Application of Soft Set in Brauer Algebra

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Abstract
In 1999, Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. In this paper, we apply the concept of soft sets to *Brauer* algebras and investigate some properties of soft sets to Brauer Algebra.

Introduction

Most of the problems in engineering, medical science, economics, environments, and so forth, have various uncertainties. The problems in system identification involve characteristics which are essentially nonprobabilistic in nature. In response to this situation, Zadeh [1] introduced fuzzy set theory as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [2]. Molodtsov [3] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields including game theory, operations research, Riemann integration, and Perron integration. At present, work on soft set theory is progressing rapidly. After Molodtsov's work, some operations and application of soft sets were studied by many researchers including Ali et al. [4], Aktaş and Çağman [5], Chen et al. [6], and Maji et al. [7]. Maji et al. [7] gave first practical application of soft sets in decision making problems. To address decision making problems based on fuzzy soft sets, Feng et al. introduced the concept of soft level sets of fuzzy soft sets and initiated an adjustable decision making scheme using fuzzy soft sets [8]. It is interesting to see that soft sets are closely related to many other soft computing models such as rough sets and fuzzy sets. Feng et al. [9] first considered the combination of soft sets, fuzzy sets, and rough sets. Using soft sets as the granulation structures, Feng et al. [10] defined soft approximation spaces, soft rough approximations, and soft rough sets, which are generalizations of Pawlak's rough set model based on soft sets. It has been proven that in some cases Feng's soft rough set model could provide better approximations than classical rough sets. The algebraic structure of soft set theories has been studied increasingly in recent years. Aktaş and Çağman [5] defined the notion of soft groups. Feng et al. [11] initiated the study of soft semirings, and soft rings were defined by Acar et al. [12]. Jun [13] introduced soft BCL -algebras, and Kazancı et al. [14] introduced soft BCL algebras. Along this direction, we apply soft set theory to Brauer algebras and investigate some of their properties. We introduce the notion of Abelian soft -algebras and investigate some of their properties.

1. Soft Set in Brauer Algebra

If $\wp$ is a Brauer Algebra and A a nonempty set, a set-valued function $F : A \to \wp(K)$ can be defined by $F(X) = \{y \in \wp \mid xRy\}, x \in A,$ where R is an arbitrary binary relation from A to $\wp$;that is, R is a subset of $A \times \wp$ unless otherwise specified. The pair F_A is then a soft set over $\wp$

1.1Definition

Let F_A be a soft set over $\wp$.Then F_A is called a soft Brauer Algebra over F_A if F(x) is Brauer sub algebra of Brauer Algebra F_A for all $x \in A.$

1.2Definition

Let F_A be a nonnull soft set over $\wp$ if F(x) is Brauer sub algebra of $\wp$ for all $x \in SuppF_A$.

1.2.1Example

Consider the Brauer Algebra $\wp = (S_3, \cdot, \Theta, e)$ on the symmetric group $S_3 = \{e, a, b, x, y, z\}$, where $e = (1), a = (123), b = (132), x = (12), y = (13), z = (23),$ and Θ are given by the following cayley table:

Θ	e	x	y	z	a	b
e	e	x	y	z	b	a
x	x	e	a	b	z	y
y	y	b	e	a	x	z
z	z	a	b	e	y	x
a	a	z	x	y	e	b
b	b	y	z	x	a	e

Let F_A be a soft set over $\wp$,where $A = \wp$ and $F : A \to P(\wp)$ is a set-valued function defined by F(e)={e},F(a)=F(b)={e,a,b},F(x)={e,x},F(Y)={e,y}, and F(z)={e,z} being Brauer sub algebra of $\wp$ for all $x \in SuppF_A$.Therefore, F_A is a soft Brauer Algebra over $\wp$.

1.2.2Example

Consider the Brauer Algebra $\wp = (S, \cdot, \Theta, e)$ on the Dihedral group G={e,a,u,v,b,x,y,z},where $u = a^2, v = a^3, x = ab, y = a^2b, z = a^3b,$ and * are given by the following cayley table:

*	e	a	u	v	b	x	y	z
e	e	v	u	a	b	x	y	z
a	a	e	v	u	x	y	z	b
u	u	a	e	v	y	z	b	x
v	v	u	a	e	z	b	x	y
b	b	x	y	z	e	v	u	a

x	x	y	z	b	a	e	v	u
y	y	z	b	x	u	a	e	v
z	z	b	x	y	v	u	a	e

Let F_A be a soft set over $\wp$, where $A = \wp$ and $F : A \to P(\wp)$ is a set –valued function defined by F(e)={e},F(a)=F(v)={e,a,u,v},F(u)={e,u},F(x)={e,x},F(y)={e,y},and F(z)={e,z} being Brauer sub-algebras of $\wp$.Therefore, F_A is a soft Brauer algebra over F_A.

1.3Lemma

Let F_A be a soft Brauer algebra over $\wp$. then

(i)if $x \in F(x) \Rightarrow x^{-1} \in F(x)$ for all $x \in A$,

(ii) if $a\Theta b \in F(x) \Rightarrow b\Theta a \in F(x)$ for all $a, b \in A$.

1.4Proposition

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$. Then, the bi-intersection $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$ if it is nonnull.

Proof

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft brauer algebras over $\wp$.

We can write $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, where $B = \cap_{i\in\wedge} A_i$ and $H(x) = \cap_{i\in\wedge} F_i(x)$ for all $x \in B$.

Let $x \in SuppH_B$. Then, $\cap_{i\in\wedge} F_i\ (x) \neq 0$, and so we have $F_i(x) \neq 0$ for all $i \in \wedge$.

Since $\{(F_i\ )_{A_i} \mid i \in \wedge\}$ is a nonempty family of soft Brauer algebras over $\wp$,it follows that $F_i(x)$ is a Brauer subalgebra of X for all $i \in \wedge$, and its intersection is also a Brauer subalgebra of $\wp$,that is, $H(x) = \cap_{i\in\wedge} F_i(x)$ is a Brauer subalgebra of $\wp$ for all $x \in SuppH_B$.

Hence, $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, is a soft Brauer algebra over $\wp$.

1.5Proposition

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$. Then, the extended intersection $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

Proof

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft brauer algebras over $\wp$.

We can write $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, where $B = \cap_{i\in\wedge} A_i$ and $H(x) = \cap_{i\in\wedge} F_i(x)$ for all $x \in B$.

Let $x \in SuppH_B$. Then, $\cap_{i\in\wedge} F_i\ (x) \neq 0$, and so we have $F_i(x) \neq 0$ for all $i \in \wedge$.

Since $\{(F_i\ )_{A_i} \mid i \in \wedge\}$ is a nonempty family of soft Brauer algebras over $\wp$,it follows that $F_i(x)$ is a Brauer subalgebraof X for all $i \in \wedge$, and its intersection is also a Brauer subalgebra of $\wp$,that is, $H(x) = \cap_{i\in\wedge} F_i(x)$ is a Brauer subalgebra of $\wp$ for all $x \in SuppH_B$.

Hence, $\overline{\cap}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, is a soft Brauer algebra over $\wp$.

1.6Proposition

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.If $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$ for all $i, j \in \wedge, x_i \in A_i$, then the restricted union $\overline{\cup}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

Proof

Suppose that $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft brauer algebras over $\wp$.

We can write $\overline{\cup}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, where $B = \cap_{i\in\wedge} A_i$ and $H(x) = \cap_{i\in\wedge} F_i(x)$ for all $x \in B.$

Let $x \in SuppH_B$. Since $SuppH_B = \cup_{i\in\wedge} Supp(F_i)_{A_i} \neq 0, F_{i_0} \neq 0$ for some $i_0 \in \wedge.$

By assumption, $\cup_{i\in\wedge} F_i(x)$ is a Brauer algebra of $\wp$ for all $x \in SuppH_B$.

Hence restricted union $\overline{\cup}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

1.7Proposition

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.Then the $\wedge -$intersection $\overline{\wedge}_{i\in\wedge}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$ if it is nonnull.

Proof

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.

We can write $\overline{\wedge}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, where $B = \Pi_{i\in\wedge} A_i$ and $H(x) = \cap_{i\in\wedge} F_i(x)$ for all $x = (x_i)_{i\in\wedge} \in B.$

Suppose that the soft set H_B is nonempty.

If $x = (x_i)_{i\in\wedge} \in SuppH_B$. $H(x) = \cap_{i\in\wedge} F_i(x) \neq 0.$

Since $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$, nonempty set $F_i(x)$ is a Brauer subalgebra of $\wp$ for all $i \in \wedge.$

It follows that $H(x) = \cap_{i\in\wedge} F_i(x)$ is a Brauer subalgebra of $\wp$ for all $x = (x_i)_{i\in\wedge} \in SuppH_B$.

Hence $\wedge -$ intersection $\overline{\wedge}_{i\in\wedge}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

1.8Proposition

Let $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.If $F_i(x_i) \subseteq F_j(x_j)$ or $F_j(x_j) \subseteq F_i(x_i)$ for all $i, j \in \wedge, x_i \in A_i$, then the $\vee$ union $\overline{\vee}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

Proof

Assume that $\{(F_i)_{A_i} \mid i \in \wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.

We can write $\overline{\vee}_{i\in\wedge}(F_i\ )_{A_i} = H_B$, where $B = \Pi_{i\in\wedge} A_i$ and $H(x) = \cup_{i\in\wedge} F_i(x)$ for all $x = (x_i)_{i\in\wedge} \in B.$

Let $x = (x_i)_{i\in\wedge} \in SuppH_B$. Then $H(x) = \cup_{i\in\wedge} F_i(x) \neq 0.$

So we have $F_{i_0} \neq 0$ for some $i_0 \in \wedge.$

By assumption, $\cup_{i\in\wedge} F_i(x)$ is a Brauer algebra of $\wp$ for all $x = (x_i)_{i\in\wedge} \in SuppH_B$.

Hence $\vee-$union $\overline{\vee}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

1.9Proposition

Let $\{(F_i)_{A_i} \mid i\in\wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.Then,the Cartesian product $\tilde{\Pi}_{i\in\wedge}(F_i)_{A_i}$ is a soft Brauer algebra over $\Pi_{i\in\wedge}\wp_i$.

Proof

Let $\{(F_i)_{A_i} \mid i\in\wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$.

We can write $\tilde{\Pi}_{i\in\wedge}(F_i\ )_{A_i}=H_B$, where $B=\Pi_{i\in\wedge}A_i$ and $H(x)=\Pi_{i\in\wedge}F_i(x)$ for all $x=(x_i)_{i\in\wedge}\in B.$

Suppose that the soft set H_B is nonnull.

If $x=(x_i)_{i\in\wedge}\in SuppH_B$, $H(x)=\Pi_{i\in\wedge}F_i(x)\neq 0.$

Since $\{(F_i)_{A_i} \mid i\in\wedge\}$ be a nonempty family of soft Brauer algebra over $\wp$, nonempty set $F_i(x)$ is a Brauer subalgebra of $\wp$ for all $i\in\wedge.$

It follows that $H(x)=\Pi_{i\in\wedge}F_i(x)$ is Brauer subalgebra of $\wp$ for all $x=(x_i)_{i\in\wedge}\in SuppH_B.$

Hence $\wedge-$ intersection $\tilde{\Pi}_{i\in\wedge}(F_i\ )_{A_i}$ is a soft Brauer algebra over $\wp$.

2.0Definition

Let F_A be a soft Brauer Algebra over $\wp$

- F_A is called the trivial soft Brauer Algebra over $\wp$ if F(x)={e} for all $x\in A.$
- F_A is called the whole soft Brauer Algebra over $\wp$ if $F(x)=\wp$ for all $x\in A.$

2.1Definition

Let F_A be a soft set over a Brauer Algebra $\wp$.Then, the inverse of F_A is denoted by $F_A{}^{-1}$ and is defined as follows $F_A{}^{-1}=\{(F(a))^{-1}:a\in A\}$, where $(F(a)^{-1}$ is called the inverse of F(a) and is defined as $(F(a))^{-1}=\{x^{-1}:x\in F(a)\}$.

2.2Theorem

Let F_A and G_B be any two soft sets over $\wp$.Then $(F_A\circ G_B)^{-1}=G_B{}^{-1}\circ F_A{}^{-1}$.

2.3Theorem

If F_A is a soft Brauer Algebra over $\wp$, then $F_A{}^{-1}=F_A$.

Note

The converse of the theorem is not true in general, and it can be seen in the following example

2.3.1Example

Consider the Brauer Algebra on the dihedral group G={e,a,u,v,b,x,y,z},which is given in example. Let F_A be a soft set over $\wp$,where $A=\wp$ and $F:A\to P(\wp)$ is a set-valued function defined by

$(F(e))^{-1}=\{e\},(F(a))^{-1}=\{e,a,u\},\left(F(u)\right)^{-1}=\{e,u\},(F(v))^{-1}=\{e,v,u\},(F(b))^{-1}=\{e,b\},(F(x))^{-1}=\{e,x\},$
$(F(y))^{-1}=\{e,y\},(F(z))^{-1}=\{e,z\}.$

Therefore , we find that $F(\alpha)=(F(\alpha))^{-1}$ for all $\alpha \in A$. Hence ${F_A}^{-1}=F_A$, but F_A is not soft Brauer Algebra over $\wp$ because $F(a)=\{e,a,u\}$ is not a Brauer sub algebra of $\wp$.

Conclusions

Presently, science and technology are featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle various types of systems containing elements of uncertainty. A large number of these models are based on an extension of the ordinary set theory, namely, soft sets. In 1999, Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty and vagueness, and many researchers have created some models to solve problems in decision making and medical diagnosis. We have applied the concept of soft set theory to K-algebras and have investigated some of their properties. The natural extension of this research work is connected with the study of (i) fuzzy soft intersection K-algebras and (ii) roughness in K-algebras.

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