The Edge-to-Vertex Strong Geodetic Number of a Graph

[1]A.L. MerlinSheela, [2]J.John, [3]M. Antony
[1]Research Scholar, Register Number: 18233232092003,
Department of Mathematics
St. Jude's College,Thoothoor – 629 176, Tamil Nadu, India.
Email: sheelagodwin@gmail.com
[2] Department of Mathematics
Government College of Engineering, Tirunelveli – 627007, Tamil Nadu, India.
Email: john@gcetly.ac.in
[3] Department of Mathematics
St. Jude's College,Thoothoor – 629 176, Tamil Nadu, India
Affiliated to ManonmaniamSundaranar University, Abishekapatti,
Tirunelveli - 627 012.

Abstract
In this paper, we introduce the concept of *edge-to-vertex strong geodetic number* $sg_{ev}(G)$ of a connected graph with at least 3 vertices. Some general properties satisfied by these concepts are studied. The edge-to-vertex strong geodetic number of certain classes of graphs is determined. It is proved that for each pair of integers k and m with $2 \leq k \leq m$, there exists a connected graph G of order $m+1$ and size m with $sg_{ev}(G) = k$. It is shown that for positive integers $r, d$and $l \geq 2$with $r \leq d \leq 2r$, there exists a connected graph $G$with $radG = r$, $diamG = d, sg_{ev}(G) = l$. Connected graphs of size $m \geq 4$with edge-to-vertex strong geodetic number $m, m-1$ or $m-2$ are characterized.

Keywords: distance, edge-to-edge distance, edge-to-vertex geodetic number, edge-to-vertex strong geodetic number.
AMS Subject Classification: 05C12

1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and m respectively. For basic definitions and terminologies, we refer to [3,4,5,6,9]. If $uv \in E(G)$, we say that u is a *neighbor* of v and denote by $N_G(v)$, the set of neighbors of v. The degree of a vertex $v \in V$is $\deg_G(v) = |N_G(v)|$. A vertex v is said to be a universal *vertex* if $\deg_G(v) = n-1$. A vertex v is called an *extreme vertex* if the subgraph induced by v iscomplete. An edge of a connected graph G is called an *extreme edge* of G if one of its ends is an extreme vertex of G.

For vertices u and v in a connected graph G, the *distance* $d(u, v)$ is the length of a shortest u–v path in G. A u–v path of length $d(u, v)$ is called a u–v *geodesic*. The *diameter* of graph is the maximum distance between the pair of vertices of G. The diameter of G is denoted by *diamG* or d. The closed interval $I[x,y]$ consists of x,y and all vertices lying on some $x-y$ geodesic of G and for a non-empty set $S \subseteq V(G)$, $I[S] = \cup_{x,y\in S} I[x,y]$. A set $S \subseteq V(G)$ in a connected graph G is a geodetic set of G if $I[S] = V(G)$. Let $\tilde{g}[x,y]$be a selected fixed $x-y$geodesic. For $S \subseteq V(G)$, we set $\tilde{I}[S] = \{\tilde{g}(x,y): x,y \in S\}$ and let $V(\tilde{I}[S]) = \bigcup_{P\in G[S]} V(P)$. If $V(\tilde{I}[S]) = V$ for some $\tilde{I}[S]$then the set S is called a *strong geodetic set* of G. The minimum cardinality of a strong geodetic set of G is called the *strong geodetic number* of G and is denoted by $sg(G)$.

For,$e = uv, f = wz \in E(G)$, the distance between e and f is $d(e,f) = min\{d(u,w), d(u,z), d(v,w), d(v,z)\}$. A $e-f$ path of length $d(e,f)$ is called a $e-f$geodesic. A set$S \subseteq E$ is called an *edge-to-vertex geodetic set* if every edge of G is either an element of S or lies on a geodesic joining a pair of edges of S. The *edge-to-vertex geodetic number* $g_{ev}(G)$ of G is the minimum cardinality of its edge-to-vertex geodetic sets and any edge-to-vertex geodetic set of cardinality$g_{ev}(G)$ is called a g_{ev}*-set* of G. The edge-to-vertex geodetic number of G is studied in [1,7,8,11].

Theorem 1.1[11]. Every end-edge of a connected graph G belongs to every edge-to-vertex geodetic set of G.

Theorem 1.2[11]. For the complete graph $K_n (n \geq 4)$,$g_{ev}(K_n) = \begin{cases} \frac{n}{2} \text{ if } n \text{ is even} \\ \frac{n+1}{2} \text{ if } n \text{ is odd.} \end{cases}$.

Theorem 1.3[11]. For the complete bipartite graph $G = K_{r,s} (2 \leq r \leq s), g_{ev}(G) = s$.

2. The Edge-to-Vertex Strong Geodetic Number of a Graph

Definition 2.1. For $f,h \in E(G)$, let$\tilde{g}_{ev}[f,h]$ be a selected fixed *e-f* geodesic. For $S \subseteq E(G)$, $\tilde{I}_{ev}[S] = \{\tilde{g}(e,f): e,f \in S\}$ and let $(\tilde{I}_{ev}[S]) = \bigcup_{P\in G[S]} V(P)$. If $V(\tilde{I}_{ev}[S]) = V$ for some $\widetilde{\tilde{I}_{ev}}[S]$, then the set S is called an edge-to-vertex strong geodetic set of G. The minimum cardinality of an edge-to-vertex strong geodetic set of G is called the *edge-to-vertex strong geodetic number* of G and is denoted by $sg_{ev}(G)$.

Example 2.2. For the graph G given in Figure 2.1, $S = \{v_1v_2, v_4v_5\}$ is a minimum edge-to-vertex geodetic set of G so that $g_{ev}(G) = 2$. Also $S_1 = \{v_1v_2, v_4v_5, v_1v_6\}$ and $S_2 = \{v_1v_2, v_4v_5, v_5v_6\}$ are the edge-to-vertex strong geodetic sets of G so that $sg_{ev}(G) = 3$.

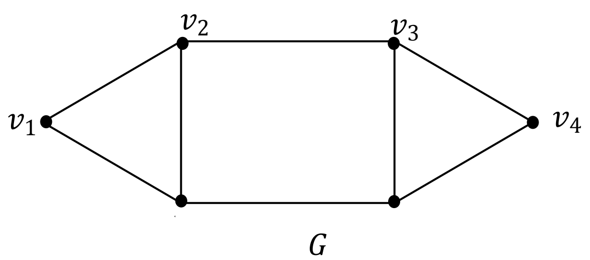


Figure 2.1

Example2.3. For the graph G given in Figure 2.1, $S_1 = \{v_1v_2, v_4v_5, v_1v_6\}$ and $S_2 = \{v_1v_2, v_4v_5, v_5v_6\}$ are two sg_{ev}-sets of G. Thus there can be more than one sg_{ev}-set of G.

Theorem 2.4. For a connected graph G of size $m \geq 2$, $2 \leq g_{ev}(G) \leq sg_{ev}(G) \leq m$.

Proof. A g_{ev}-set needs at least two edges and therefore, $g_{ev}(G) \geq 2$. Every edge-to-vertex strong geodetic set of G is an edge-to-vertex geodetic set of G and so$2 \leq g_{ev}(G) \leq sg_{ev}(G)$. Also, the set of all edges of G is an edge-to-vertex strong geodetic set of G so that $sg_{ev}(G) \leq m$. Thus$2 \leq g_{ev}(G) \leq sg_{ev}(G) \leq m$. ■

Remark 2.5. The bounds in Theorem 2.4 are sharp. For the path P with at least 3 vertices, $g_{ev}(P) = 2$. For the cycle $G = C_5$, $g_{ev}(G) = sg_{ev}(G) = 3$. For the star $G = K_{1,m} (m \geq 3)$, $sg_{ev}(G) = m$.

Theorem 2.6. If x is an extreme vertex of a connected graph G, then every edge-to-vertex strong geodetic set contains at least one extreme edge that is incident with$x$.

Proof. Let x be an extreme vertex of G. Let $e_1, e_2, \ldots, e_k$ be the edges incident with x. Let S be any edge-to-vertex strong geodetic set of G. We claim that $e_i \epsilon S$ for some $i (1 \leq i \leq k)$. Otherwise, $e_i \notin S$ for any $i (1 \leq i \leq k)$. Since S is an edge-to-vertex strong geodetic set and the vertex $x$is not incident with any element of S, x lies on exactly one geodesic joining two elements say $h, f \in S$. Let $h = x_1x_2$ and $f = x_lx_m$. Then$x \neq x_1, x_2, x_l, x_m$and $d(h, f) \geq 1$. Assume without loss of generality that $P: x_0 = x_1, y_1, y_2, \ldots, y_t, y_{t+1} = x, y_{t+2}, \ldots, y_{s-1}, y_s = x_l$ be a h–f geodesic, where $y_1 \neq x_2$ and $y_{s-1} \neq x_m$. Since x is an extreme vertex, y_t and y_{t+2} are adjacent and so $Q: x_0 = x_1, y_1, y_2, \ldots, y_t, y_{t+1}, y_{t+2}, y_{t+3}, \ldots, y_{s-1}, y_s = x_l$ is a shorter h–f path than P, which is a contradiction. Hence,$e_i \in S$ for some $i (1 \leq i \leq k)$. ■

Remark 2.7. For the graph G given in Figure 2.1, $S = \{v_1v_2, v_4v_5, v_1v_6\}$ is an edge-to-vertex strong geodetic set of Gwhich does not contain the extreme edge v_3v_4. Thus all theextreme edges of a graph need not belong to an edge-to-vertex stronggeodetic set of G.

The following corollary shows that there are certain edges in a connected graph G that are edge-to-vertex stronggeodetic edges of$G$.

Corollary 2.8. Everyend-edge of a connected graph Gbelongs toevery edge-to-vertex stronggeodetic set of G.

Proof. This follows from Theorem 2.6. ■

Theorem 2.9. IfG is any connected graph of size m with number of end-vertices$k$,then $max\{2, k \leq sg_{ev}(G) \leq m\}$.

Proof. This follows from Theorem 2.6 and Corollary 2.8. ■

Theorem2.10. Let$G$be a tree with k end vertices. Then$sg_{ev}(G) = k$.

Proof. This follows from Corollary 2.8 ■

Theorem 2.11. For the cycle $C_n (n \geq 4)$, $sg_{ev}(C_n) = 3$.

Proof. Since$n \geq 4$, $sg_{ev}(C_n) \geq 3$. Suppose that n is even. Let $n = 2k$ and let $C_n: v_1, v_2, v_3, \ldots, v_k, v_{k+1,\ldots}, v_{2k}, v_1$be the cycle. Then$v_{k+1}$isthe antipodal vertex of $v_1$and

v_{k+2} is the antipodal vertex of v_2. Let $v_1, v_2, v_3, \ldots, v_k, v_{k+1}$ be a fixed geodesic. Let $v_{k+2}, v_{k+3}, \ldots, v_{2k}, v_1$be a $v_{k+1} - v_1$geodesic. Then $S = \{v_1v_2, v_{k+1}v_{k+2}\} \cup \{v_{k+2}v_{k+3}\}$ is an edge-to-vertex strong geodesic set of G so that$sg_{ev}(C_n) = 3$.Suppose that n is odd.Let $n = 2k + 1$ and let$C_p: v_1, v_2, v_3, \ldots, v_k, v_{k+1,\ldots}, v_{2k}, v_{2k+1}, v_1$be the cycle.Let v_{k+1} and v_{k+3} be the antipodal vertices of v_1. Then$S_1 = \{v_1v_2, v_{k+1}v_{k+2}, v_{k+2}v_{k+3}\}$ is an edge-to-vertex strong geodetic set of G so that$sg_{ev}(C_n) = 3$. ■

Theorem2.12. For the wheel $G = W_n = K + C_{n-1} (n \geq 6)$,

$$sg_{ev}(G) = \begin{cases} \left\lfloor \dfrac{n+1}{2} \right\rfloor; n-1 \; is \; even \\ \left\lceil \dfrac{n+1}{2} \right\rceil; \; n-1 \; is \; odd \end{cases}$$

Proof. Let $V(K_1) = x, V(C_{n-1}) = \{v_1, v_2, \ldots, v_{n-1}\}$. Let us assume that $n-1$ is even. Let $n-1 = 2k$. Then $S = \{v_1v_2, v_4v_5, \ldots\ v_{k+1}v_{k+2}, \ldots v_{2k-1}v_{2k}, xv_1\}$ is an edge-to-vertex strong geodetic set of G and so $sg_{ev}(G) \geq \frac{n-1}{2} + 1 = \left\lfloor \frac{n+1}{2} \right\rfloor$. We prove that $sg_{ev}(G) = \left\lfloor \frac{n+1}{2} \right\rfloor$. On the contrary, suppose that $sg_{ev}(G) < \left\lfloor \frac{n+1}{2} \right\rfloor$. Then there exists a sg_{ev}-set S' such that $|S'| \leq \left\lfloor \frac{n+1}{2} \right\rfloor$. Hence there exists $f \in S$ such that $f \notin S'$. Let $f = uv$, then either u or $v \notin \bar{I}[S']$, which is a contradiction. Therefore,$sg_{ev}(G) = \left\lfloor \frac{n+1}{2} \right\rfloor$.

Now, let us assume that $n-1$ is odd. Let $n-1 = 2k+1$. Then $S = \{v_1v_2, v_4v_5, \ldots\ v_{k+1}v_{k+2}, \ldots v_{2k-1}v_{2k}, xv_1\}$ is an edge-to-vertex strong geodetic set of G and so $sg_{ev}(G) \geq \frac{n-1}{2} + 1 = \left\lceil \frac{n+1}{2} \right\rceil$. We prove that $sg_{ev}(G) = \left\lceil \frac{n+1}{2} \right\rceil$. On the contrary, suppose that $sg_{ev}(G) < \left\lceil \frac{n+1}{2} \right\rceil$. Then there exists a sg_{ev} - set S'' such that $|s''| \leq \left\lceil \frac{n+1}{2} \right\rceil$. Hence there exists $h \in S$ such that $h \notin S''$. Let $h = u'v'$, then either u' or $v' \notin \bar{I}[S'']$, which is a contradiction. Therefore$sg_{ev}(G) = \left\lceil \frac{n+1}{2} \right\rceil$. ■

Theorem 2.13. For the complete graph $K_n (n \geq 4), sg_{ev}(K_n) = \begin{cases} \frac{n}{2} \text{ if } n \text{ is even} \\ \frac{n+1}{2} \text{ if } n \text{ is odd.} \end{cases}$.

Proof. Since every edge-to-vertex strong geodetic set is an edge-to-vertex geodetic set of G, the result follows from Theorem 1.2 ■

Theorem 2.14. Forthe complete bipartite graph $G = K_{m,n} (2 \leq m \leq n), sg_{ev}(G) = n$.

Proof. Since every edge-to-vertex strong geodetic set is an edge-to-vertex geodetic set of G, the result follows from Theorem 1.3. ■

In the following we characterize graphs G for which $sg_{ev}(G) = 2, m-1 \; or \; m$.

Theorem 2.15. For a connected graph of order $n \geq 2, sg_{ev}(G) = 2$ if and only if $G = P_n (n \geq 2)$.

Proof. Let $G = P_n (n \geq 2)$. Then by Theorem 2.10, $sg_{ev}(G) = 2$. Conversely, let $sg_{ev}(G) = 2$. We prove that $G = P_n$. On the contrary, suppose that $G \neq P_n$. Then G is not a tree. Therefore, G contains a cycle say, C. Let $S = \{e, f\}$ be a $sg_{ev}(G)$-set of G. Let $e = uv$ and $f = xy$. Without loss of generality, let us assume that $d(u, x)$ is the shortest $e - f$ path. Let $P: u = u_0, u_1, u_2, \ldots, u_l = x$ be the shortest $e - f$ path. We fix P.

Then there exists at least one vertex, say w in $V(S) \setminus V(P)$. Then S is not a $sg_{ev}(G)$-set of G, which is a contradiction. Therefore, $G = P_n$. ■

Theorem 2.16. If G is a connected graph such that it is not a star, then $sg_{ev}(G) \leq m - 1$.

Proof. Let e be an edge such that it is not an end-edge of G. Then $S = E(G) - \{e\}$ is an edge-to-vertex strong geodetic set of G so that $sg_{ev}(G) \leq m - 1$. ■

Remark 2.17. The bound in Theorem 2.16 is sharp. For double star G given in Figure 2.2, $sg_{ev}(G) = 4 = m - 1$.

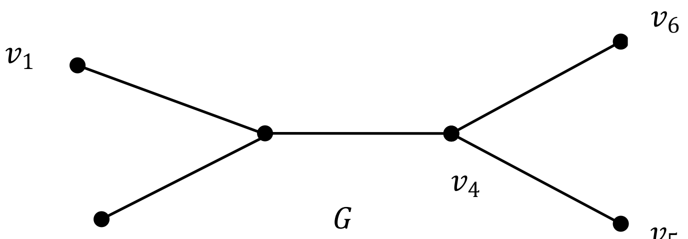


Figure 2.2

Theorem 2.18. For any connected graph G, $sg_{ev}(G) = m$ if and only if G is a star.

Proof. Let G be a star. Then by Theorem 2.10, sg_{e} m. Conversely, let $sg_{ev}(G) = m$. If G is not a star, then by Theorem 2.16, $sg_{ev}(G) \leq m - 1$, which is a contradiction. Therefore, G is a star. ■

Theorem 2.19. Let G be a connected graph which is not a tree. Then, $sg_{ev}(G) \leq m - 2$ $(m \geq 4)$.

Proof. If the graph G is a cycle $C_n (n \geq 4)$, then by Theorem 2.11, $sg_{ev}(G) \leq m - 2$. If the graph G is not a cycle, let $C: x_1, x_2, x_3, \ldots, x_k, x_1 (k \geq 3)$ be a smallest cycle in G and let x be a vertex such that x is not on C and x is adjacent to x_1, say. Now, $S = E(G) - \{x_1x_2, x_1x_k\}$ is an edge-to-vertex strong geodetic set so that $sg_{ev}(G) \leq m - 2$. ■

Remark 2.20. The bound in Theorem 2.19 is sharp. For the graph G given in Figure 2.3, $S = \{v_1v_2, v_2v_5, v_3v_4\}$ is an edge-to-vertex strong geodetic set of G so that $sg_{ev}(G) = 2 = m - 2$.

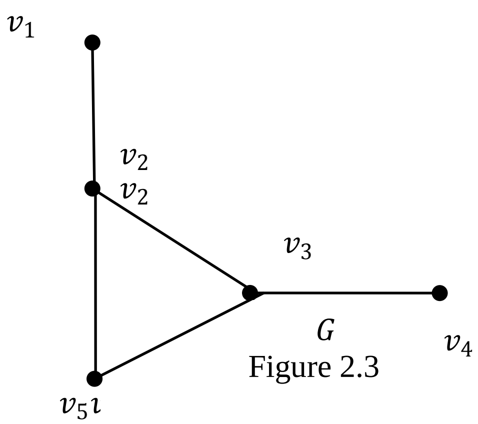


Figure 2.3

Theorem 2.21. For a connected graph $G$with $m \geq 2$, $sg_{ev}(G) \leq m - d + 2$, where d is the diameter of G.

Proof. Let x and y be vertices of G for which $d(x,y) = d$, where d is the diameter of G and let $P: x = x_0, x_1, x_2, \dots, x_d = y$ be a $x-y$ path of length d. Let $e_i = x_{i-1}y_i (1 \leq i \leq d)$.Let $S = E(G) - \{v_1v_2, v_2v_3, \dots, v_{d-2}v_{d-1}\}$. Let u be any vertex of G. If $u = v_i (1 \leq i \leq d-1)$, then u lies on the $e_1 - e_d$geodesic $P_1: v_1, v_2, \dots, v_{d-1}$.If $u \neq vi (1 \leq i \leq d-1)$, then u is incident with an edge of S. Therefore, S is an edge-to-vertex strong geodetic set of G. Consequently, $g_{ev}(G) \leq |S| = m - d + 2$. ■

Remark 2.22. The bound in Theorem 2.21 is sharp. For the star $G = K_{1,m} (m \geq 2)$, $d = 2$ and $sg_{ev}(G) = m$, by Theorem 2.10 so that $sg_{ev}(G) = m - d + 2$.

We give below a characterization theorem for trees.

Theorem 2.23. For any nontrivial tree $T$with $m \geq 2$, $sg_{ev}(T) = m - d + 2$if and only if$T$ is a caterpillar.

Proof. Let $P: x = x_0, x_1, x_2, \dots, x_{d-1}, x_d = y$ be a diametral path of length d. Let $e_i = x_{i-1}x_i (1 \leq i \leq d)$ be the edges of the diametral path P. Let k be the number of end edges of T and l be the number of internal edges of T other than $e_i (2 \leq i \leq d-1)$.Then $d - 2 + l + k = m$. By Theorem2.10, $sg_{ev}(T) = k$ and so $sg_{ev}(T) = m - d + 2 - l$.Hence $sg_{ev}(T) = m - d + 2$ if and only if $l = 0$, if and only if all internal vertices of$T$ lie on the diametral path P, if and only if$T$ is a caterpillar. ■

Theorem 2.24. For any connected graph $G$with $m \geq 3$, $sg_{ev}(G) = m - 1$ if and only if G is either C_3 or a double star.

Proof. If $G$is C_3, then $sg_{ev}(G) = 2 = m - 1$. If G is a double star, then by Theorem 2.10, $sg_{ev}(G) = m - 1$. Conversely, let $sg_{ev}(G) = m - 1$. Let $m = 3$. If G is a tree, then $G = P_4 or\ K_{1,3}$. For $G = K_{1,3}$, by Theorem 2.18, $sg_{ev}(G) = 3 = m$, which is a contradiction. If $G = P_4$, it is a double star and by Theorem 2.10, $sg_{ev}(G) = 2 = m - 1$. If G is not a tree, then $G = C_3$, which satisfies the requirements of the theorem. Thus the theorem follows.

Let $m \geq 4$. If G is not a tree, then by Theorem 2.19, $sg_{ev}(G) \leq m - 2$, which is a contradiction. Hence G is a tree. Now it follows from Theorem 2.21 that $d \leq 3$. Therefore $d = 2$ or 3. If $d = 2$, then G is the star $K_{1,m}$. By Theorem 2.10, $sg_{ev}(G) = m$, which is contradiction to the hypothesis. If $d = 3$, then G is a double star, which satisfies the requirements of the theorem. Thus the theorem is proved. ■

3. Realization results

In the following, we give some realization results.

The following theorem shows the existence of the edge-to-vertex strong geodetic number of a graph.

Theorem 3.1. For each pair of integers k and $m$with $2 \leq k \leq m$, there exists a connected graph G of order $m + 1$ and size m with $sg_{ev}(G) = k$.

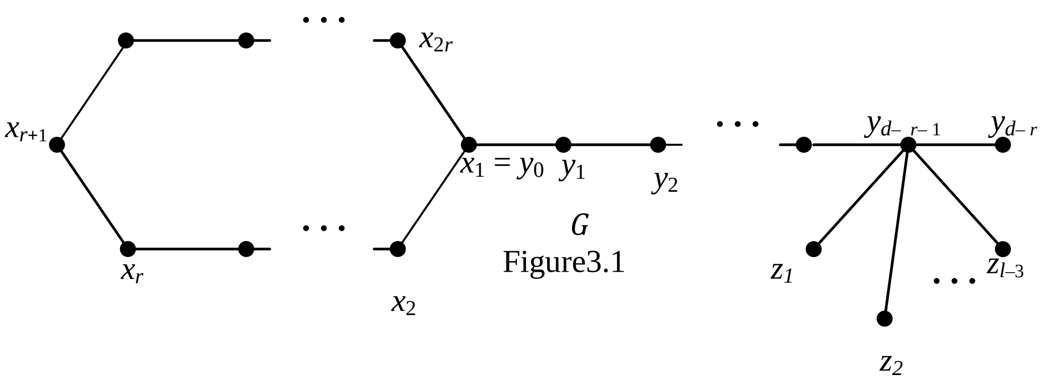
Proof. For $2 \leq k \leq m$, let P be a path of order $m–k + 3$. Let G be the graph obtainedfrom P by adding $k–2$ new vertices to P and joining them to any cut-vertex of

P.Clearly, G is a tree of order $m+1$ and size m with k end-edges and so by Theorem2.10, $sg_{ev}(G) = k$. ■

For every connected graph, $radG \leq diamG \leq 2radG$.Ostrand[10] showed that every two positive integers a and$b$ with $a \leq b \leq 2a$ are realizable as the radius and diameter, respectively, of some connected graph. Now, Ostrand's theorem can be extended so that the edge - to- vertex strong geodetic number can also be prescribed, when $a \leq b < 2a$.

Theorem 3.2. For positive integers $r, d$and $l \geq 3$ with $r \leq d < 2r$, there exists a connected graph $G$with $radG = r$, $diamG = d$ and $sg_{ev}(G) = l$.

Proof. When $r = 1$, let $G = K_{1,l}$. By Theorem 2.10, $sg_{ev}(G) = l$. Let $r \geq 2$.Let $C_{2r}: x_1, x_2, \ldots, x_{2r}, x_1$be a cycle of order $2r$ and let $P_{d-r+1}: y_0, y_1, y_2, \ldots, y_{d-r}$ be a pathof order $d–r + 1$. Let H be the graph obtained from C_{2r} and y_0 in P_{d-r+1} by identifying x_1 in C_{2r} and $y_0$in P_{d-r+1}. Now, add $(l–3)$ new vertices $z_1, z_2, \ldots, z_{l-3}$ to H and join each vertex $z_i (1 \leq i \leq l–3)$ to the vertex y_{d-r+1} and obtain the graph G of Figure 3.1. Then $radG = r$and $diamG = d$. Let $S = \{y_{d-r-1}z_1, y_{d-r-1}z_2, \ldots, y_{d-r-1}z_{l-3}, y_{d-r-1}, y_{d-r}\}$ be the set of end-edges of G. By Corollary 2.8, S is contained in every edge-to-vertex strong geodetic set of G. It is clear that S is not an edge-to-vertex strong geodetic set of G. It is also seen that $S \cup \{e\}$, where $e \in E(G) \setminus S$ is not an edge-to-vertex strong geodetic set of G. However, the set $S_1 = S \cup \{x_r x_{r+1}, x_{r+1} x_{r+2}\}$ is an edge-to-vertex strong geodetic set of G so that $sg_{ev}(G) = l– 3 + 3 = l$. ■



G
Figure3.1

In view of Theorem 2.4, we have the following realization result.

Theorem.3.3. For any positive integers a and b with $2 \leq a < b$and $b > 2a$, there exists a graph G such that $g_{ev}(G) = a$ and $sg_{ev}(G) = b$.

Proof. Let $P: x, y$ be a path on two vertices and $P_i: u_i, v_i (1 \leq i \leq b - a + 1)$ be a copy of a path on two vertices. Let G be the graph obtained from P and $P_i (1 \leq i \leq b - a + 1)$ by adding the new vertices $z, z_1, z_2, \ldots\ldots z_{a-1}$ and introducing the edges $yu_i, zv_i (1 \leq i \leq b - a + 1)$and $zz_i (1 \leq i \leq a - 1)$. The graph G is shown in Figure 3.2.

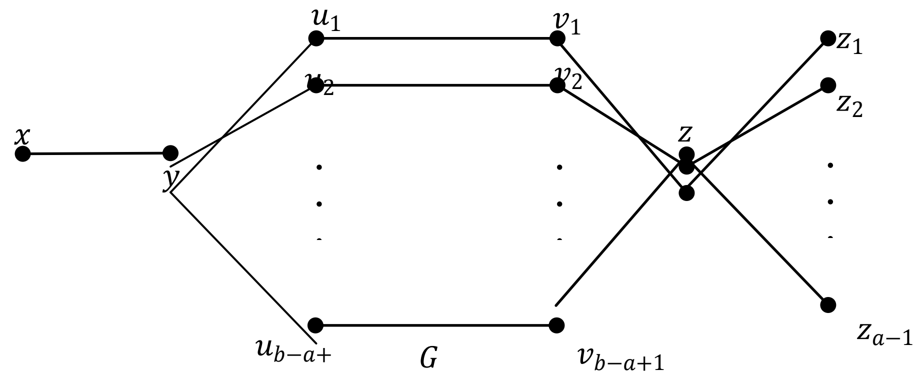


Figure3.2

First we prove that $g_{ev}(G) = a$. Let $Z = \{xy, zz_1, zz_2, \ldots . zz_{a-1}\}$ be the set of end edges of G. Then by Theorem1.1,Z is a subset of every edge-to-vertex geodetic set of G and so $g_{ev}(G) \geq a$. Since every vertex of G is incident with an element of Z or lies on a geodesic joining a pair of edges of Z, Z is an edge-to-vertex geodetic set of G so that $g_{ev}(G) = a$.

Next we prove that $sg_{ev}(G) = b$. Now fix the $xy - zz_1$geodesic $P: y, u_1, v_1, z$. By Corollary 2.8, Z is a subset of every edge-to-vertex strong geodetic set of G. Since the vertices $u_i, v_i (2 \leq i \leq b - a + 1)$ do not lie on a geodesic joining pair edges of Z, Z is not an edge-to-vertex strong geodetic set of G. It is easily observed that the edge $u_i v_i (2 \leq i \leq b - a + 1)$ belongs to every edge-to-vertex strong geodetic set of $G$and so $sg_{ev}(G) \geq b - a + a - 1 + 1 = b$.Let $S = Z \cup \{u_2 v_2, u_3 v_3, \ldots . , u_{b-a+1}$

$v_{b-a+1}\}$. Then S is an edge-to-vertex strong geodetic set of G so that $sg_{ev}(G) = b$. ■

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