1}, \tau_k) = p(e_k | \tau_k)$$

$$p(\tau_k | e_{1:k}) = \frac{p(e_k | \tau_k) p(\tau_k | e_{1:k-1})}{p(e_k | e_{1:k-1})}$$

$$bel(\tau_k) = p(\tau_k | e_{1:k})$$

$$\overline{bel(\tau_k)} = p(\tau_k | e_{1:k-1})$$

However  $\int P(e_k | e_{1:k-1})$  is usually a normalized constant, so:

$$bel(\tau_k) = \eta p(e_k | \tau_k) \overline{bel(\tau_k)}$$

By using a weighted sum of random samples, the Bayesian particle filter technique may approximate the posterior probability density of placement and navigation during the whole motion process.

$$p(\tau_{0:k} | e_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(\tau_{0:k} - \tau_{0:k}^i)$$

Among them,  $\{w_k^i\}$  is the normalised weight of particles, N is the number of particles,  $\{\tau_k^i\}$  and  $\delta(\cdot)$  is the Dirac-Delta function. 'Particles' are a sequence of samples acquired via Monte Carlo random sampling at k moments. (Seeger, 2004) As a result, the particle set and its weight value  $\{\tau_k^i, w_k^i\}$  may be used to roughly depict the posterior probability density  $P(\tau_k | e_{1:k-1})$  of the system, as illustrated in Figure 4.

Normalized constant  $P(\tau_k | e_{1:k-1})$  is generally unknown, so the particle set cannot be directly sampled from posterior probability density  $P(\tau_k | e_{1:k-1})$ . The importance density function  $q(\tau_k | e_{1:k-1})$ , which is easier to sample, is usually used to sample through importance sampling. The importance density  $q(\tau_k | e_{1:k-1})$ , can be expressed as:

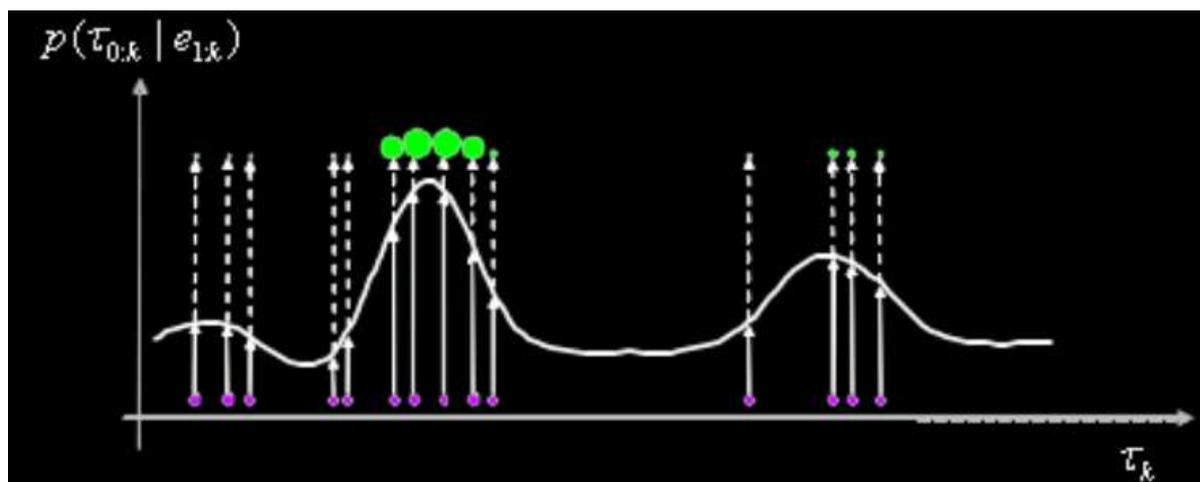


Figure 4: Distribution of the posterior probability density

As the normalised constant  $P(\tau_k|e_{1:k-1})$  is often unknown, the posterior probability density  $P(\tau_k|e_{1:k-1})$  cannot be directly sampled for the particle set.(Trivedi & Balakrishnan, 2013) The

most common method of importance sampling uses the simpler to sample significance density function  $q(\tau_k|e_{1:k-1})$ . The expression for the significance density  $q(\tau_k|e_{1:k-1})$  is:

$$q(\tau_{0:k} | e_{1:k}) \approx \sum_{i=1}^N \delta(\tau_{0:k} - \tau_{0:k}^i)$$

Thus, the particle weight meets:

$$w_k^j \propto \frac{p(\tau_{0:k}^j | e_{1:k})}{q(\tau_{0:k}^j | e_{1:k})}$$

If the importance density may be divided into:

$$q(\tau_{0:k} | e_{1:k}) = q(\tau_k | \tau_{0:k-1}, e_{1:k})q(\tau_{0:k-1} | e_{1:k-1})$$

The updating formula for weight may be calculated using the recursive Bayesian estimating approach as follows:

$$w_k^j \propto \frac{p(e_k | \tau_k^j)p(\tau_k^j | \tau_{k-1}^j)p(\tau_{0:k-1}^j | e_{1:k-1})}{q(\tau_k^j | \tau_{0:k-1}^j, e_{1:k})q(\tau_{0:k-1}^j | e_{1:k-1})} = w_{k-1}^j \frac{p(e_k | \tau_k^j)p(\tau_k^j | \tau_{k-1}^j)}{q(\tau_k^j | \tau_{0:k-1}^j, e_{1:k})}$$

If  $q(\tau_k|e_{1:k-1})$ , the importance density function only depends on  $\tau$  and  $e_k$ , and it is unnecessary to store the historical values of particle sets and observations.(Han & Zhang, 2007) The weight updating formula of the above equation can be simplified as:

It is not essential to preserve the historical values of particle sets and observations if  $q(\tau_k|e_{1:k-1})$  since the importance density function only relies on  $\tau$  and  $e_k$ . The weight updating formula in the previously stated equation is as follows:

$$w_k^j \propto w_{k-1}^j \frac{p(e_k | \tau_k^j)p(\tau_k^j | \tau_{k-1}^j)}{q(\tau_k^j | \tau_{k-1}^j, e_k)}$$

The posterior probability density of the state may be iteratively calculated in order to acquire the statistical data of the state expectation by calculating the weight of the particle set generated

by sequential sampling(Li et al., 2018) of the importance density function in accordance with the preceding formula:

$$q(\tau_k^i | \tau_{k-1}^i, e_k) = p(\tau_k^i | \tau_{k-1}^i)$$

$$w_k^j \propto w_{k-1}^j p(e_k | \tau_k^j)$$

After normalizing weight:

$$\tilde{w}_k^j = w_k^j / \sum_{i=1}^N w_k^i$$

One way to express the posterior probability density is as follows:

$$p(\tau_k | e_{1:k}) \approx \sum_{i=1}^N \tilde{w}_k^i \delta(\tau_k - \tau_k^i)$$

When the particle number is  $N \rightarrow \infty$ , the large number theorem can ensure that the above equation can approximate the real posterior probability  $P(\tau_k | e_{1:k-1})$ , so as to complete the random error estimation in the navigation and positioning of micro platform.

The following equation may approximate the genuine posterior probability  $P(\tau_k | e_{1:k-1})$  when the particle number is  $N \rightarrow \infty$ , completing the random error estimate in the navigation and placement of the micro platform.

## 2. Conclusions

A technique for systematic error estimation and correction in underwater cooperative formation navigation and placement is put forward in this research. The motion model of an underwater micro-platform is created by integrating the Kalman filter method with different sensor data carried by the micro-platform itself. The mistake of the closed-loop path of the short-distance eight-character navigation prior to the coordination of the micro-platform is what distinguishes the error connection between navigation and positioning at the same time. A quick calibration and correction approach for systematic error and random error in cooperative placement is provided, which may increase the precision of underwater cooperative formation navigation. It is based on multivariate linear regression equation and Bayesian particle filter method.

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