SOME NEW CONTINUOUS FUNCTIONS IN INTUITIONISTIC TOPOLOGICAL SPACES

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ABSTRACT
The purpose of this article is to introduce and study the concept of intuitionistic $\hat{w}$-continuous functions in intuitionistic topological spaces. We investigate the fundamental properties of these functions and examine their relationships with other existing types of intuitionistic continuous functions. Our findings provide new insights into the structure and behavior of intuitionistic continuity, contributing to a broader understanding of intuitionistic topological spaces.

Keywords: Intuitionistic sets, $\mathfrak{T}\hat{w}$- closed, $\mathfrak{T}\hat{w}$- open, Intuitionistic $\hat{w}$-continuous.

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INTRODUCTION

The concept of intuitionistic sets in topological spaces was first introduced by Coker [2] in 1996. He also introduced the concept of intuitionistic points and investigated some fundamental properties of closed sets in intuitionistic topological spaces. The concept of intuitionistic continuity plays a major role in Intuitionistic Topology. In 2009, Younis Yaseen, J. and Asmaa Raouf, G.[5] introduced the concepts of Generalization Closed Sets and Generalized Continuity on Intuitionistic Topological Spaces. In 2012, Duraisamy, C. and Dhavamani, M. [9] introduced the notion of intuitionistic non-continuous functions in Intuitionistic Topological Spaces.

In this article, we define a new function, namely **intuitionistic $\hat{w}$- continuous** in intuitionistic topological spaces and discuss its properties. The following definitions and results are essential to proceed further.

2 PRELIMINARIES

Definition 2.1 [2]: Let $\mathcal{M}$ be a non-empty set. **An intuitionistic set** (shortly $\mathfrak{IS}$) $\mathcal{A}$ is an object having the form $\mathcal{A} = < \mathcal{M}, \mathcal{A}_1, \mathcal{A}_2 >$ Where $\mathcal{A}_1, \mathcal{A}_2$ are subsets of $\mathcal{M}$ satisfying $\mathcal{A}_1 \cap \mathcal{A}_2 = \varphi$. The set $\mathcal{A}_1$ called the set of members of $\mathcal{A}$, while $\mathcal{A}_2$ is called set of nonmembers of $\mathcal{A}$.

Definition 2.2 [2]: An intuitionistic topology (shortly $\mathfrak{I}\tau$) on a non-empty set $\mathcal{M}$ is a family $\mathfrak{I}\tau$ of $\mathfrak{I}$S in $\mathcal{M}$ satisfying following axioms.

1) $\tilde{\varphi}, \ \widetilde{\mathcal{M}} \in \mathfrak{I}\tau$
2) $G_1 \cap G_2 \in \mathfrak{I}\tau$, for any $G_1, G_2 \in \mathfrak{I}\tau$
c. $\cup G_i \in \mathfrak{I}\tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \mathfrak{I}\tau$.

Here the pair $(\mathcal{M}, \mathfrak{I}\tau)$ is called an **Intuitionistic Topological Space** (shortly $\mathfrak{I}TS$ $(\mathcal{M}, \mathfrak{I}\tau)$) and any $\mathfrak{I}S$ is called an **Intuitionistic Open Set** (shortly $\mathfrak{I}OS$) in $\mathcal{M}$. The complement $\mathcal{A}^C$ of $\mathfrak{I}OS$ is called an **Intuitionistic Closed Set** (shortly $\mathfrak{I}CS$) in $\mathcal{M}$.

Definition 2.3 [2]: Let $(\mathcal{M}, \mathfrak{I}\tau_1)$, $(\mathbb{Y}, \mathfrak{I}\tau_2)$ be two nonempty sets and $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be a function.

(a) If $\mathfrak{B} = < \mathbb{Y}, \mathfrak{B}_1, \mathfrak{B}_2 >$ is a $\mathfrak{I}S$ in $\mathbb{Y}$, then the ***preimage*** of $\mathfrak{B}$ under $\mathfrak{F}$, denoted by $\mathfrak{F}^{-1}(\mathfrak{B})$ is the $\mathfrak{I}S$ in $\mathcal{M}$ defined by $\mathfrak{F}^{-1}(\mathfrak{B}) = < \mathcal{M}, \ \mathfrak{F}^{-1}(\mathfrak{B}_1), \ \mathfrak{F}^{-1}(\mathfrak{B}_2) >$.

(b) If $\mathcal{A} = < \mathcal{M}, \mathcal{A}_1, \mathcal{A}_2 >$ is $\mathfrak{I}S$ in $\mathcal{M}$, then the ***image*** of $\mathcal{A}$ under $\mathfrak{F}$, denoted by $\mathfrak{F}(\mathcal{A})$, is the $\mathfrak{I}S$ in $\mathbb{Y}$ defined by $\mathfrak{F}(\mathcal{A}) = < \mathbb{Y}, \mathfrak{F}(\mathcal{A}_1), \mathfrak{F} - (\mathcal{A}_2) >$, where $\mathfrak{F} - (\mathcal{A}_2) = (\mathfrak{F} ((\mathcal{A}_2)^c))^c$.

Corollary 2.4 [2]: Let $\mathcal{A}$, $\mathcal{A}_i (i \in J)$ be $\mathfrak{I}S$ in $\mathcal{M}$ and $\mathfrak{B}$, $\mathfrak{B}_j$ $(j \in K)$ be $\mathfrak{I}S$ in $\mathbb{Y}$ and $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be a function. Then

a. $\mathcal{A}_1 \subseteq \mathcal{A}_2 \Rightarrow \mathfrak{F}(\mathcal{A}_1) \subseteq \mathfrak{F}(\mathcal{A}_2)$
b. $\mathfrak{B}_1 \subseteq \mathfrak{B}_2 \Rightarrow \mathfrak{F}^{-1}(\mathfrak{B}_1) \subseteq \mathfrak{F}^{-1}(\mathfrak{B}_2)$
c. $\mathcal{A} \subseteq \mathfrak{F}^{-1}(\mathfrak{F} (\mathcal{A}))$ and if $\mathfrak{F}$ is injective, then $\mathcal{A} = \mathfrak{F}^{-1}(\mathfrak{F} (\mathcal{A}))$.
d. $\mathfrak{F} (\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{B}$ and if $\mathfrak{F}$ is surjective, then $\mathfrak{F} (\mathfrak{F}^{-1} (\mathfrak{B})) = \mathfrak{B}$.
e. $\mathfrak{F}^{-1}(\cup \mathfrak{B}_j) = \cup \ \mathfrak{F}^{-1}(\mathfrak{B}_j)$
f. $\mathfrak{F}^{-1}(\cap \mathfrak{B}_j) = \cap \ \mathfrak{F}^{-1}(\mathfrak{B}_j)$
g. $\mathfrak{F} (\cup \mathcal{A}_i) = \cup \ \mathfrak{F}(\mathcal{A}_i)$
h. $\mathfrak{F} (\cap \mathcal{A}_i) \subseteq \cap \ \mathfrak{F}(\mathcal{A}_i)$, and if $\mathfrak{F}$ is injective, then $\mathfrak{F}(\cap \mathcal{A}_i) = \cap \ \mathfrak{F}(\mathcal{A}_i)$
i. $\mathfrak{F}^{-1}(\tilde{\mathbb{y}}) = \widetilde{\mathcal{M}}$
j. $\mathfrak{F}^{-1}(\tilde{\varphi}) = \tilde{\varphi}$
k. $\mathfrak{F}(\widetilde{\mathcal{M}}) = \widetilde{\mathbb{Y}}$, if $\mathfrak{F}$ is onto
l. $\mathfrak{F}(\tilde{\varphi}) = \tilde{\varphi}$
m. If $\mathfrak{F}$ is onto, then $\overline{\mathfrak{F}(\mathcal{A})} \subseteq \mathfrak{F}(\bar{\mathcal{A}})$ and if, furthermore, $\mathfrak{F}$ is 1-1, we have $\overline{\mathfrak{F}(\mathcal{A})} = \mathfrak{F}(\bar{\mathcal{A}})$.
n. $\mathfrak{F}^{-1}(\overline{\mathfrak{B}}) = \overline{\mathfrak{F}^{-1}(\mathfrak{B})}$
o. $\mathfrak{B}_1 \sqsubset \mathfrak{B}_2 \Rightarrow \mathfrak{F}^{-1}(\mathfrak{B}_1) \sqsubset \mathfrak{F}^{-1}(\mathfrak{B}_2)$.

Definition 2.5[2]: Let $(\mathcal{M}, \mathfrak{I}\tau_1)$ and $(\mathbb{Y}, \mathfrak{I}\tau_2)$ be two $\mathfrak{I}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be a function. Then $\mathfrak{F}$ is said to be an **intuitionistic continuous shortly $\mathfrak{I}$- continuous)** if the inverse image of every intuitionistic set of $\mathbb{Y}$ is intuitionistic set of $\mathcal{M}$.

Definition 2.6 [8]: Let $(\mathcal{M}, \mathfrak{I}\tau_1)$ and $(\mathbb{Y}, \mathfrak{I}\tau_2)$ be two $\mathfrak{I}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be called an **intuitionistic semi closed continuous (shortly $\mathfrak{I}s$ -CS continuous)** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{I}$-semi closed in $\mathcal{M}$. **Definition 2.7 [5]:** Let $(\mathcal{M}, \mathfrak{I}\tau_1)$ and $(\mathbb{Y}, \mathfrak{I}\tau_2)$ be two $\mathfrak{I}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \to (\mathbb{Y}, \mathfrak{I}\tau_2)$ be called an **intuitionistic g- continuous (shortly $\mathfrak{I}g$- continuous)** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{I}g$- closed in $\mathcal{M}$.

Definition 2.8[5]: Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two $\mathfrak{T}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be called an **intuitionistic sg- continuous (shortly $\mathfrak{T}sg$- continuous)** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{T}sg$- closed in $\mathcal{M}$.

Definition 2.9[6]: Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two $\mathfrak{T}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be called an **intuitionistic α- continuous (shortly $\mathfrak{T}\alpha$- continuous)** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{T}\alpha$- closed in $\mathcal{M}$.

Definition 2.10[5]: Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two $\mathfrak{T}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be called an **intuitionistic gp- continuous** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{T}gp$- closed in $\mathcal{M}$.

Definition 2.11[5]: Let $(\mathcal{M}, \mathfrak{T}\tau_1)$ and $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two $\mathfrak{T}TS$ and let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be called an **intuitionistic w- continuous (shortly $\mathfrak{T}w$- continuous)** if for every intuitionistic closed set V of $\mathbb{Y}$, $\mathfrak{F}^{-1}(V)$ is $\mathfrak{T}w$- closed in $\mathcal{M}$.

3. INTUITIONISTIC $\widehat{w}$-CONTINUOUS FUNCTIONS

Definition 3.1: Let $(\mathcal{M}, \mathfrak{T}\tau_1)$, $(\mathbb{Y}, \mathfrak{T}\tau_2)$ be two $\mathfrak{T}TS$. Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be a function. Then $\mathfrak{F}$ is said to be an **intuitionistic $\widehat{w}$-continuous** function (shortly **$\mathfrak{T}\widehat{w}$-continuous**), if the inverse image of every intuitionistic closed set in $\mathbb{Y}$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$.

Example 3.2: Let $\mathcal{M} = \{k, g\}$ with the family $\mathfrak{T}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \{k\}, \varphi>, <\mathcal{M}, \{k\}, \{g\}>\}$. $\mathfrak{T}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \{g\}>, <\mathcal{M}, \{g\}, \varphi>, <\mathcal{M}, \{k\}, \{g\}>, <\mathcal{M}, \varphi, \{k\}>, <\mathcal{M}, \varphi, \varphi>\}$. Let $\mathbb{Y} = \{a, b\}$ with $\mathfrak{T}\tau_2 = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \{b\}, \varphi>, <\mathbb{Y}, \varphi, \{a\}>\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ as $\mathfrak{F}(k) = a$, $\mathfrak{F}(g) = b$. Then $\mathfrak{F}^{-1}(<\mathbb{Y}, \{b\}, \varphi>) = <\mathcal{M}, \{g\}, \varphi>$, $\mathfrak{F}^{-1}(<\mathbb{Y}, \varphi, \{a\}>) = <\mathcal{M}, \varphi, \{k\}>$ and $\mathfrak{F}^{-1}(\mathbb{Y}) = \mathcal{M}$. Hence inverse image of every intuitionistic closed set in $\mathbb{Y}$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- continuous function.

Theorem 3.3: Every $\mathfrak{T}$- continuous function is $\mathfrak{T}\widehat{w}$- continuous but not conversely.

Proof: Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be $\mathfrak{T}$- continuous. Let $\mathcal{A}$ be a $\mathfrak{T}$- closed set in $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}$- closed in $\mathcal{M}$. But every $\mathfrak{T}$- closed set is $\mathfrak{T}\widehat{w}$-closed set. Therefore, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- continuous.

Example 3.4: Let $\mathcal{M} = \{k, l\}$ with $\mathfrak{T}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 = <\mathcal{M}, \varphi, \{k\}>$, $\mathcal{A}_2 = <\mathcal{M}, \{l\}, \varphi>$. $\mathfrak{T}\widehat{w}$- CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \{l\}, \varphi>, <\mathcal{M}, \varphi, \{l\}>, <\mathcal{M}, \{l\}, \{k\}>\}$. $\mathfrak{T}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \varphi, \{l\}>, <\mathcal{M}, \{l\}, \varphi>\}$. Let $\mathbb{Y} = \{a, b\}$ with $\mathfrak{T}\tau_2 = \{\tilde{\varphi}, \tilde{\mathbb{y}}, <\mathbb{Y}, \{b\}, \{a\}>, <\mathbb{Y}, \varphi, \{a\}>\}$. $\mathfrak{T}$-CS $(\mathbb{Y}, \mathfrak{T}\tau_2) = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \{a\}, \varphi>, <\mathbb{Y}, \{a\}, \{b\}>\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ as $\mathfrak{F}(k) = b$, $\mathfrak{F}(l) = a$. Here $\mathfrak{F}$ is not $\mathfrak{T}$- continuous, since $\mathfrak{F}^{-1}(<\mathbb{Y}, \{a\}, \{b\}>) = <\mathcal{M}, \{l\}, \{k\}>$ is not $\mathfrak{T}$-closed in $(\mathcal{M}, \mathfrak{T}\tau_1)$.

Theorem 3.5: Every $\mathfrak{T}g$- continuous function is $\mathfrak{T}\widehat{w}$-continuous but not conversely. **Proof:** Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be $\mathfrak{T}g$- continuous. Let $\mathcal{A}$ be $\mathfrak{T}$- closed set of $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}g$- closed in $\mathcal{M}$. But every $\mathfrak{T}g$- closed set is $\mathfrak{T}\widehat{w}$-closed set. Therefore, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$-continuous.

Example 3.6: Let $\mathcal{M} = \{a, b, c\}$ with $\mathfrak{T}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5, \mathcal{A}_6\}$ where $\mathcal{A}_1 = <\mathcal{M}, \varphi, \varphi>$, $\mathcal{A}_2 = <\mathcal{M}, \varphi, \{a\}>$, $\mathcal{A}_3 = <\mathcal{M}, \varphi, \{c\}>$, $\mathcal{A}_4 = <\mathcal{M}, \{b\}, \varphi>$, $\mathcal{A}_5 = <\mathcal{M}, \varphi, \{a, c\}>$, $\mathcal{A}_6 = <\mathcal{M}, \{b\}, \{a\}>$. $\mathfrak{T}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \varphi, \{a\}>, <\mathcal{M}, \varphi, \{c\}>, <\mathcal{M}, \{b\}, \varphi>, <\mathcal{M}, \{b\}, \{c\}>, <\mathcal{M}, \{b\}, \{a\}>, <\mathcal{M}, \{a, b\}, \{c\}>, <\mathcal{M}, \{b, c\}, \{a\}>, <\mathcal{M}, \{b\}, \{a, c\}>, <\mathcal{M}, \{a, b\}, \varphi>, <\mathcal{M}, \{b, c\}, \varphi>, <\mathcal{M}, \varphi, \{a, c\}>\}$. $\mathfrak{T}g$- CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \varphi, \{a\}>, <\mathcal{M}, \varphi, \{c\}>, <\mathcal{M}, \{b\}, \varphi>, <\mathcal{M}, \{b\}, \{c\}>, <\mathcal{M}, \{b\}, \{a\}>, <\mathcal{M}, \varphi, \{a, c\}>, <\mathcal{M}, \{a, b\}, \varphi>\}$. Let $\mathbb{Y} = \{p, y, u\}$ with $\mathfrak{T}\tau_2 = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \varphi, \{y, u\}>, <\mathbb{Y}, \{p, u\}, \varphi>\}$. $\mathfrak{T}$- CS $(\mathbb{Y}, \mathfrak{T}\tau_2) = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \{y, u\}, \varphi>, <\mathbb{Y}, \varphi, \{p, u\}>\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ as $\mathfrak{F}(a) = p$, $\mathfrak{F}(b) = y$, $\mathfrak{F}(c) = u$. Here $\mathfrak{F}^{-1}(<\mathbb{Y}, \{y, u\}, \varphi>) = <\mathcal{M}, \{b, c\}, \varphi>$ is not $\mathfrak{T}g$-closed in $(\mathcal{M}, \mathfrak{T}\tau_1)$.

Theorem 3.7: Every $\mathfrak{T}s$- continuous function is $\mathfrak{T}\widehat{w}$-continuous but not conversely.

Proof: Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ be $\mathfrak{T}s$- continuous. Let $\mathcal{A}$ be $\mathfrak{T}$- closed set in $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}s$- closed in $\mathcal{M}$. Since every $\mathfrak{T}s$- closed set is $\mathfrak{T}\widehat{w}$-closed set, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$-continuous.

Example 3.8: Let $\mathcal{M} = \{\mathfrak{f}, \mathfrak{q}\}$ with $\mathfrak{T}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, L_1, L_2, L_3, L_4\}$ where $L_1 = <\mathcal{M}, \varphi, \varphi>$, $L_2 = <\mathcal{M}, \{\mathfrak{f}\}, \varphi>$, $L_3 = <\mathcal{M}, \varphi, \{\mathfrak{q}\}>$, $L_4 = <\mathcal{M}, \{\mathfrak{f}\}, \{\mathfrak{q}\}>$. $\mathfrak{T}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>, <\mathcal{M}, \{\mathfrak{f}\}, \varphi>, <\mathcal{M}, \{\mathfrak{q}\}, \varphi>, <\mathcal{M}, \{\mathfrak{q}\}, \{\mathfrak{f}\}>\}$. $\mathfrak{T}s$- CS $(\mathcal{M}, \mathfrak{T}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, <\mathcal{M}, \varphi, \varphi>\}$. Let $\mathbb{Y} = \{\mathfrak{f}, \mathfrak{q}\}$ with $\mathfrak{T}\tau_2 = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \varphi, \varphi>, <\mathbb{Y}, \varphi, \{\mathfrak{q}\}>\}$. $\mathfrak{T}$- CS $(\mathbb{Y}, \mathfrak{T}\tau_2) = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, <\mathbb{Y}, \varphi, \varphi>, <\mathbb{Y}, \{\mathfrak{q}\}, \varphi>\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ as $\mathfrak{F}(\mathfrak{f}) = \mathfrak{q}$, $\mathfrak{F}(\mathfrak{q}) = \mathfrak{f}$. Here $\mathfrak{F}^{-1}(<\mathbb{Y}, \{\mathfrak{q}\}, \varphi>) = <\mathcal{M}, \{\mathfrak{f}\}, \varphi>$ is not $\mathfrak{T}s$-closed in $(\mathcal{M}, \mathfrak{T}\tau_1)$.

Theorem 3.9: Every $\mathfrak{I}sg$- continuous function is $\mathfrak{I}\widehat{w}$-continuous but not conversely.
Proof: Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ be $\mathfrak{I}sg$- continuous. Let $\mathcal{A}$ be $\mathfrak{I}$- closed set in $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}sg$- closed in $\mathcal{M}$. Since every $\mathfrak{I}sg$- closed set is $\mathfrak{I}\widehat{w}$-closed set, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{I}\widehat{w}$- continuous.
Example 3.10: Let $\mathcal{M} = \{i, j, k\}$ with $\mathfrak{I}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, C_1, C_2, C_3, C_4, C_5\}$ where $C_1 = < \mathcal{M}, \varphi, \varphi >, C_2 = < \mathcal{M}, \varphi, \{i\} >, C_3 = < \mathcal{M}, \varphi, \{k\} >, C_4 = < \mathcal{M}, \{j\}, \varphi >$, $C_5 = < \mathcal{M}, \varphi, \{i, k\} >$. $\mathfrak{I}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{I}\tau_1) = \{ \tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{i\} >, < \mathcal{M}, \varphi, \{k\} >, < \mathcal{M}, \{j\}, \varphi >, < \mathcal{M}, \{j\}, \{k\} >, < \mathcal{M}, \{j\}, \{i\} >, < \mathcal{M}, \{i, j\}, \{k\} >, < \mathcal{M}, \{j, k\}, \{i\} >, < \mathcal{M}, \{j\}, \{i, k\} >, < \mathcal{M}, \{i, j\}, \varphi >, < \mathcal{M}, \{j, k\}, \varphi >, < \mathcal{M}, \varphi, \{i, k\} >\}$. $\mathfrak{I}sg$- CS $(\mathcal{M}, \mathfrak{I}\tau_1) = \{ \tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{i\} >, < \mathcal{M}, \varphi, \{k\} >, < \mathcal{M}, \{j\}, \varphi >, < \mathcal{M}, \{j\}, \{k\} >, < \mathcal{M}, \{j\}, \{i\} >, < \mathcal{M}, \varphi, \{i, k\} >, < \mathcal{M}, \{i, j\}, \varphi >\}$. Let $\mathbb{Y} = \{\ell, m, n\}$ with $\mathfrak{I}\tau_2 = \{ \tilde{\varphi}, \widetilde{\mathbb{Y}}, < \mathbb{Y}, \varphi, \{m, n\} >, < \mathbb{Y}, \{\ell, n\}, \varphi >\}$. $\mathfrak{I}$- CS $(\mathbb{Y}, \mathfrak{I}\tau_2) = \{ \tilde{\varphi}, \widetilde{\mathbb{Y}}, < \mathbb{Y}, \{m, n\}, \varphi >, < \mathbb{Y}, \varphi, \{\ell, n\} >\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ as $\mathfrak{F}(i) = \ell, \mathfrak{F}(j) = m, \mathfrak{F}(k) = n$. Here $\mathfrak{F}^{-1}$ $(< \mathbb{Y}, \{m, n\}, \varphi >) = < \mathcal{M}, \{j, k\}, \varphi >$ is not $\mathfrak{I}sg$-closed in $(\mathcal{M}, \mathfrak{I}\tau_1)$.
Theorem 3.11: Every $\mathfrak{I}\alpha$- continuous function is $\mathfrak{I}\widehat{w}$-continuous but not conversely.
Proof: Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ be $\mathfrak{I}\alpha$- continuous. Let $\mathcal{A}$ be $\mathfrak{I}$- closed set in $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}\alpha$- closed in $\mathcal{M}$. Since every $\mathfrak{I}\alpha$- closed set is $\mathfrak{I}\widehat{w}$-closed set, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{I}\widehat{w}$- continuous.
Example 3.12: Let $\mathcal{M} = \{\mathfrak{x}, \mathfrak{w}, \mathfrak{y}\}$ with the family $\mathfrak{I}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \{\mathfrak{x}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \{\mathfrak{x}\} >, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{x}, \mathfrak{y}\} >\}$. $\mathfrak{I}\widehat{w}CS$ $(\mathcal{M}, \mathfrak{I}\tau_1) = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{x}\} >, < \mathcal{M}, \varphi, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \varphi >, < \mathcal{M}, \{\mathfrak{w}\}, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \{\mathfrak{x}\} >, < \mathcal{M}, \{\mathfrak{y}, \{\mathfrak{x}, \mathfrak{w}\}\} >, < \mathcal{M}, \{\mathfrak{w}, \}, \{\mathfrak{x}, \mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{x}, \mathfrak{w}\}, \varphi >, < \mathcal{M}, \{\mathfrak{w}, \mathfrak{y}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{x}, \mathfrak{y}\} >$. $\mathfrak{I}\alpha$-CS $(\mathcal{M}, \mathfrak{I}\tau_1) = \{ \tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{x}\} >, < \mathcal{M}, \varphi, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \varphi >, < \mathcal{M}, \{\mathfrak{w}\}, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}\}, \{\mathfrak{x}\} >, < \mathcal{M}, \{\mathfrak{y}\}, \{\mathfrak{x}\} >, < \mathcal{M}, \{\mathfrak{x}, \mathfrak{w}\}, \{\mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}, \}, \{\mathfrak{x}, \mathfrak{y}\} >, < \mathcal{M}, \{\mathfrak{w}, \mathfrak{y}\}, \{\mathfrak{x}\} >, < \mathcal{M}, \{\mathfrak{x}, \mathfrak{w}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{x}, \mathfrak{y}\} >, < \mathcal{M}, \varphi, \{\mathfrak{x}, \mathfrak{w}\} >, < \mathcal{M}, \{\mathfrak{w}, \mathfrak{y}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{w}, \mathfrak{y}\} >\}$. Let $\mathbb{Y} = \{ s, t, u \}$ with $\mathfrak{I}\tau_2 = \{\tilde{\varphi}, \widetilde{\mathbb{Y}}, < \mathbb{Y}, \{t, u\}, \{s\} > < \mathbb{Y}, \{u\}, \{s, t\} >, < \mathbb{Y}, \{t\}, \{s, u\} >\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ as $\mathfrak{F}(\mathfrak{x}) = s$, $\mathfrak{F}(\mathfrak{w}) = t$ and $\mathfrak{F}(\mathfrak{y}) = u$. Then $\mathfrak{F}^{-1}(< \mathbb{Y}, \{u\}, \{s, t\} >) = < \mathcal{M}, \{\mathfrak{y}\}, \{\mathfrak{x}, \mathfrak{w}\} >$ which is not $\mathfrak{I}\alpha$- closed set in $(\mathcal{M}, \mathfrak{I}\tau_1)$.Therefore, $\mathfrak{F}$ is not $\mathfrak{I}\alpha$- continuous.
Theorem 3.13: Every $\mathfrak{I}w$- continuous function is $\mathfrak{I}\widehat{w}$-continuous but not conversely.
Proof: Let $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ be $\mathfrak{I}w$- continuous. Let $\mathcal{A}$ be $\mathfrak{I}$- closed set in $\mathbb{Y}$. Then $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}w$- closed in $\mathcal{M}$. Since every $\mathfrak{I}w$- closed set is $\mathfrak{I}\widehat{w}$-closed set, $\mathfrak{F}^{-1}(\mathcal{A})$ is $\mathfrak{I}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}$ is $\mathfrak{I}\widehat{w}$-continuous.
Example 3.14: Let $\mathcal{M} = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$ with $\mathfrak{I}\tau_1 = \{\tilde{\varphi}, \widetilde{\mathcal{M}}, B_1, B_2, B_3, B_4, B_5, B_6\}$ where $B_1 = < \mathcal{M}, \varphi, \varphi >, B_2 = < \mathcal{M}, \varphi, \{\mathfrak{a}\} >$, $B_3 = < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{a}\} >$, $B_4 = < \mathcal{M}, \varphi, \{\mathfrak{c}\} >$, $B_5 = < \mathcal{M}, \{\mathfrak{b}\}, \varphi >, B_6 = < \mathcal{M}, \varphi, \{\mathfrak{a}, \mathfrak{c}\} >$. $\mathfrak{I}\widehat{w}$-CS $(\mathcal{M}, \mathfrak{I}\tau_1) = \{ \tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \{i, j\}, \{k\} >, < \mathcal{M}, \varphi, \{\mathfrak{a}\} >, < \mathcal{M}, \varphi, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \varphi >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{a}\} >, < \mathcal{M}, \{\mathfrak{c}\}, \{\mathfrak{a}\} >, < \mathcal{M}, \{\mathfrak{a}, \mathfrak{b}\}, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{a}, \mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}, \mathfrak{c}\}, \{\mathfrak{a}\} >, < \mathcal{M}, \{\mathfrak{a}, \mathfrak{b}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{a}, \mathfrak{b}\} >, < \mathcal{M}, \varphi, \{\mathfrak{b}, \mathfrak{c}\} >, < \mathcal{M}, \varphi, \{\mathfrak{a}, \mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}, \mathfrak{c}\}, \varphi >$. $\mathfrak{I}w$- CS $(\mathcal{M}, \mathfrak{I}\tau_1) = \{ \tilde{\varphi}, \widetilde{\mathcal{M}}, < \mathcal{M}, \varphi, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{a}\} >, < \mathcal{M}, \varphi, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \varphi >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{a}\} >, < \mathcal{M}, \{\mathfrak{a}, \mathfrak{b}\}, \{\mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{b}, \mathfrak{c}\}, \{\mathfrak{a}\} >, < \mathcal{M}, \{\mathfrak{b}\}, \{\mathfrak{a}, \mathfrak{c}\} >, < \mathcal{M}, \{\mathfrak{a}, \mathfrak{b}\}, \varphi >, < \mathcal{M}, \varphi, \{\mathfrak{a}, \mathfrak{c}\} >, < \mathcal{M}, \varphi, \{\mathfrak{a}, \mathfrak{b}\} >, \mathcal{M}, \{\mathfrak{b}, \mathfrak{c}\}, \varphi >\}$. Let $\mathbb{Y} = \{\ell, m, n\}$ with $\mathfrak{I}\tau_2 = \{ \tilde{\varphi}, \widetilde{\mathbb{Y}}, < \mathbb{Y}, \{\ell\}, \{m, n\} >, < \mathbb{Y}, \{\ell, m\}, \{n\} >, < \mathbb{Y}, \{\ell, n\}, \{m\}\}$. $\mathfrak{I}CS$ $(\mathbb{Y}, \mathfrak{I}\tau_2) = \{ \tilde{\varphi}, \widetilde{\mathbb{Y}}, < \mathbb{Y}, \{m, n\}, \{\ell\} >, \mathbb{Y}, \{n\}, \{\ell, m\} >, < \mathbb{Y}, \{m\}, \{\ell, n\} >\}$. Define $\mathfrak{F} : (\mathcal{M}, \mathfrak{I}\tau_1) \rightarrow (\mathbb{Y}, \mathfrak{I}\tau_2)$ as $\mathfrak{F}(\mathfrak{a}) = \ell, \mathfrak{F}(\mathfrak{b}) = m, \mathfrak{F}(\mathfrak{c}) = n$. Here $\mathfrak{F}$ is not $\mathfrak{I}w$- continuous since $\mathfrak{F}^{-1}$ $(< \mathbb{Y}, \{n\}, \{\ell, m\} >) = < \mathcal{M}, \{\mathfrak{c}\}, \{\mathfrak{a}, \mathfrak{b}\} >$ is not $\mathfrak{I}w$-closed in $(\mathcal{M}, \mathfrak{I}\tau_1)$.

Remark 3.15: The following diagram shows the relationships of $\mathfrak{I}\widehat{w}$- continuous with other existing intuitionistic continuous functions.

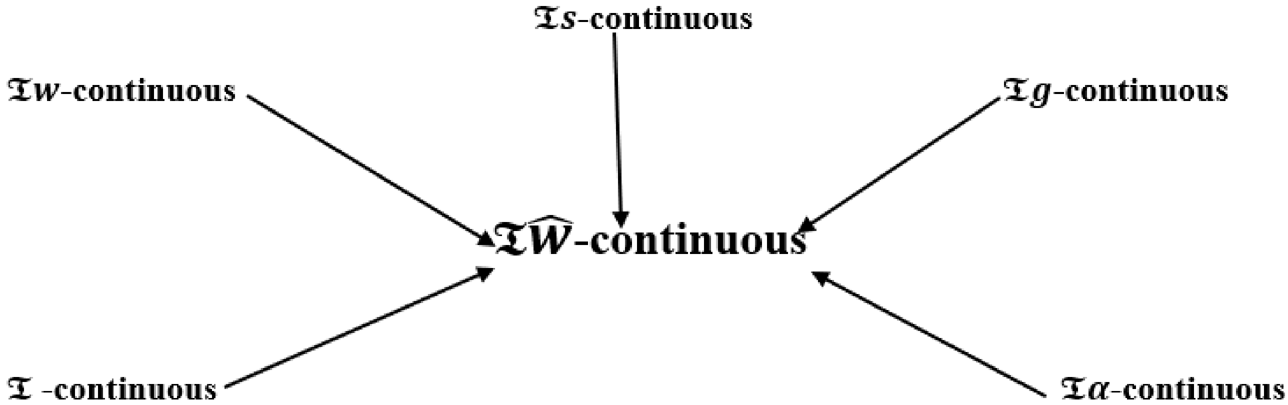

Fig. 1

Theorem 3.16: A function $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ is a $\mathfrak{T}\widehat{w}$-continuous iff the inverse image of every $\mathfrak{T}$-open set in $\mathbb{Y}$ is $\mathfrak{T}\widehat{w}$- open in $\mathcal{M}$.

Proof: Let $\mathcal{K}$ be a $\mathfrak{T}$- open set in $\mathbb{Y}$. Then $\mathbb{Y} - \mathcal{K}$ is $\mathfrak{T}$- closed set in $\mathbb{Y}$. Since $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$-continuous, $\mathfrak{F}^{-1}(\mathbb{Y} - \mathcal{K})$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. That is, $\mathfrak{F}^{-1}(\mathbb{Y} - \mathcal{K}) = \mathcal{M} - \mathfrak{F}^{-1}(\mathcal{K})$ is $\mathfrak{T}\widehat{w}$-closed in $\mathcal{M}$. Hence $\mathfrak{F}^{-1}(\mathcal{K})$ is $\mathfrak{T}\widehat{w}$-open in $(\mathcal{M}, \mathfrak{T}\tau_1)$. Conversely, let $\mathcal{L}$ be a $\mathfrak{T}$- closed set in $\mathbb{Y}$. Then $\mathbb{Y} - \mathcal{L}$ is $\mathfrak{T}$- open in $\mathbb{Y}$. Hence $\mathfrak{F}^{-1}(\mathbb{Y} - \mathcal{L}) = \mathcal{M} - \mathfrak{F}^{-1}(\mathcal{L})$ is $\mathfrak{T}\widehat{w}$- open in $\mathcal{M}$. Hence $\mathfrak{F}^{-1}(\mathcal{L})$ is $\mathfrak{T}\widehat{w}$- closed in $\mathcal{M}$ and $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$-continuous.

Theorem 3.17: A function $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ is a $\mathfrak{T}\widehat{w}$-continuous iff $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$ for every subset $\mathfrak{B}$ of $(\mathbb{Y}, \mathfrak{T}\tau_2)$.

Proof: By the given hypothesis $\mathfrak{B} \subseteq \mathbb{Y}$. Then $\mathfrak{T}int\,(\mathfrak{B})$ is $\mathfrak{T}$- open in $\mathbb{Y}$. Since $\mathfrak{F}$ is $\mathfrak{T}\widehat{w}$- continuous, $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B}))$ is $\mathfrak{T}\widehat{w}$- open in $\mathcal{M}$. Hence it follows that $\mathfrak{T}\widehat{w}\; int \quad \mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) = \mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B}))$. Also $\mathfrak{T}int\ (\mathfrak{B}) \subseteq \mathfrak{B}$. Then $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{B})$ which implies $\mathfrak{T}\widehat{w}\; int(\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B}))) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$. Hence $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$. Conversely, let $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$ for every subset $\mathfrak{B}$ of $\mathbb{Y}$. Let $\mathfrak{B}$ be $\mathfrak{T}$- open in $\mathbb{Y}$. Hence $\mathfrak{T}int\,(\mathfrak{B}) = \mathfrak{B}$. Given $\mathfrak{F}^{-1}(\mathfrak{T}int\,(\mathfrak{B})) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$, that is $\mathfrak{F}^{-1}(\mathfrak{B}) \subseteq \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$. Also $\mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{B})$. Hence it follows that $\mathfrak{F}^{-1}(\mathfrak{B}) = \mathfrak{T}\widehat{w}\; int\,(\mathfrak{F}^{-1}(\mathfrak{B}))$ which implies that $\mathfrak{F}^{-1}(\mathfrak{B})$ is $\mathfrak{T}\widehat{w}$- open in $\mathcal{M}$ for every subset $\mathfrak{B}$ of $\mathbb{Y}$. Hence $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}$-continuous.

Theorem 3.18: A function $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ is a $\mathfrak{T}\widehat{w}$-continuous iff $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$ for every subset $\mathfrak{B}$ of $(\mathbb{Y}, \mathfrak{T}\tau_2)$.

Proof: Let $\mathfrak{B} \subseteq \mathbb{Y}$ and $\mathfrak{T}cl(\mathfrak{B})$ be $\mathfrak{T}$- closed in $\mathbb{Y}$. Hence $\mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$ is $\mathfrak{T}\widehat{w}$- closed in $\mathcal{M}$. Therefore, $\mathfrak{T}\widehat{w}\; cl(\mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B})) = \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$. Since $\mathfrak{B} \subseteq \mathfrak{T}cl(\mathfrak{B})$, $\mathfrak{F}^{-1}(\mathfrak{B}) \subseteq \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$, $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{T}\widehat{w}\; cl(\mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B})) = \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$. Hence $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$. Conversely, let $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{T}cl(\mathfrak{B}))$ for every subset $\mathfrak{B} \subseteq \mathbb{Y}$. Let $\mathfrak{B}$ be $\mathfrak{T}$- closed in $\mathbb{Y}$, then $\mathfrak{T}cl(\mathfrak{B}) = \mathfrak{B}$. Therefore, the given hypothesis becomes $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) \subseteq \mathfrak{F}^{-1}(\mathfrak{B})$. But $\mathfrak{F}^{-1}(\mathfrak{B}) \subseteq \mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B}))$. Hence $\mathfrak{T}\widehat{w}\; cl\,(\mathfrak{F}^{-1}(\mathfrak{B})) = \mathfrak{F}^{-1}(\mathfrak{B})$. Thus $\mathfrak{F}^{-1}(\mathfrak{B})$ is $\mathfrak{T}\widehat{w}$- closed in $\mathcal{M}$ for every $\mathfrak{T}$- closed $\mathfrak{B}$ in $\mathbb{Y}$. Hence $\mathfrak{F} : (\mathcal{M}, \mathfrak{T}\tau_1) \to (\mathbb{Y}, \mathfrak{T}\tau_2)$ is $\mathfrak{T}\widehat{w}$-continuous.

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