CLAY COATINGS AND ELECTRICAL CONDUCTIVITY IN CONSTRICTED CHANNELS

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This study investigates the impact of varying pore throat constriction in 3D straight channels on the electrical conductivity of fluid-filled, clay-containing porous rocks through simulation. The research identifies a critical ratio of clay conductance to fluid conductance that dictates the regime of electrical conductance behaviour. A non-linear increase in electrical conductance is observed when the clay-to-fluid conductance ratio exceeds the critical ratio, whereas a linear relationship is maintained below this critical ratio. A modified form of Archie’s law relating effective conductivity and porosity has been proposed for the clay coated channels.

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1 Introduction

The combination of electrical conductivity of the clay matrix material and the pore fluid determines the overall electrical conductivity of the porous soil and rocks. Electrical conductivity serves as a crucial parameter in soil science, providing valuable insights into soil properties and composition. It is influenced by a combination of factors that contribute to its variability and significance in soil mapping.

Clay content and the degree of saturation affect electrical conductivity in porous rocks. Clay-rich porous media, particularly those with high organic matter content, exhibit a high cation exchange capacity. This capacity enhances the retention of positively charged ions, leading to increased electrical conductivity levels, akin to the impact of salinity. The presence of water in the porous media is essential for electrical conductivity, as water contains ions that facilitate the conduction of electricity. Dry porous media typically display lower electrical conductivity values compared to moist ones due to the absence of water [1]. Porosity plays a vital role in determining soil electrical conductivity [2]. Higher porosity allows for greater water retention, thereby enhancing electrical conductivity. Porous media with high clay content tend to have higher electrical conductivity values than sandier ones due to their superior moisture retention capabilities. Compaction influences electrical conductivity by reducing air gaps and increasing the density of the porous media.

The close relationship between electrical conductivity, moisture content, and porous media properties underscores the utility of electrical conductivity in soil mapping and property assessment. By understanding the dynamics of electrical conductivity and its influencing factors, researchers and practitioners can effectively utilize this parameter to map soil variations, assess its quality, and make informed decisions in agriculture, environmental management, and land use planning. It also enables engineers to evaluate the safety and stability of construction projects built on or within rock structures, and monitor groundwater contamination by tracing the movement of fluids through rock formations.

The Archie's law [3,4], widely used in the oil and gas industry, is well-known for predicting the electrical conductivity of clean rocks and soils but it doesn't work well for clay coated rocks [5,6]. In compacted porous reservoirs, the shape of component-mineral grains and pores plays a more significant role in current flow than its reliance on the degree of cementation [7]. The cementation exponent yielding the formation factor has been explored by several researchers for the practical application of Archie's law in estimating water and hydrocarbon saturation in porous rocks [8].

The surface conductivity of reservoir rocks exhibits a non-linear relationship with fluid salinities, significantly impacting the effective conductivity of the porous system. The presence of clay minerals and variations in channel macro-geometry further complicate this relationship, emphasizing the need for sophisticated models that capture these intricate dynamics. Reservoir rocks often contain clay minerals, which can affect the overall conductivity of the porous system through surface conduction [9]. Additionally, the shape and size of the channels within the porous medium influence surface conductivity, leading to variations in effective electrical conductivity. Even small changes in channel geometry can have a substantial impact on the electrical conductivity of the system, highlighting the non-linear nature of this relationship.

Various theoretical approaches have been developed to model the electrical conductivity of fractal porous media [10,11,12]. Finite-difference discretization has been employed to determine the effective resistivity of porous media at the pore scale [13]. Finite-element methods to construct 3-D fractured rock studies [14] have revealed that the level of liquid saturation in rocks needs to be considered in determining the electrical resistivity in fractured rock samples. In complex porous media with heterogeneous pore-space distributions and multiple interactions, including chemical and physical processes, the challenge of providing a comprehensive model persists. Despite efforts by researchers [15,16] no single model has emerged as universally accurate for modelling the electrical conductivity of such systems.

This work explores the impact of varied constriction in a straight, clay-coated channel on effective electrical conductivity. A cuboid-shaped tube embedded within a rock matrix is considered for the study. The tube is constricted in the middle along its length. Factors such as the composition of clay minerals, the fraction of walls coated with clay, and the salinity of the fluid within the porous structure predominantly affect the conductivity of the channel. Laplace's equation is solved using the finite difference method to determine the voltage distribution in the system. Ohm's Law is then applied to determine effective conductance, and thereby the conductivity, from the cross-section and length. The novelty of this work lies in proposing a modified version of Archie's law, tailored to clay-coated channels with varied constricted pore throats. This study also investigates the relative importance of fluid to clay conductance. However, it does not address the effects of pH, temperature of the fluid, and stress on the soil causing compaction on electrical conductivity.

2 Methodology

Clay minerals play a crucial role in altering the electrical conductivity of rocks by establishing an additional conductive pathway owing to their superior conductivity compared to most common rock-forming minerals. The inclusion of clay particles generates a unique conductive route within the rock. This research delves into the electrical conductivity of rocks, underscoring the importance of clay minerals in facilitating current flow. Clay aggregates may exist within grains or as coatings on grain surfaces. In environments with high salinity, ions form an electrical double layer along surfaces, where their mobility is constrained by electrostatic forces. Within samples saturated with saline solutions, conduction primarily occurs through the pore space. In rocks harbouring clay, conductivity is influenced by both pore space conduction and the interplay between fluids and minerals. The electrical conductivity of porous rocks is shaped by factors such as porosity, channel configuration, grain size, and morphology.

2.1 Structure Generation

A three-dimensional straight channel featuring symmetrically positioned constrictions of varying widths and equal lengths is considered to be contained within a rock. The fluctuation in constriction suggests a direct correlation between the channel width and the porosity. The pore space is divided into cubic nodes or cells in three dimensions.

At the interface of the fluid and the rock, the pore layer is assigned a different conductance value, distinct from the rest of the pore cells, only if clay is present (Fig.1a). The fraction of the pore-rock interface that is coated with clay, is chosen randomly from a probability distribution p between 0 and 1.0. The rock has no conductivity. It is presumed that the conductance remains uniform within every cell. The resultant conductance between two neighbouring pore cells, each possessing individual conductance values g_i and g_j is computed as $g_{ij} = g_i g_j/(g_i + g_j)$.

The methodology employed to solve for electrical conductivity is founded on Knudsen and Fazekas [17].

2.2 Algorithm for Electrical Conductivity

At the inlet and outlet layers, false nodes are strategically positioned with potentials $V_{in} = 1$ and $V_{out} = 0$, respectively, serving as boundary conditions. Applying Kirchhoff's current law, the solution of V_i of a pore cell in terms of its neighbouring cells (Fig.1b) can be framed as

$$V_i = \frac{\sum_{j=1}^{j=6} g_{ij} V_j}{\sum_{j=1}^{j=6} g_{ij}} \qquad (1)$$

where j runs over all the 6 nearest neighbour cells in three-dimensions. Equation Eq.(1) is iteratively applied until the voltage at each internal node reaches convergence.

Once steady state is achieved, the current I_{net} remains constant across the sample's cross-section. I_{net} can be determined by summing the differences in potentials between the false cells at the outlet/inlet and its adjacent layer. Finally, the effective conductance *G* can be obtained from the following equation:

$$G = I_{net}/(V_{in} - V_{out}) \qquad (2)$$

where, $V_{in} - V_{out}$ =1.

The effective electrical conductivity σ of the system is determined from the relation given below.

$$\sigma = \frac{GL}{A_{avg}} \qquad (3)$$

where, A_{avg} is the average cross section and L is the length of the system.

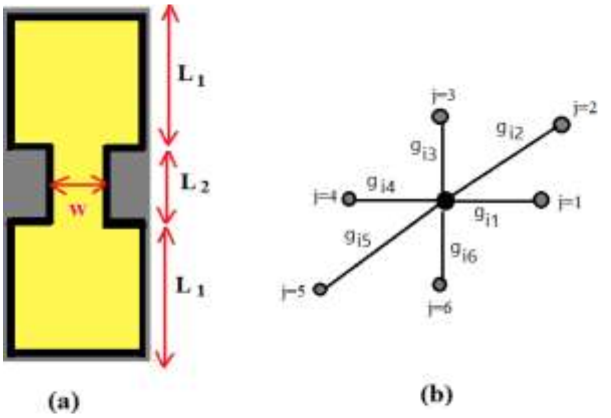

Figure 1: (a) Schematic diagram of a vertical section of the 3D channel. Grey, black and yellow denotes rock, clay interface and pore fluid respectively. (b) Schematic diagram of six neighbouring cells surrounding a pore cell *i* in 3D, and their associated conductance.

3 Results and Discussion

In the first part of this work, the electrical conductivity of the clay-coated channel is investigated for varying proportions of clay in the constricted channels, with clay distribution controlled by a probability (*p*).

The clay conductance g_c and fluid conductance g_f is kept fixed. The conductance values for clay, fluid, and rock are kept constant at g_c =100, g_f =10 and g =0, respectively. These values are selected based on the understanding that clay ions, generally have higher conductance compared to the fluid. The effective electric conductivity σ, for each structure is calculated using equation Eq.(3), based on *G* determined by equation Eq.(2), with inputs from Table(1).

Table 1: Structure dimensions and porosity

w	$A_{avg}(units^2)$	$L(units)$	ϕ
(a) 50	2500	250	1.0
(b) 40	2320	250	0.928
(c) 30	2180	250	0.872
(d) 20	2080	250	0.832
(e) 10	2020	250	0.808

The widths of the channels at the region of constriction varies as *w=10, 20, 30, 40 and 50 units* while the length *L₁* and *L₂* are fixed as 100 and 50 units (Fig.1a). The total length of the sample is *2 L₁+ L₂ =250 units* for all cases. The porosity of the channel without constriction (*w=50* units) is set to unity. The porosity for the constricted channels for *w* ranging from *10 to 40 units* is assigned proportionally based on their pore volume. Figure 2 shows the variation of *σ* determined following the procedure described in subsection 2.2 with *φ* for *p* ranging from 0 to 1.0. The data shows overall good agreement with the following equation:

$$\sigma = \{6 + \exp(1.8p^{1.25})\}(\varphi - 0.808)^{0.21} \qquad (4)$$

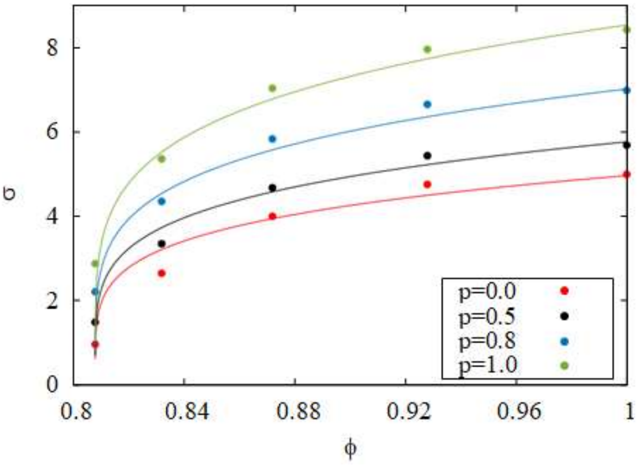

Figure 2: Variation of effective electrical conductivity with variation in porosity; fluid conductance, clay conductance are kept constant. Clay content is varied by fraction *p.*

In the second part of this study, the variation of effective conductance with fluid conductance is conducted for a fully clay-coated channel (*p*=1.0) across three structures: *w =10,30,50 units*, to cover the whole range of porosity. The conductance values for clay and rock are kept constant at g_c=100 and $g = 0$ respectively. The fluid conductance is varied over a wide range, from $g_f = 1$ to $g_f = 500$. Two distinct regimes can be identified from the plots.

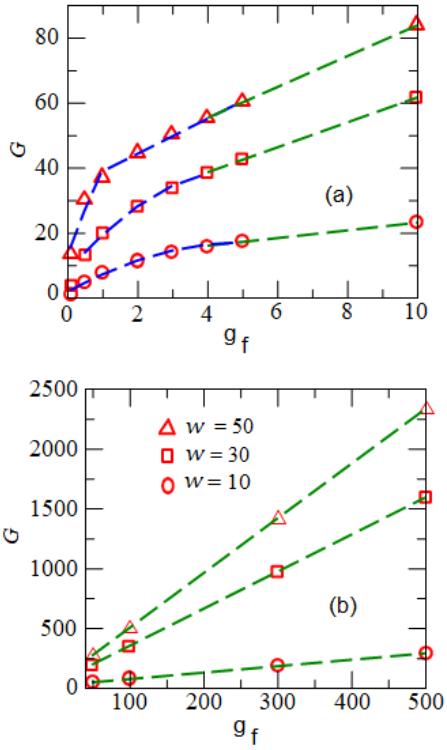

Figure 3: Variation of effective electrical conductance with variation in fluid conductance for : (a) $g_f < g_c$ (b) $g_c >= g_f$. The channels are fully clay coated. The blue lines and green lines represent the quadratic fit and linear fit, respectively, to the red simulation points.

Figure 3 demonstrates that G scales non-linearly with g_f in the regime g_f<<g_c whereas for g_f~g_c and g_f>g_c, G exhibits a linear relationship with g_f. The observations indicate that when g_f < 4 (Fig.3a), surface conductivity along the clay lines channel walls become the primary paths of electrical transport. In this regime the simulation points could be fitted using a quadratic law of the form $G = Ag_f{}^2 + Bg_f + C$. The parameters are enlisted in Table 2. A linear fit shows a good agreement for 4< g_f <10. When g_f ~ g_c and beyond, G versus g_f is a linear fit with the form $G = mg_f + c$ (Table 2). However, the magnitudes of the linear fit parameters, m_1 and c_1 for 4< g_f <10 differ noticeably from m_2 and $c_2$corresponding to g_f~g_c and g_f>g_c, which can be attributed to the transition from the quadratic to linear regime. In the linear regime, the effective conductivity is primarily determined by the influence of pore fluid conductivity, which predominates over surface conductivity. Since the g_c and g_f values are arbitrary, the ratio of these parameters is significant in determining the critical value at which the transition from the non-linear to the linear regime in σ occurs. In this case, the shift in the regime is observed at a critical ratio of g_c to g_f of ~25.

Table 2: Quadratic and linear fit

w	C	B	A	c_1	m_1	c_2	m_2
(a) 50	12.2	31.00	-4.20	36.45	4.75	36.91	4.60
(b) 30	7.50	12.75	1.25	23.20	3.85	35.32	3.12
(c) 10	1.34	6.15	-0.60	11.20	1.20	16.70	0.54

4 Conclusions

The impact of varying the pore throat in a 3D straight channel on electrical conductivity σ, of fluid-filled rocks has been investigated through simulation. This research stands out by examining how the constriction of pore throat within the macro geometry of a straight transport channel affects the overall σ of clay-containing porous rocks. It evaluates the relative significance of fluid conductance versus clay conductance.

The main findings of the work may be summarized as below.

- Existence of a critical ratio of clay conductance to fluid conductance. For a fully clay coated channel wall, electrical conductance (and thereby conductivity), increase non-linearly with fluid conductance as long the ratio of clay conductance to fluid conductance > 25.
- Below this critical ratio, effective electrical conductance vary linearly with fluid conductance.
- The surface conductivity is the main contributing factor to effective electrical conductivity in the non-linear regime, whereas in the linear regime, fluid conductivity is the controlling factor.

- Effective conductivity versus porosity is shown to obey a modified form of Archie's law for clay content ranging from p =0 to 1.0.

Electrical conductivity in porous media depends on various factors such as temperature, pH, and stress-induced compaction. Future research will incorporate these factors to develop a robust algorithm suitable for more realistic scenarios.

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