Spatial Temporal Variations of Water Hyacinth in a Delayed Fisheries System

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Abstract

Water hyacinth (Eichhornia crassipes) reveals intermittent recurring patterns of decrease and proliferation with specialist environmental and regulatory effects. This analysis intended to screen the intensity of water hyacinth impact on fishery system. The dynamics of a fisheries system in an aquatic environment with two zones: water hyacinth zone and free zone are examined in this study. Fish harvesting is permitted in both zones, and fish migration is permitted from the water hyacinth zone to the free zone but not the other way around. The dynamics of stability are shown in this study when a discrete time delay is included in the fish death rate due to oxygen depletion and water pollution produced by water hyacinth. The analysis concludes that there is a requirement for supported checking of the obtrusive macro-phytes close by environment displaying concentrates on utilizing the accessible time series information to obviously distinguish the biological variables that initiative water hyacinth dynamics and forecast all the more definitively its effect on the delayed fisheries system. To validate the analytical conclusions, numerical simulations are used.

Keywords: Water hyacinth, Fishery, Diffusion, Delay, Stability

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1. Introduction

The water hyacinth (Eichhornia crassipes fig.1, 2, 3) is a sort of aquatic obtrusive and free-drifting macrophyte plant. In spite of the fact that it is local to Brazil, it has a worldwide dispersion and is a disturbance in the aquatic climate. At the point when the plants are put into the normal natural surroundings, they rapidly expansion in inclusion. Water hyacinth plants can resist both acidic and basic conditions, although impartial water bodies support more bubbly development [1]. The shoot system covers the outer layer of water body to catch the daylight there by hindering the section of daylight into water, which is needed by green growth and different creatures present in the water to make due. This prompts a multiplication in the development of algal populace and along these lines upsets the environmental equilibrium [2]. The developed plant comprise of long, pendant roots, rhizomes, stolons, leaves, inflorescences and natural product bunches. The plants are up to 1 m high albeit; 40 cm is the typical estimated tallness. The inflorescence permits 6-10 lily-like blossoms, every one being 4-7 cm in width. The stems and leaves contain air-filled tissue which gives the plant its considerable lightness. Hyacinth has procured the monikers "the wonderful fallen angel" and "the weed from hell"[3].

Fig. 1 (ref. [14])

Fig. 2 (ref [14])

Fig. 3(ref [14])

Fish is a significant species variety in the oceanic biological system. Quantitative models of fish populace blend in with essential models that attention on and inspect just the bio-mass of the populace cycle, otherwise called development and populace. Just the previously mentioned factor joins and changes the effect of outside impacts like as rivalry for food supply and predation. To fisheries scientists, the age structure into a homogeneous stock, just as lumping together various species, is logical. Many investigations have connected water hyacinth contamination to fish creation [4-11]. They discussed what water hyacinth means for fish development. The effect of water hyacinth on the fish business is critical. It has different degrees of contamination, going from low to direct to high. The hyacinth mats considerably affect fishing action since it takes more time to get to the fishing grounds. The mats are likewise perilous on the grounds that they block light, definitely lessen oxygen levels, and permit harmful gases like hydrogen sulphide and smelling salts to get away. Thus, amphibian biodiversity is being lost.

Tragically, numerous lakes and water bodies especially in emerging countries don't have solid and predictable estimations spatial just as volumetric of sea-going vegetation. Expanded wealth of sea-going vegetation frequently brings about a decrease in water profundity and unfortunate abundance. Abundance can be considerably improved by human exercises, for example, contribution of supplements and natural matter into water bodies from the encompassing catchment regions, removal of modern and homegrown emanating. Expanded supplement stacking brings about improved supplement obsession prompting expanded macrophtic biomass [12], overabundance collections of phytoplankton biomass and the consumption of disintegrated oxygen in base waters [13] which along these lines prompts fish and other sea-going creature's demise. While happening in bounty, amphibian vegetation may subsequently bring about a diminished sporting worth of lake by blocking route and decreasing admittance to coastlines. Data on the spatial conveyance of sea-going weeds is required to comprehend the development of weed attack, decide impacted regions and assess the proficiency of control measures and the executives activities. It is at this level that accessibility of robotized, continuous information becomes basic.

In this manner, we adopting two zone methods to safeguard the fisheries population, which is under threat from water hyacinth, is a superior option. A zone free of water hyacinth, known as a hyacinth free zone, is developed. The catch ability coefficient falls as the abundance level of water hyacinth increases in this chapter, and the density of fish is exactly proportional to the abundance of fish at time (t). Furthermore, it is hypothesized that the prevalence of water hyacinth causes biological impacts such as fish death and migration. The oxygen loss and pollution induced by the growth of water hyacinth will not result in the death of the fish population right away, but it will take some time. This population shows the impact of the time lag on the dynamics of this system.

2. Water Hyacinth Model Formulation

The mathematical formulation of water hyacinth model is represented by the following a reaction diffusion equations and physical interpretation of the parameters given Table 1

Notation	Interpretation of the parameters	Parameter values
$W(t)$	Fish population in Water hyacinth region	30 (initial value)
$F(t)$	Fish population in Water hyacinth free region	40 (initial value)
ρ	Rate at which water hyacinth reduces fish catch	0.32
Ψ	Migration rate	0.19
r	Death rate due to oxygen depletion	0.45

s	Pollution rate	0.36
c_1	Catchability coefficient in water hyacinth region	0.004
c_2	Catchability coefficient in water hyacinth free region	0.002
e_1	Harvesting effort in water hyacinth region	20
e_2	Harvesting effort in water hyacinth free region	10
α	Intrinsic growth rate in water hyacinth region	0.6
β	Intrinsic growth rate in water hyacinth free region	1.5
m	Carrying capacity in water hyacinth region	600
n	Carry capacity in water hyacinth in free region	300
τ	Time delay	1.549
D_W	Diffusion coefficient in water hyacinth zone	30
D_F	Diffusion coefficient in water hyacinth free zone	60

Table.1 Physical interpretation of the parameter

$$\begin{cases} W_t = D_W W_{ss} + H_1(W,F) \\ F_t = D_F F_{ss} + H_2(W,F) \end{cases} \tag{1}$$

with the additional states of the following conditions

$$W(s,t) > 0, F(s,t) > 0 \text{ in } 0 \le s \le L, L > 0$$

$$\frac{\partial W(0,t)}{\partial t} = \frac{\partial P(L,t)}{\partial t} = \frac{\partial W(0,t)}{\partial t} = \frac{\partial F(L,t)}{\partial t} = 0$$

(2)

and corresponding ODE system $\begin{cases} W^1 = H_1(W,F) \\ F^1 = H_2(W,F) \end{cases}$ (3)

where
$$\begin{cases} H_1(W,F) = \alpha W\left(1 - \frac{W}{m}\right) - (1-\rho)c_1 e_1 W - \Psi W - rW(t-\tau) - sW(t-\tau) \\ H_2(W,F) = \beta F\left(1 - \frac{F}{n}\right) + \psi F - c_2 e_2 F \end{cases} \tag{4}$$

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Further both the variables W and F are non-negative and the parameters are assumed to be positive.

3. Analysis of water hyacinth system with delay and without Diffusion:

3.1. Equilibrium Analysis: The steady states of the system (2) are the solutions of the steady state equations.

$$\begin{cases} \alpha W\left(1-\dfrac{W}{m}\right)-(1-\rho)c_1e_1W-\Psi W-rW(t-\tau)-sW(t-\tau)=0 \\ \beta F\left(1-\dfrac{F}{n}\right)+\psi F-c_2e_2F=0 \end{cases}$$

The possible equilibrium points are obtained by $E_0(0,0)$(In the absence of both the zones), $E_1(\gamma,0)$(In the presence of water hyacinth zone), $E_2(0,\delta^*)$(In the presence of free zone of water hyacinth), $E_3(\gamma^*,\delta^*)$(In the presence of both the zones i.e., the interior equilibrium).

Case (i): The population is extinct, and this insignificant stable state exists at all times.

Case (ii): If γ is positive solution of $W'(t)=0$ then $\gamma=(m/\alpha)\left[\rho c_1 e_1+(\alpha-c_1e_1-\psi-r-s)\right]$

This positive steady state exists only when

$$\rho c_1 e_1+\alpha > c_1 e_1+\psi+r+s \tag{5}$$

Case (iii): If δ is the positive solution of $F'(t)=0$ then $\delta=n(\beta-c_2e_2)/\beta$. This positive steady state exists only when

$$\beta > c_2 e_2 \tag{6}$$

Case (iv): If (γ^*,δ^*) are the positive solutions of $W'(t)=0$ and $F'(t)=0$ then

$$\gamma^*=(m/\alpha)(\rho c_1 e_1+\alpha-c_1e_1-\psi-r-s)$$

$$\delta^{*2}+G\delta^*+H=0 \tag{7}$$

where $G=-n(\beta-c_2e_2)/\beta$; $H=-(\psi mn/\alpha\beta)\left(\alpha+\rho c_1e_1-(c_1e_1+\psi+r+s)\right)$

From the biological point of view, consider the interior equilibrium $E_3(\gamma^*,\delta^*)$.

Let $W=\gamma-\gamma^*, F=\delta-\delta^*$ be the perturbed variables of the system (3)

3.2. Stability Analysis

3.2.1. Stability analysis in the absence of delay : The characteristic equation of the system (3) is

$$\chi^2+X\chi+Y=0 \tag{8}$$

where $X=P+R=\dfrac{\alpha\gamma^*}{m}+\dfrac{\beta\delta^*}{n}+\dfrac{\psi\gamma^*}{\delta^*}$, $Y=Q+T=\dfrac{\alpha\beta}{mn}\gamma^*\delta^*+\dfrac{\alpha\psi}{m}\dfrac{\gamma^{*2}}{\delta^*}$, here $X>0, Y>0$. So, the Eigen values of the characteristic equation are either real and negative or complex conjugate with negative real part. Hence, the system (3) is locally asymptotically stable.

Theorem 1: The system (3) is locally asymptotically stable at $E_3(\gamma^*,\delta^*)$ if the equation (8) has both the roots with negative real parts.

Theorem 2: The unique interior equilibrium point $E_3(\gamma^*,\delta^*)$ is globally asymptotically stable for the ODE system (2).

Proof: In this, we are concerned with the global stability of the model system (3) in a positively invariant set() . To do this let us construct Lyapunov function as follows

$$\Pi=\left[W-W^*-W^*\log\left(\frac{W}{W^*}\right)\right]+l_1\left[F-F^*-F^*\log\left(\frac{F}{F^*}\right)\right] \tag{9}$$

where l_1 is a positive constant, to be chosen suitably later on. Differentiating Π with respect to t, we get,

$$\frac{d\Pi}{dt}=\frac{\left(W-W^*\right)}{W}\frac{dW}{dt}+l_1\frac{\left(F-F^*\right)}{F}\frac{dF}{dt}$$

Choosing $l_1=\frac{n}{\beta}$, a little algebraic manipulation yields,

$$\frac{d\Pi}{dt}=-\frac{\alpha}{m}\left(W-W^*\right)^2-\left(F-F^*\right)^2<0$$

.

Therefore $E_3\left(\gamma^*,\delta^*\right)$ is globally asymptotically stable.

3.2.2. Stability analysis in the presence of delay: The characteristic equation of the linear system is given by

$$\Delta\left(\chi,\tau\right)=\chi^2+P\chi+Q+e^{-\chi\tau}\left(R\chi+U\right)=0 \quad (10)$$

Let $\chi\left(\tau\right)=\theta\left(\tau\right)+i\eta\left(\tau\right)$ be a root of the characteristic equation (10)**.** Let τ be a particular value of the delay such that $\theta\left(\tau\right)=0,\eta\left(\tau\right)>0$**.** Substituting $\chi=i\eta$ in (10), the equation (10) becomes

$$\left(-\eta^2+i\eta P+Q\right)+e^{-i\eta\tau}\left(Ri\eta+U\right)=0 \quad (11)$$

separating the real and imaginary parts

$$\begin{cases}\eta^2-Q=U\cos\eta\tau+R\eta\sin\eta\tau\\ P\eta=U\sin\eta\tau-R\eta\cos\eta\tau\end{cases} \quad (12)$$

squaring and adding equation (11), we get $\eta^4+K\eta^2+L=0$ (13)

where $K=P^2-2Q-R^2;L=Q^2-U^2$

Lemma 1: If $K=P^2-2Q-R^2>0$ then the equation (13) does not have any real solutions.

Theorem 2: If $K>0$, $L<0$ then the equilibrium point $E_3\left(\gamma^*,\delta^*\right)$ is locally asymptotically stable for all $\tau\geq 0$.

Lemma 2: If $K>0$, $L<0$ then the equation (13) has a unique positive root, it is η_0^2 and let the corresponding τ be τ_0.

Lemma3: If $K<0,L>0$ and $K^2-4L>0$ then the equation (13) has two positive roots. Let them be $\eta_\pm^2$ and the corresponding τ is $\tau^\pm$.

Eliminating $\sin\eta\tau$ from (12)

$$\tau^\pm=\frac{1}{\eta}\arccos\left(\frac{\eta^2\left(U-PR\right)-QU}{U^2+\eta^2R^2}\right)+\frac{2n\pi}{\eta} \quad (14)$$

where $n = 0,1,2,...etc.$,

Thus, η may be η_0 or $\eta_{\pm}$ .The above said positive roots, either from Lemma 2 or from Lemma 3, satisfy all the equations from (12)-(14).

4. Analysis of the Water Hyacinth System with Diffusion

4.1. Diffusion analysis in one dimensional space: Under the beginning and boundary conditions of the system, we investigate the fish population in the water hyacinth system, as well as the free zone of the water hyacinth (1). We investigated the linearized version of the system about $E_3\left(\gamma^*,\delta^*\right)$ to study the influence of diffusion on the system..

Let $W=\gamma^*+w;\ F=\delta^*+f$ be the perturbed variables of the system (1). Here $\left(w,f\right)$ are slight increments of $\left(W,F\right)$ about the stability point $E_3\left(\gamma^*,\delta^*\right)$ and assume that the solution in the form of $w=\Omega_1 e^{\xi t}e^{j\phi t};\ f=\Omega_2 e^{\xi t}e^{j\phi t}$ where ξ and ϕ represents the wave number and frequency respectively.

$$\begin{cases} \dfrac{\partial w}{\partial t}=-\dfrac{\alpha W^*}{m}w-D_1\phi^2 w \\ \dfrac{\partial f}{\partial t}=-\dfrac{\beta F^*}{n}f-D_2\phi^2 f \end{cases} \tag{15}$$

Then the characteristic equation of the system (15) is given by

$$\mu^2+\Theta_1\mu+\Theta_2=0 \tag{16}$$

where $\Theta_1=\dfrac{\alpha W^*}{m}+D_1\phi^2+\dfrac{\beta F^*}{n}+D_2\phi^2$;

$$\Theta_2=\frac{\alpha\beta W^*F^*}{mn}+\frac{\alpha W^*}{m}D_2\phi^2+\frac{\beta F^*}{n}D_1\phi^2+D_1D_2\phi^4;$$

According to Routh-Hurwitz criteria, the system (1) is locally stable if $\Theta_1>0$ and $\Theta_2>0$. Our main goal is to figure out what constitutes unsteadiness of diffusive system (1). If one of the roots of an equation (12) is non-negative, the system (1) becomes unstable. The fact that a root is positive is $\Theta_1>0$ i.e., $\phi^2>-\dfrac{1}{D_1+D_2}\left[\dfrac{\alpha W^*}{m}+\dfrac{\beta F^*}{n}\right]$ a necessary condition.

Since the wavenumber ϕ is real and positive, the above statement can be executed if $\dfrac{\alpha W^*}{m}+\dfrac{\beta F^*}{n}<0$. Therefore, the conditions required for system diffusion instability are:

$$\frac{\alpha W^*}{m}+\frac{\beta F^*}{n}<0 \tag{17}$$

Sufficient conditions for one of the roots of equation (15) to be positive are: $\Theta_2\left(\phi^2\right)<0$

Hence the state for diffusive unsteadiness is given by $\dfrac{\Omega_1}{D_1}+\dfrac{\Omega_2}{D_2}>\varsigma$ (18)

where $\Omega_1 = \dfrac{\alpha}{2mF^*}$; $\Omega_2 = \dfrac{\beta}{2nW^*}$; $\varsigma = \dfrac{\alpha\beta}{mn}$. So (17) and (18) gives the unsteadiness conditions for the system (1). Hence the system (16) pushes into an unstable oscillation.

Theorem 3:

(i) If the internal stability $E_3\left(\gamma^*,\delta^*\right)$ of system (3) is globally stable, then the same steady state of system (1) at the initial and boundary conditions is also globally stable.

(ii) If the internal stability $E_3\left(\gamma^*,\delta^*\right)$ of the system (3) is unstable, the system (1) can be stabilized in the same steady state under the condition (2) by appropriately increasing the diffusion coefficient.

Proof: Let $W(s,t)=W$, $F(s,t)=F$ for $s\in[0,R]$ and $t\in[0,\infty)$

Consider the function $\Pi_1(t)=\int_0^R \Pi(W,F)\,ds$ Where $\Pi(W,F)$ is defined in equation (9)

$$\frac{d\Pi_1}{dt}=\int_0^R\left(\frac{\partial\Pi}{\partial W}\frac{\partial W}{\partial t}+\frac{\partial\Pi}{\partial F}\frac{\partial F}{\partial t}\right)ds=\Delta_1+\Delta_2 \tag{19}$$

Where $\Delta_1=\int_0^R \frac{d\Pi}{dt}ds$ and $\Delta_2=\int_0^R\left(D_W\frac{\partial\Pi}{\partial W}\frac{\partial^2 W}{\partial s^2}+D_F\frac{\partial\Pi}{\partial F}\frac{\partial^2 F}{\partial s^2}\right)ds$ (20)

Using the boundary conditions (2), we obtain

$$\begin{aligned}\Delta_2 &= -D_W\int_0^R\frac{\partial^2\Pi}{\partial W^2}\left(\frac{\partial W}{\partial s}\right)^2 ds - D_F\int_0^R\frac{\partial^2\Pi}{\partial F^2}\left(\frac{\partial F}{\partial s}\right)^2 ds\\ &= -D_W\int_0^R\frac{\dot{W}}{W^2}\left(\frac{\partial W}{\partial s}\right)^2 ds - D_F\int_0^R\frac{\dot{F}}{F^2}\left(\frac{\partial F}{\partial s}\right)^2 ds\end{aligned} \tag{21}$$

From (19)-(21), obviously we observed that if $\Delta_1<0$ then $\Pi_1^1(t)$ is non-positive. If $\Delta_1>0$, at that point it tends to be noticed that by expanding the dissemination coefficients D_W and D_F sufficiently large, $\Pi_1^1(t)$ can be made non-positive.

4.2. Diffusion analysis in two dimensional space

The PDE system (1) reduces in two dimensional spaces

$$\begin{cases}W_t = D_W\nabla^2 W + R(W,F)\\ F_t = D_F\nabla^2 F + Z(W,F)\end{cases} \tag{22}$$

The Laplacian operator $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$ is defined in the two dimensional space. We analyse the system (22) under the subsequent primary and frontier conditions

$$\begin{cases} W(s,r,t)>0, F(r,s,t)>0 \text{ where } 0\le r,s\le J, J>0 \\ \dfrac{\partial W}{\partial \Upsilon}(0,t)=\dfrac{\partial W}{\partial \Upsilon}(J,t)=\dfrac{\partial F}{\partial \Upsilon}(0,t)=\dfrac{\partial F}{\partial \Upsilon}(J,t)=0 \text{ where } (s,r)\in\partial\wp, t>0 \end{cases} \quad (23)$$

where Υ is the outward normal of $\partial\wp$. This section shows that the results of Theorem 2 are still valid.

Consider $\prod_2(t)=\iint_{\wp}\prod(W,F)\,d\Im$, where $\prod$ which is same as specified in equation (9) is.

If one differentiates $\prod_2$ according to the time along the solution of the system (20), one obtains

$$\frac{d\prod_2}{dt}=\Delta_1+\Delta_2 \quad (24)$$

Where $\Delta_1=\iint_{\wp}\frac{d\prod}{dt}d\Im$, $\Delta_2=\iint_{\wp}D_W\frac{\partial\prod}{\partial W}\nabla^2W+D_F\frac{\partial\prod}{\partial F}\nabla^2F$

$$\iint_{\wp}X\nabla^2Y d\Im=\int_{\partial\wp}X\frac{\partial Y}{\partial\square}-\iint(\nabla X,\nabla Y)d\Im$$

Furthermore, under an examination like Dubey and Hussain [14], we prove that

$$\begin{cases} \iint_{\wp}D_W\left(\dfrac{\partial\prod}{\partial W}\nabla^2W\right)d\Im=-\iint_{\wp}D_W\dfrac{\partial^2\prod}{\partial W^2}\left[\left(\dfrac{\partial W}{\partial s}\right)^2+\left(\dfrac{\partial W}{\partial r}\right)^2\right]d\Im\le 0 \\ \iint_{\wp}D_F\left(\dfrac{\partial\prod}{\partial F}\nabla^2F,\right)d\Im=-\iint_{\wp}D_W\dfrac{\partial^2\prod}{\partial F^2}\left[\left(\dfrac{\partial F}{\partial s}\right)^2+\left(\dfrac{\partial F}{\partial r}\right)^2\right]d\Im\le 0 \end{cases} \quad (25)$$

The above two expressions gives $\Delta_2\le 0$.We observe that if $\Delta_1\le 0$, then $\frac{d\prod_2}{dt}\le 0$. Thus, the system (22) is globally asymptotically stable without diffusion at the inner equilibrium point $E_3(\gamma^*,\delta^*)$, then the system (22) remains globally asymptotically stable in the presence of diffusion

5. Numerical Simulations

Numerical simulation has been carried out in this section relating to the system (1) about the steady state $E_3(\gamma^*,\delta^*)$ and accordingly it is solved numerically. We have designed the time series evaluations with and without delay, diffusion on a normal finite domain with zero flux boundary conditions.

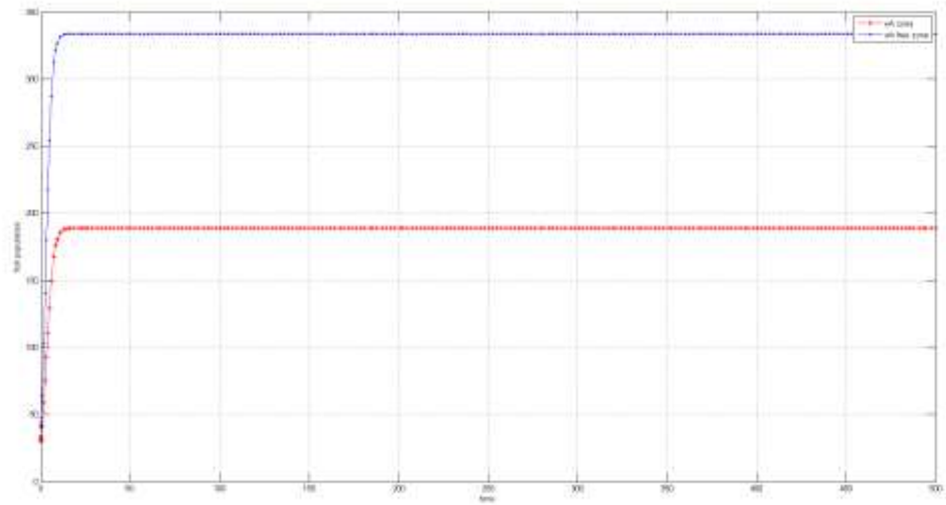

Fig.4

Fig.4 represents time series evolution of fish population in the two zones showing stable oscillation of the population towards $E_3\left(\gamma^*, \delta^*\right)$ with the attributes of table 1.

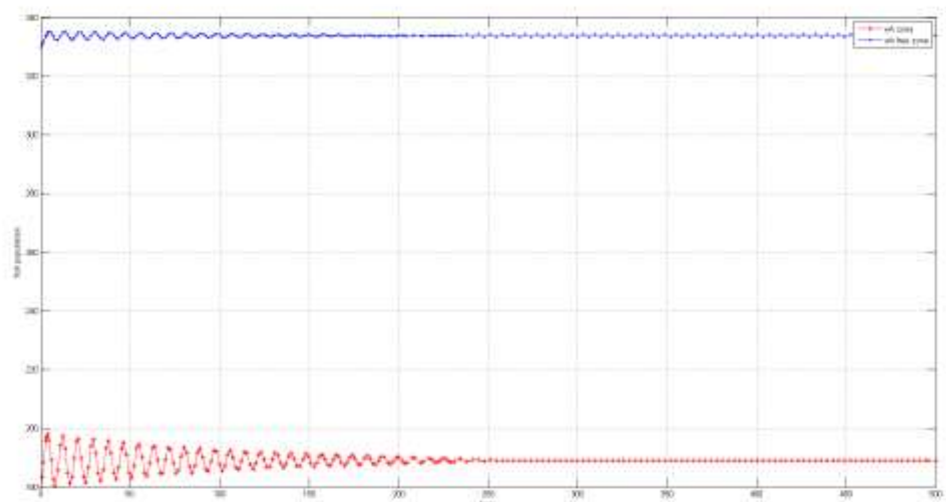

Fig.5

Fig.5 represents time series evolution of fish population in the two zones showing stable oscillation of the population when $\tau = 1.53$ with the attributes of table 1.

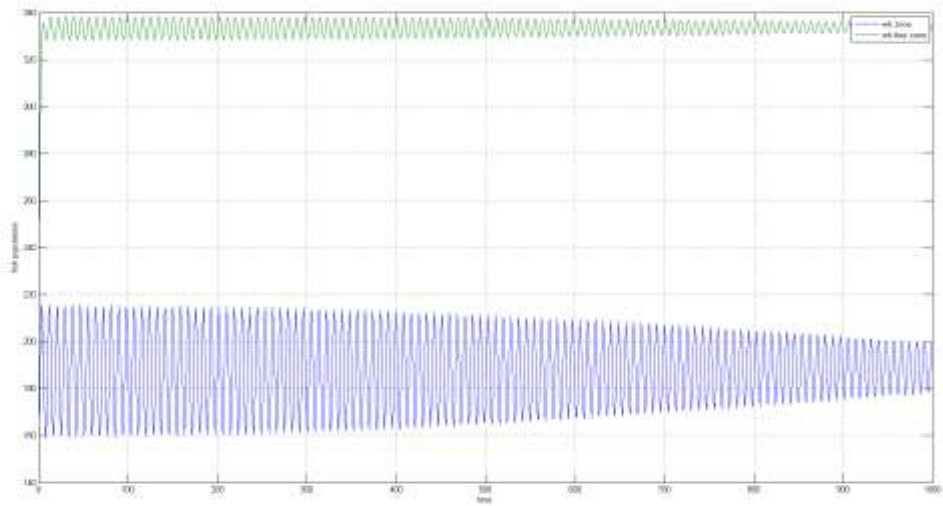

Fig.6

Fig.6 represents time series evolution of fish population in the two zones showing unstable oscillation of the population when $\tau = 1.549$ with the attributes of table 1.

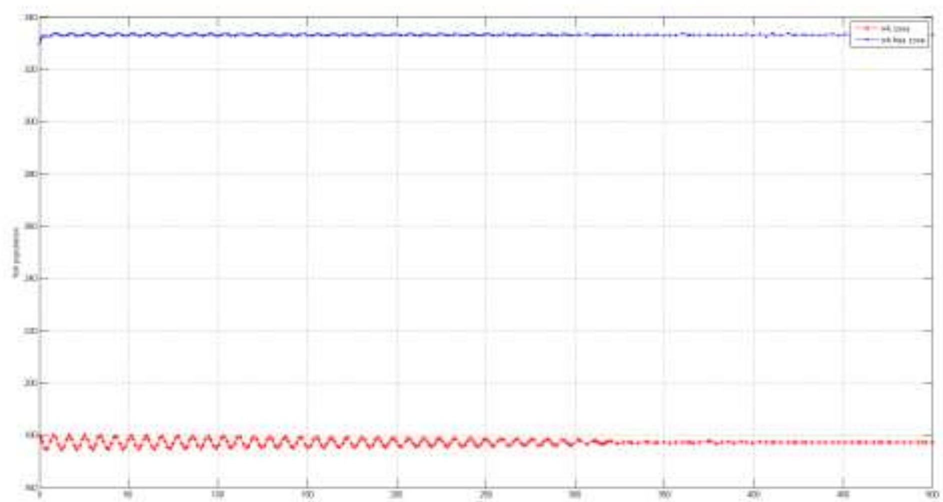

Fig.7

Fig.7 represents time series evolution of fish population in the two zones showing stable oscillation of the population when $\tau = 1.48$ with the attributes of table 1.

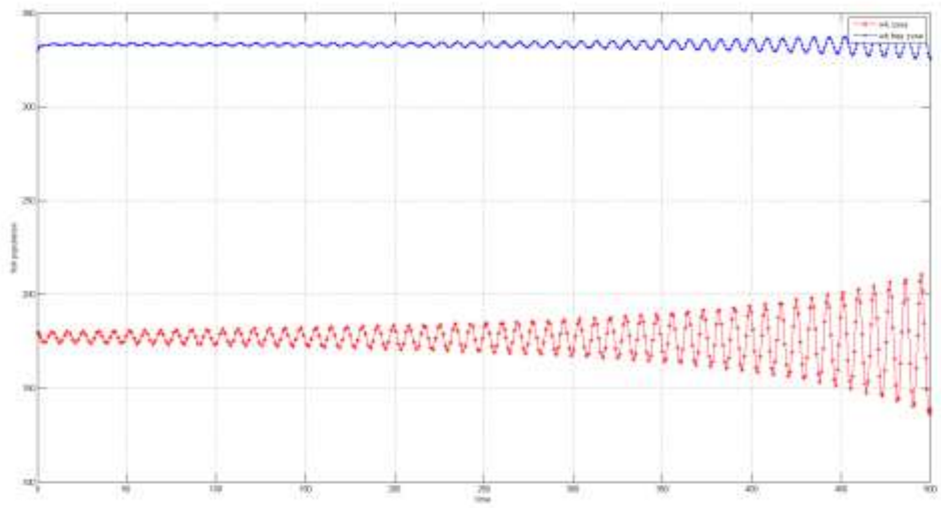

Fig.8

Fig.8 represents time series evolution of fish population in the two zones showing unstable oscillation of the population when $\tau = 1.495$ with the attributes of table 1.

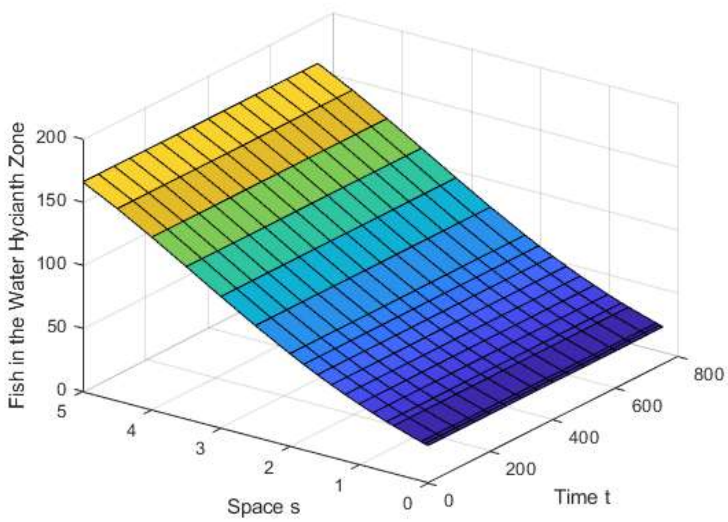

Fig.9

Fig.9 represents fish movement in the water hyacinth zone with reference to time and space variables along with the attributes of table 1.

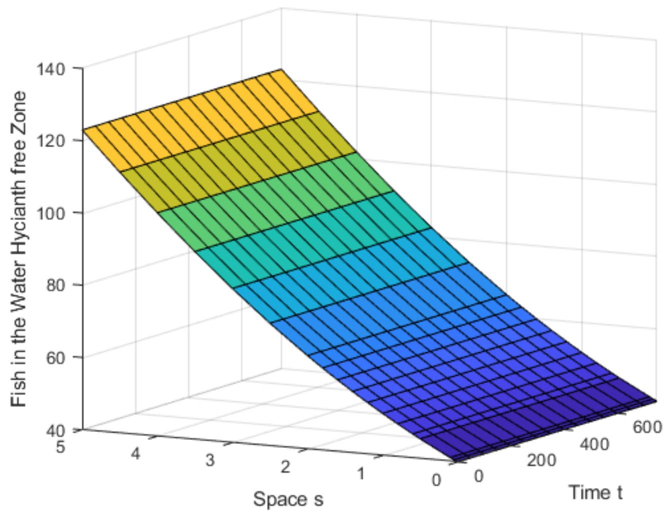

Fig.10

Fig.10 represents fish movement in the water hyacinth free zone with reference to time and space variable along with the attributes of table 1.

6. Concluding remarks

This study looked into the effect of water hyacinth on the balance of fish biomass densities. The equilibrium of the model was explored. Water hyacinth has a considerable negative impact on fish stocks, according to the study. Slight modifications to the water hyacinth model parameters had a significant impact on the model's equilibrium. The fish population in the water hyacinth zone was declining in the equilibrium. Because water hyacinth poses a risk of extinction for fish, it should be eradicated using environmentally acceptable ways. We have discussed the local and global stability of the system. Also, we analyzed the movement of fish species with reference to time and space variables by choosing a suitable set of diffusion coefficients. The findings and methodological framework presented here will be beneficial in future research to study the ramifications for specific real-world systems.

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