A DIFFERENT APPROACH ON FUZZY ERLANG QUEUING MODEL

P. Yasodai[1], W. Ritha[2]

Article History: Received: 12.12.2022 **Revised:** 29.01.2023 **Accepted:** 15.03.2023

Abstract

In many real-world scenarios, fuzzy queuing models have a greater impact than naturally occurring crisp queues. In this research paper, the mathematical models in which the L-R method and the Alpha cut method are considered the major ones. A specific mathematical model is formulated with the given assumptions. The formulation is derived for LR fuzzy numbers. All fuzzy quantities are taken as triangular fuzzy numbers. The solutions are established, and the fuzzy results determine the impact of fuzzy on real-life problems.

Keywords: Fuzzy queuing model, Triangular fuzzy number, L-R fuzzy number, Erlang-k distribution, L-R method, Alpha-cuts Method, Interval arithmetic, Significant measures.

[1,2]Department of Mathematics Holy Cross College (Autonomous) Affiliated to Bharathidasan University Tiruchirappalli – 620 002 Tamilnadu, India

DOI: 10.31838/ecb/2023.12.s2.124

1. Introduction

Erlang developed the queuing theory in 1903 as a consequence of high telephone traffic. In practical situations service rate and arrival rate have many uncertainties. Such uncertainties can be resolved by using fuzzy set theory. Fuzzy queuing model was first introduced by R.J. Li and E.S. Lee [6]. Further developed this model by many authors. Using Zadeh's Extension Principle, W. Ritha and Lilly Robert [9] proposed fuzzy queues with priority discipline. Fuzzy N policy queues with infinite capacity have been researched by W. Ritha and Menon [10]. R. Srinivasan [11] exectued DSW Algorithm for the brief description of his fuzzy queuing model. Kaufmann [5] was the first to propose the L-R technique, which may be utilised to establish the performance measures of fuzzy queues. Later, Zimmermann [12] refined this L-R method.

J.P. Mukeba, R. Mabela, and B. Ulungu [7] used the L-R technique to generate performance metrics for the Single server fuzzy queuing model. Using the alpha-cuts method, the majority of these studies are focused to discovering system performance measurements. In this instance, we use a novel method dubbed the L-R method, which is fundamentally based on L-R fuzzy arithmetic, to compute the number of clients and the waiting time in the Multiserver Erlang queuing model $FM\,/\,FE_K\,/\,S:\infty\,/\,FCFS$. A Numerical illustration provided an efficiency analysis of this model.

2. Preliminaries

2.1. Fuzzy set

A fuzzy set $A=\{(x,\phi_{\tilde{A}}(x):x\in E)\}$ is concluded by a membership function $\phi_{\tilde{A}}$ mapping from elements of a universe of discourse E to the unit interval [0,1] (i.e) $\phi_{\tilde{A}}:E\to[0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\phi_{\tilde{A}}(x)$ is called the membership value of $x\in E$ in the fuzzy set $\tilde{A}$.

2.2. α – cut

Let $\tilde{A}$ be a subset in the universe E. The α – cut of $\tilde{A}$ noted $\tilde{A}_\alpha$ is a classical set defined as follows: $\tilde{A}=\{x\in E\,/\,\phi_{\tilde{A}}(x)\ge\alpha\}$

2.3. L-R fuzzy number:

A fuzzy number $\tilde{P}$ is said to be a L-R fuzzy number if only if there exists three real numbers p, $r_1>0, r_2>r_1$ and two positive, continuous and decreasing functions L and R from ℝ to [0,1], such that

$$L(0)=R(0)=1$$
$$L(1)=0,\qquad L(x)>0,\qquad \lim_{x\to\infty}L(x)=0$$
$$R(1)=0,\qquad R(x)>0,\qquad \lim_{x\to\infty}R(x)=0$$

$$\eta_{\tilde{P}}(x)=\begin{cases} L\left(\dfrac{p-x}{r_1}\right) & if\ \ x\in[p-r_1,p] \\ R\left(\dfrac{x-p}{r_2}\right) & if\ \ x\in[p,p+r_2] \\ 0 & otherwise \end{cases}$$

The L-R representation of the fuzzy numbers $\tilde{P}$ is $\tilde{P}=\langle p,r_1,r_2\rangle_{LR}$, where p is called the mean/mode/modal value of $\tilde{P}$, r_1 and r_2 are the left spread and right spread of $\tilde{P}$. Conventionally, $\langle p,0,0\rangle_{LR}$ is the ordinary real number 'p'; also called fuzzy singleton.

2.4. Arithmetic of L-R fuzzy Numbers:

Suppose there are two L-R fuzzy numbers of the same type $\tilde{P}=\langle p,r_1,r_2\rangle_{LR}$ and $\tilde{Q}=\langle q,r_3,r_4\rangle_{LR}$

1. $\tilde{P}+\tilde{Q}=\langle p+q, r_1+r_3, r_2+r_4\rangle_{LR}$
2. $\tilde{P}-\tilde{Q}=\langle p-q, r_1+r_4, r_2+r_3\rangle_{LR}$
3. $\tilde{P}.\tilde{Q}=\langle p.q,\ pr_3+q\,r_1-r_1r_3,\ pr_4+qr_2+r_2r_4\rangle_{LR}$

4. $$\frac{\tilde{P}}{\tilde{Q}} = \frac{\langle p, r_1, r_2 \rangle_{LR}}{\langle q, r_3, r_4 \rangle_{LR}} = \left\langle \frac{p}{q}, \frac{pr_4}{q(q+r_4)} + \frac{r_1}{q} - \frac{r_1 r_4}{q(q+r_4)}, \frac{pr_3}{q(q-r_3)} + \frac{r_2}{q} + \frac{r_2 r_3}{q(q-r_3)} \right\rangle_{LR}$$

2.5. Triangular fuzzy number:

A fuzzy number $\tilde{P}$ is said to be triangular fuzzy number if and only if there exists three real numbers $s_1 < s_2 < s_3$, such that:

$$\phi_{\tilde{P}}(x) = \begin{cases} \left(\dfrac{x - s_1}{s_2 - s_1}\right), & if \ \ s_1 \le x \le s_2 \\ \left(\dfrac{s_3 - x}{s_3 - s_2}\right), & if \ \ s_2 \le x \le s_3 \\ 0, & otherwise \end{cases}$$

Denoted by $\tilde{P} = (s_1, s_2, s_3)$ or $\tilde{P} = (s_1 / s_2 / s_3)$

Remark:

A triangular fuzzy number $\tilde{P} = (s_1 / s_2 / s_3)$ is always an L-R fuzzy number.
In L-R representation,

$$\tilde{P} = (s_1 / s_2 / s_3) = \langle s_2, s_2 - s_1, s_3 - s_2 \rangle_{LR}$$

for $L(x) = R(x) = \max(0, 1 - x)$

Let T_1 and T_2 be two interval numbers defined by ordered pairs of real numbers with lower upper bounds $T_1 = [r_1, r_2],\ r_1 \le r_2$ and $T_2 = [r_3, r_4],\ r_3 \le r_4$. Define a general arithmetic property with the symbol $*$, where $* = [+, -, ., \div]$ symbolically the operation.
$T_1 * T_2 = [r_1, r_2] * [r_3, r_4]$ represents another interval. The interval calculation depends on the magnitude and signs of the elements r_1, r_2, r_3, r_4.

2.6. Interval analysis arithmetic:

i. $[r_1, r_2] + [r_3, r_4] = [r_1 + r_3, r_2 + r_4]$

ii. $[r_1, r_2] - [r_3, r_4] = [r_1 - r_4, r_2 - r_3]$

iii. $[r_1, r_2].[r_3, r_4] = [\max(r_1 r_3, r_1 r_4, r_2 r_3, r_2 r_4),\ \min(r_1 r_3, r_1 r_4, r_2 r_3, r_2 r_4)]$

iv. $[r_1, r_2] \div [r_3, r_4] = \left[\min\left(\frac{r_1}{r_3}, \frac{r_1}{r_4}, \frac{r_2}{r_3}, \frac{r_2}{r_4}\right),\ \max\left(\frac{r_1}{r_3}, \frac{r_1}{r_4}, \frac{r_2}{r_3}, \frac{r_2}{r_4}\right) \right]$ provided that $0 \notin [r_3, r_4]$.

v. $$\alpha[r_1, r_2] = \begin{cases} [\alpha r_1, \alpha r_2], & for\ \alpha > 0 \\ [\alpha r_2, \alpha r_1], & for\ \ \alpha < 0 \end{cases}$$

3. Mathematical Model Formulation

Consider a queuing system in k-phases multiple server facility, denoted by $FM / FE_K / S$ in which arrival occurs as exponential distribution with fuzzy rate $\tilde{\lambda}$ and service time according to Erlang's k distribution with fuzzy rate $\tilde{\mu}$. It consists of 's' number of service channels and a chain of 'k' similar stages gaining an average service time $\frac{1}{s\mu}$ individually. The service discipline is first come first served and the system capacity are infinite. The Execution proportions of this queuing model are

Expected number of customers waiting in the queue

$$\tilde{L}_q = \frac{(k+1)\tilde{\lambda}^2}{2ks\tilde{\mu}\left(s\tilde{\mu}-\tilde{\lambda}\right)}$$

Expected time a customer waits in the queue

$$\tilde{W}_q = \frac{(k+1)\tilde{\lambda}}{2ks\tilde{\mu}\left(s\tilde{\mu}-\tilde{\lambda}\right)}$$

Expected number of customers waiting in the system

$$\tilde{L}_s = \tilde{L}_q + \frac{\tilde{\lambda}}{s\tilde{\mu}}$$

Expected time a customer waits in the system

$$\tilde{W}_s = \tilde{W}_q + \frac{1}{s\tilde{\mu}}$$

3.1. Expected "Number of Customers" and their "Waiting Time" in a Fuzzy queue: L-R Method Interpretation

Consider a multiple server Erlang queue $FM / FE_K / S$ whose arrival and service rates are positive triangular fuzzy numbers noted respectively $\tilde{\lambda} = (f_1 / f_2 / f_3)$ and $\tilde{\mu} = (g_1 / g_2 / g_3)$ with $f_3 < g_1$.

(i) Number of Customers

Number of customers in the queue $\tilde{N}_c$ can be estimated by replacing $\tilde{\lambda}$ and $\tilde{\mu}$ with their suitable L-R fuzzy values, $\tilde{\lambda} = \langle f_2,\ f_2 - f_1,\ f_3 - f_2\rangle_{LR}$ and $\tilde{\mu} = \langle g_2,\ g_2 - g_1,\ g_3 - g_2\rangle_{LR}$

in $\tilde{L}_q$.

$$\tilde{L}_q = \frac{(k+1)\tilde{\lambda}^2}{2ks\tilde{\mu}\left(s\tilde{\mu}-\tilde{\lambda}\right)}$$

$$\tilde{N}_c = \frac{(k+1)\langle f_2,\ f_2 - f_1,\ f_3 - f_2\rangle_{LR}\langle f_2,\ f_2 - f_1,\ f_3 - f_2\rangle_{LR}}{2ks\langle g_2,\ g_2 - g_1,\ g_3 - g_2\rangle_{LR}\left(s\langle g_2,\ g_2 - g_1,\ g_3 - g_2\rangle_{LR} - \langle f_2,\ f_2 - f_1,\ f_3 - f_2\rangle_{LR}\right)}$$

$$= \frac{(k+1)\left\langle f_2^{\,2},\ f_2^{\,2} - f_1^{\,2},\ f_3^{\,2} - f_2^{\,2}\right\rangle_{LR}}{2ks\left\langle g_2(sg_2 - f_2),\ g_2(sg_2 - f_2) - g_1(sg_1 - f_3),\ g_3(sg_3 - f_1) - \mu_2(sg_2 - f_2)\right\rangle_{LR}}$$

$$= \frac{(k+1)}{2ks}\left\langle \frac{f_2^{\,2}}{g_2(sg_2 - f_2)},\ \frac{f_2^{\,2}}{g_2(sg_2 - f_2)} - \frac{f_1^{\,2}}{g_3(sg_3 - f_1)},\ \frac{f_3^{\,2}}{g_1(sg_1 - f_3)} - \frac{f_2^{\,2}}{g_2(sg_2 - f_2)}\right\rangle_{LR}$$

$$= \langle N_1, N_2, N_3\rangle_{LR}$$

With

$$N_1 = \frac{(k+1)}{2ks}\left[\frac{f_2^{\,2}}{g_2(sg_2 - f_2)}\right]$$

$$N_2 = \frac{(k+1)}{2ks}\left[\frac{f_2^{\,2}}{g_2(sg_2 - f_2)} - \frac{f_1^{\,2}}{g_3(sg_3 - f_1)}\right] \text{ and}$$

$$N_3 = \frac{(k+1)}{2ks}\left[\frac{f_3^{\,2}}{g_1(sg_1 - f_3)} - \frac{f_2^{\,2}}{g_2(sg_2 - f_2)}\right]$$

Where N_1, N_2 and N_3 are the three positive real numbers which is the approximate values of mode, Left spread and right spread of $\tilde{N}$. The lower and upper bound of $\tilde{N}$ are $N_1 - N_2$ and $N_1 - N_3$ respectively.

(ii) Expected Waiting Tim

Expected waiting time in the queue $\tilde{T}_w$ can be computed by replacing $\tilde{\lambda}$ and $\tilde{\mu}$ with their suitable L-R fuzzy values such as $\tilde{\lambda} = \langle f_2, f_2 - f_1, f_3 - f_2 \rangle_{LR}$ & $\tilde{\mu} = \langle g_2, g_2 - g_1, g_3 - g_2 \rangle_{LR}$ in $\tilde{W}_q$, we get

$$\tilde{W}_q = \frac{(k+1)\tilde{\lambda}}{2ks\tilde{\mu}(s\tilde{\mu} - \tilde{\lambda})}$$

$$\tilde{T}_w = \frac{(k+1)\langle f_2, f_2 - f_1, f_3 - f_2 \rangle_{LR}}{2ks\langle g_2, g_2 - g_1, g_3 - g_2 \rangle_{LR}\left(s\langle g_2, g_2 - g_1, g_3 - g_2 \rangle_{LR} - \langle f_2, f_2 - f_1, f_3 - f_2 \rangle_{LR}\right)}$$

$$= \frac{(k+1)\langle f_2, f_2 - f_1, f_3 - f_2 \rangle_{LR}}{2ks\left\langle g_2(sg_2 - f_2), \mu_2(sg_2 - f_2) - g_1(sg_1 - f_3), g_3(sg_3 - f_1) - g_2(sg_2 - f_2) \right\rangle_{LR}}$$

$$= \frac{(k+1)}{2ks}\left\langle \frac{f_2}{g_2(sg_2 - f_2)}, \frac{f_2}{g_2(sg_2 - f_2)} - \frac{f_1}{g_3(sg_3 - f_1)}, \frac{f_3}{g_1(sg_1 - f_3)} - \frac{f_2}{g_2(sg_2 - f_2)} \right\rangle_{LR}$$

$$= \langle T_1, T_2, T_3 \rangle_{LR}$$

$$T_1 = \frac{(k+1)}{2ks}\left[\frac{f_2}{g_2(sg_2 - f_2)}\right]$$

With $T_2 = \frac{(k+1)}{2ks}\left[\frac{f_2}{g_2(sg_2 - f_2)} - \frac{f_1}{g_3(sg_3 - f_1)}\right]$ *and*

$$T_3 = \frac{(k+1)}{2ks}\left[\frac{f_3}{g_1(sg_1 - f_3)} - \frac{f_2}{g_2(sg_2 - f_2)}\right]$$

These three positive real numbers T_1, T_2 and T_3 are approximate values of the mode, Left spread and right spread of $\tilde{T}$.

4. Numerical Example

Consider a $FM / FE_K / S$ queue, where both the arrival rate and service rate are Triangular fuzzy numbers represented by $\tilde{\lambda} = (14/15/16)$ and $\tilde{\mu} = (17/18/19)$ with k=3 phases with the servers s=2. Computing the performance measures are as follows:

4.1. Alpha – Cut Method Solution

The alpha-cuts method comprises the following steps:

The alpha cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ are

$\lambda_\alpha = [\alpha + 14, -\alpha + 16]$ and

$\mu_\alpha = [\alpha + 17, -\alpha + 19]$

(i) The membership functions of L_q, L_s, W_s *and* W_q are not in conventional forms. Utilizing the Parametric Nonlinear Programming mathematical programming tool, this alpha-cuts method appeals to the problem (NLP). To solve this issue, performance measures must be in alpha-cut form.

(ii) Substitute $\tilde{\lambda} = \lambda_\alpha$ and $\tilde{\mu} = \mu_\alpha$ in characteristic metrics.

(a) Expected number of customers in the queue

$$\tilde{L}_{q_\alpha} = \frac{(k+1)\lambda_\alpha^{\ 2}}{2ks\mu_\alpha\left(s\mu_\alpha - \lambda_\alpha\right)}$$

$$= \frac{4[\alpha+14, -\alpha+16][\alpha+14, -\alpha+16]}{6\left(2[\alpha+17, -\alpha+19]\right)\left(2[\alpha+17, -\alpha+19]-[\alpha+14, -\alpha+16]\right)}\left(u\sin g\ parametric\ NLP\right)$$

$$= \left[\frac{2\left(\alpha^2+28\alpha+196\right)}{3\left(6\alpha^2-162\alpha+912\right)}, \frac{2\left(\alpha^2-32\alpha+256\right)}{3\left(6\alpha^2+138\alpha+612\right)}\right]$$

(b) Expected waiting time in the queue

$$\tilde{W}_{q_\alpha} = \frac{(k+1)\lambda_\alpha}{2ks\mu_\alpha\left(s\mu_\alpha - \lambda_\alpha\right)}$$

$$= \frac{4[\alpha+14, -\alpha+16]}{6\left(2[\alpha+17, -\alpha+19]\right)\left(2[\alpha+17, -\alpha+19]-[\alpha+14, -\alpha+16]\right)}\left(u\sin g\ parametric\ NLP\right)$$

$$= \left[\frac{2(\alpha+14)}{3\left(6\alpha^2-162\alpha+912\right)}, \frac{2(-\alpha+16)}{3\left(6\alpha^2+138\alpha+612\right)}\right]$$

(c) Expected number of customers in the system

$$\tilde{L}_{s_\alpha} = \tilde{L}_{q_\alpha} + \frac{\lambda_\alpha}{s\mu_\alpha}$$

$$= \left[\frac{2\left(\alpha^2+28\alpha+196\right)}{3\left(6\alpha^2-162\alpha+912\right)}, \frac{2\left(\alpha^2-32\alpha+256\right)}{3\left(6\alpha^2+138\alpha+612\right)}\right] + \frac{[\alpha+14, -\alpha+16]}{2[\alpha+17, -\alpha+19]}$$

$$= \left[\frac{2\left(\alpha^2+28\alpha+196\right)}{3\left(6\alpha^2-162\alpha+912\right)} + \frac{\alpha+14}{-2\alpha+38}, \frac{2\left(\alpha^2-32\alpha+256\right)}{3\left(6\alpha^2+138\alpha+612\right)} + \frac{-\alpha+16}{2\alpha+34}\right]$$

(d) Expected waiting time in the system

$$\tilde{W}_{s_\alpha} = \tilde{W}_{q_\alpha} + \frac{1}{s\mu_\alpha}$$

$$= \left[\frac{2(\alpha+14)}{3\left(6\alpha^2-162\alpha+912\right)}, \frac{2(-\alpha+16)}{3\left(6\alpha^2+138\alpha+612\right)}\right] + \frac{[1, 1]}{2[\alpha+17, -\alpha+19]}$$

$$= \left[\frac{2(\alpha+14)}{3\left(6\alpha^2-162\alpha+912\right)} + \frac{1}{-2\alpha+38}, \frac{2(-\alpha+16)}{3\left(6\alpha^2+138\alpha+612\right)} + \frac{1}{2\alpha+34}\right].$$

Finally, substitute $\alpha = 0$ and $\alpha = 1$ to obtain triangular fuzzy numbers

$$\tilde{L}_{q_\alpha} = (0.1433 / 0.1984 / 0.2789)$$

$$\tilde{W}_{q_\alpha} = (0.0102 / 0.0132 / 0.0174)$$

$$\tilde{L}_{s_\alpha} = (0.5117 / 0.6151 / 0.7495)$$

$$\tilde{W}_{s_\alpha} = (0.0365 / 0.0410 / 0.0468)$$

Which give following membership functions,

$$\eta_{\tilde{L}_{q\alpha}}(x)=\begin{cases}\dfrac{x-0.1433}{0.0551}, & if\ 0.1433\le x\le 0.1984\\ \dfrac{0.2789-x}{0.0805}, & if\ 0.1984\le x\le 0.2789\\ 0, & otherwise\end{cases}$$

$$\eta_{\tilde{W}_{q\alpha}}(x)=\begin{cases}\dfrac{x-0.0102}{0.0030}, & if\ 0.0102\le x\le 0.0132\\ \dfrac{0.0174-x}{0.0042}, & if\ 0.0132\le x\le 0.0174\\ 0, & otherwise\end{cases}$$

$$\eta_{\tilde{L}_{s\alpha}}(x)=\begin{cases}\dfrac{x-0.5117}{0.1034}, & if\ 0.5117\le x\le 0.6151\\ \dfrac{0.7495-x}{0.1344}, & if\ 0.6151\le x\le 0.7495\\ 0, & otherwise\end{cases}$$

And

$$\eta_{\tilde{W}_{s\alpha}}(x)=\begin{cases}\dfrac{x-0.0365}{0.0045}, & if\ 0.0365\le x\le 0.0410\\ \dfrac{0.0468-x}{0.0058}, & if\ 0.0410\le x\le 0.0468\\ 0, & otherwise\end{cases}$$

Final results using α - cuts method

1. The number of customers in the queue is approximately between 0.1433 and 0.2789
2. The waiting time in the queue is approximately between 0.0102 and 0.0174
3. The number of customers in the system is approximately between 0.5117 and 0.7495
4. The waiting time in the system is approximately between 0.0365 and 0.0468

4.2. L-R Method Solution

The L-R method solution comprises the following steps

(i) Find the L-R representations of fuzzy numbers $\tilde{\lambda}$ and $\tilde{\mu}$

$\tilde{\lambda}=\langle 15,1,1\rangle_{lR}$ and $\tilde{\mu}=\langle 18,1,1\rangle_{lR}$

(ii) Substituting $\tilde{\lambda}=\langle 15,1,1\rangle_{lR}$ and $\tilde{\mu}=\langle 18,1,1\rangle_{lR}$ in characteristic formulas

(a) Expected number of customers in the queue

$$\tilde{L}_q = \frac{(k+1)\lambda^2}{2ks\mu(s\mu-\lambda)}$$

$$= \frac{4\langle 15,1,1\rangle_{LR}\langle 15,1,1\rangle_{LR}}{6\left(2\langle 18,1,1\rangle_{LR}\left(2\langle 18,1,1\rangle_{LR}-\langle 15,1,1\rangle_{LR}\right)\right)}$$

$$= \frac{2\langle 225,29,31\rangle_{LR}}{3\langle 36,2,2\rangle_{LR}\langle 21,\ 3,\ 3\rangle_{LR}}$$

$$= \langle 0.1984, 0.0551, 0.0804\rangle_{LR}$$

$$= (0.1433 / 0.1984 / 0.2788)$$

(b) Expected waiting time in the queue

$$\tilde{W}_q = \frac{(k+1)\tilde{\lambda}}{2ks\tilde{\mu}(s\tilde{\mu}-\tilde{\lambda})}$$

$$= \frac{4\langle 15,1,1\rangle_{LR}}{6\left(2\langle 18,1,1\rangle_{LR}\left(2\langle 18,1,1\rangle_{LR}-\langle 15,1,1\rangle_{LR}\right)\right)}$$

$$= \frac{2\langle 15,1,1\rangle_{LR}}{3\langle 756,144,156\rangle_{LR}}$$

$$= \langle 0.0132, 0.0030, 0.0042\rangle_{LR}$$

$$= (0.0102 / 0.0132 / 0.0174)$$

(c) Expected number of customers in the system

$$\tilde{L}_s = \tilde{L}_q + \frac{\tilde{\lambda}}{s\tilde{\mu}}$$

$$= \langle 0.1984, 0.0551, 0.0804\rangle_{LR} + \frac{\langle 15,1,1\rangle_{LR}}{2\langle 18,1,1\rangle_{LR}}$$

$$= \langle 0.6151,\ 0.1033, 0.1343\rangle_{LR}$$

$$= (0.5188 / 0.6151 / 0.7494)$$

(d) Expected waiting time in the system

$$\tilde{W}_s = \tilde{W}_q + \frac{1}{s\tilde{\mu}}$$

$$= \langle 0.0132, 0.0030, 0.0042\rangle_{LR} + \frac{1}{2\langle 18,1,1\rangle_{LR}}$$

$$= \langle 0.0410, 0.0045, 0.0058\rangle_{LR}$$

$$= (0.0365 / 0.0410 / 0.0468)$$

Final results using L-R Method

1. The number of customers in the queue is approximately between 0.1433 and 0.2788, Mean value is 0.1984
2. The waiting time in the queue is approximately between 0.0102 and 0.0174.
3. The number of customers in the system is approximately between 0.5118 and 0.7494, Mean value is 0.6151
4. The waiting time in the system is approximately between 0.0365 and 0.0468

The most possible waiting time in the queue is 0.0132(≃ 0.8 minutes).

The most possible waiting time in the system is 0.0410($\simeq$ 2.5 minutes).

2. Conclusion

Fuzzy queuing models have the ability to recognise, analyse, shape, interpret, and make use of ambiguous and uncertain facts and information. In this chapter, we analyse the L-R Method, a new tool for locating performance measurements of Erlang multiserver queue such as $L_q, L_s, W_s \text{ and } W_q$. In Comparison to the α –cut method, L-R Method is more comfortable, flexible, and supple. Fuzzy queues overcome the issue of crisp queues. The strategy suggested in this study can be used to resolve many existing scenarios where there are uncertainties in the queuing model.

3. References

Buckley, J. J., & Qu, Y. (1990). On using α -cuts to evaluate fuzzy equations. Fuzzy sets and systems. 38(3), 309-312.

Donald Gross., John F. Shortle., James M. Thompson., & Carl M. Harris. (2008).

Fundamentals of Queueing Theory. Fourth Edition, Wiley Series in Probability and Statistics.

Dubosis, D., & Prade, H. (1980). Fuzzy sets and Systems: Theory and Applications, Academic Press, New York.

Hendry M. Prade. (1980). An outline of fuzzy or possibilistic models for queueing systems. Fuzzy Sets, Springer, 147-153.

Kaufmann, A., & Gupta, M. M. (1991). Introduction to fuzzy Arithmetic: Theory and Applications. Van Nostrand Reinhold, New York.

Li, R. J., & Lee, E. S. (1989). Analysis of Fuzzy Queues. Computers and Mathematics With Applications, 17(7), 1143-1147.

Mukeba, J. P., Mabela, B., & Ulungu, B. (2015). Computing fuzzy queueing performance measures by L-R Method. Journal of fuzzy set valued Analysis. 1, 57- 67.

Negi, D. S., & Lee, E. S. (1992). Analysis of Simulation of Fuzzy Queues. Fuzzy sets and Systems, 46(3), 321-330.

Ritha, W., & Lilly Robert. (2009). Application of fuzzy set theory to retrial queues. International Journal of Algorithms, Computing and Mathematics, 2(4), 9-18.

Ritha, W., & Sreelekha Menon, B. (2011). Fuzzy N Policy queues with Infinite Capacity. Journal of Physical Sciences, 15, 73-82.

Srinivasan, R. (2014). Fuzzy Queueing Model using DSW Algorithm. International Journal of Advanced Research in Mathematics and Computer Applications, 1(1), 57-62.

Zimmermann, H. Z. (2001). Fuzzy Set Theory and its Applications. Springer Science +Bussiness Media, New-York, Fourth Edition.