STUDY OF TWO-UNIT WARM STANDBY SYSTEM WITH DIFFERENT FAILURE MODES AND INSTRUCTIONS

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Abstract

In this paper, we have analyzed a two-unit warm standby system with various types of failure and instruction. There are three types of failures that have been considered: operating unit or the warm standby unit failure, failure due to common error and the failure due to human error. There is repair faculty available for the repair of the failed unit. In case the repairman is unable to repair the failed unit, an expert repairman is called for the repair of the failed unit. Using semi-Markov process and regenerative point technique, various reliability measures have been derived. Also, the comparison analysis has been drawn graphically.

Keywords: Failed unit, repairman, instruction, MTSF, Availability.

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1. Introduction

A lot of work has been done in the field of reliability and redundant systems. In this paper, we are extending the work in redundant systems by taking a two-unit identical system in warm standby with three types of failure. The three types of failure are operating unit failure or the warm standby unit failure, failure due to common error and the failure due to human error. There is a repair faculty available at all times, so that the system can be taken care of as soon as the failure occurs. Now, there is a possibility that repairman is unable to repair the system by himself. In such a case, expert repairman is called and expert repairman helps via instruction and if need be, he also repairs the system himself.

In their analysis of a two-unit warm standby system with instructions at need, G.S. Mokaddis, Y.M. Ayed, and H.S. Al-Hajeri (2013) took into account two repairmen: an expert and an assistant. The assistant was only called in when the expert repairman was preoccupied with fixing a malfunctioning unit, as they regarded the main repair faculty to be skilled repairmen. The instructions for fixing the malfunctioning equipment may or may not be required by the assistant repairman, and various probabilities were taken into account. The implications of random replacement times were taken into account by G. Levitin, L. Xing, and Y. Dai (2015) when assessing and optimizing 1-out-of-N warm standby systems. When components fail to do a mission task within the allotted mission time, the system is said to have failed. They also examined the duration required to transition the unit from warm standby to operational. A stochastic model for a two-unit hot standby database system comprising an operational (main) unit and a hot standby unit was created by A. Manocha, G. Taneja, S. Singh, and R. Rishi (2019). The primary unit acted as the production unit and stayed in sync with the hot standby unit via the online transfer of archive redo logs. The primary unit's data was simultaneously saved in the hot standby unit. Various scenarios of the primary database failing were considered. In order to prevent data loss, a database administrator (DBA) randomly checked the standby unit to determine if any redo log files had been modified or created. The system that Lalji Munda and Gulshan Taneja (2023) examined consisted of three units: a warm standby, a cold standby, and an operating unit. They reasoned that when the main unit (operational) fails, the cold standby, when engaged, transforms to warm standby, and vice versa for warm standby. They computed a number of performance metrics using the Markov process and regeneration point technique. They also derived cut-off values for the failure rate, activation rate, revenue cost, and cost per repairman visit in order to ascertain the system's optimal profit.

2. Description of model and Assumptions:

i. System is made up of two identical units. One unit is operating and the other is kept as warm standby.
ii. If there is a failure in one unit, the standby unit will be in operation automatically and the failed unit will go under repair.
iii. The system is in failed state when both the units malfunction.
iv. After the repair, the system acts like a new one.
v. The time to failure for each unit is in exponential distribution and the repair time and instruction time are in arbitrary distribution.
vi. All the random variables are mutually independent.

3. Nomenclature

P probability that the repairman repairs system without instructions

q probability that the repairman fails to repair system without instructions

λ_0 constant failure rate of the operative unit

O operative unit

WS warm standby

λ_1 constant failure rate of the warm standby unit

λ_2 constant failure rate of the system due to common cause failure

λ_3 constant failure rate of the system due to human error

$g_a(t)$ p.d.f. of repair time of failed unit by assistant repairman

$g_e(t)$ p.d.f. of repair time of failed unit by expert repairman

$g_2(t)$p.d.f. of repair time of failed unit due to common error

$g_3(t)$ p.d.f. of repair time of failed unit due to human error

$G_a(t)$ c.d.f. of repair time of failed unit by assistant repairman

$G_e(t)$ c.d.f. of repair time of failed unit by expert repairman

$G_2(t)$ c.d.f. of repair time of failed unit due to common error

$G_3(t)$ c.d.f. of repair time of failed unit due to human error

F_{uwi} failed unit waiting for repair while expert is giving instructions

F_{uwe} repair by the repairman is continued from the previous state while
instructions are still being given

F_{ura} failed unit under repair of assistant

F_{ure} failed unit under repair of expert

F_{uRa} failed unit under repair of assistant when repair is continued from the previous state

F_{uRe} failed unit under repair of expert when repair is continued from the previous state

F_{uw} failed unit waiting for repair after getting instructions

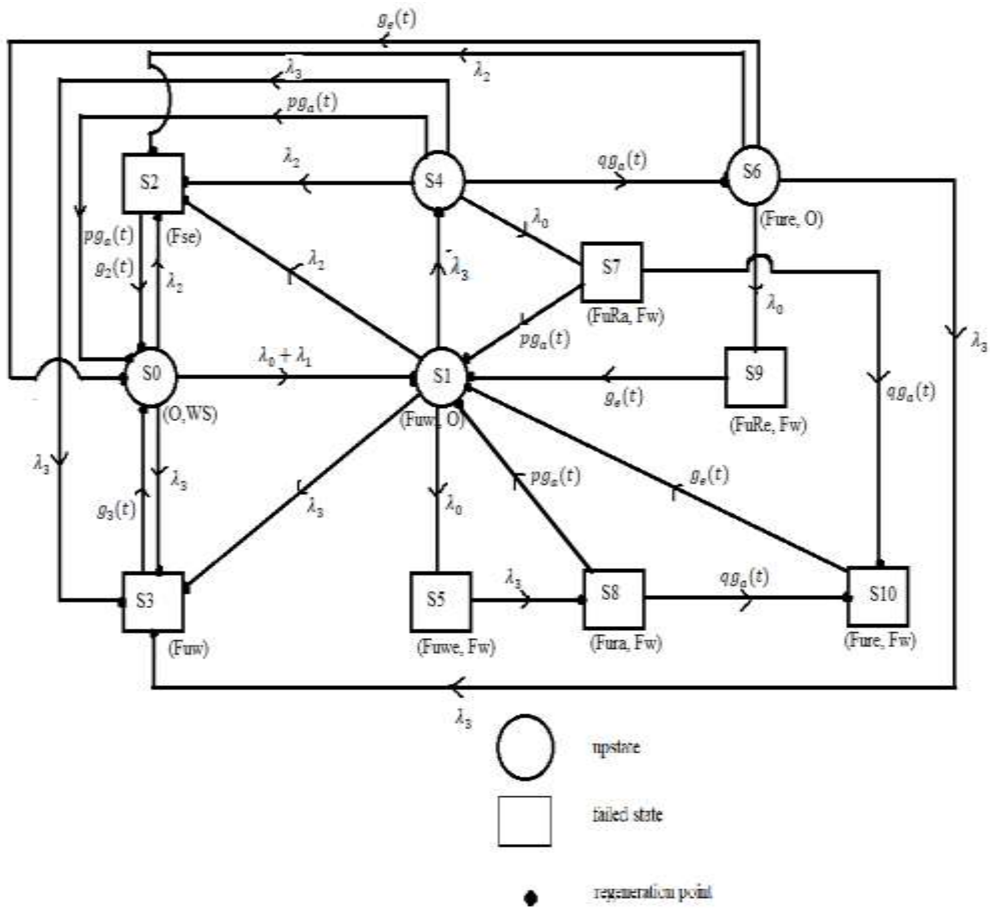

Fig. 1 (Transition Diagram)

4. Transition probabilities

The transition probabilities are:

$dQ_{01}(t) = (\lambda_0 + \lambda_1)e^{-(\lambda_0+\lambda_1+\lambda_2+\lambda_3)t}dt$

$dQ_{02}(t) = \lambda_2 e^{-(\lambda_0+\lambda_1+\lambda_2+\lambda_3)t}dt$

$dQ_{03}(t) = \lambda_3 e^{-(\lambda_0+\lambda_1+\lambda_2+\lambda_3)t}dt$

$dQ_{12}(t) = \lambda_2 e^{-(\lambda_0+\lambda_2+2\lambda_3)t}dt$

$dQ_{13}(t) = \lambda_3 e^{-(\lambda_0+\lambda_2+2\lambda_3)t}dt$

$dQ_{14}(t) = \lambda_3 e^{-(\lambda_0+\lambda_2+2\lambda_3)t}dt$

$dQ_{15}(t) = \lambda_0 e^{-(\lambda_0+\lambda_2+2\lambda_3)t}dt$

$$dQ_{18}^{(5)}(t) = \left(\lambda_0 e^{-(\lambda_0+\lambda_2+2\lambda_3)t} © \lambda_3 e^{-\lambda_3 t}\right)dt = \frac{\lambda_0\lambda_3}{\lambda_0+\lambda_2+\lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_0+\lambda_2+2\lambda_3)t}\right]dt$$

$dQ_{20}(t) = g_2(t)dt$

$dQ_{30}(t) = g_3(t)dt$

$dQ_{40}(t) = pe^{-(\lambda_0+\lambda_2+\lambda_3)t}g_a(t)dt$

$dQ_{42}(t) = \lambda_2 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_a(t)}dt$

$dQ_{43}(t) = \lambda_3 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_a(t)}dt$

$dQ_{46}(t) = qe^{-(\lambda_0+\lambda_2+\lambda_3)t}g_a(t)dt$

$dQ_{47}(t) = \lambda_0 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_a(t)}dt$

$$dQ_{41}^{(7)}(t) = \left(\lambda_0 e^{-(\lambda_0+\lambda_2+\lambda_3)t} © p\right)g_a(t)dt = \frac{p\lambda_0}{\lambda_0+\lambda_2+\lambda_3}\left[1 - e^{-(\lambda_0+\lambda_2+\lambda_3)t}\right]g_a(t)dt$$

$$dQ_{4,10}^{(7)}(t) = \left(\lambda_0 e^{-(\lambda_0+\lambda_2+\lambda_3)t} © p\right)g_a(t)dt = \frac{q\lambda_0}{\lambda_0+\lambda_2+\lambda_3}\left[1 - e^{-(\lambda_0+\lambda_2+\lambda_3)t}\right]g_a(t)dt$$

$dQ_{60}(t) = e^{-(\lambda_0+\lambda_2+\lambda_3)t}g_e(t)dt$

$dQ_{62}(t) = \lambda_2 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_e(t)}dt$

$dQ_{63}(t) = \lambda_3 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_e(t)}dt$

$dQ_{69}(t) = \lambda_3 e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_e(t)}dt$

$$dQ_{61}^{(9)}(t) = \left(\lambda_0 e^{-(\lambda_0+\lambda_2+\lambda_3)t} © 1\right)g_e(t)dt = \frac{\lambda_0}{\lambda_0+\lambda_2+\lambda_3}\left[1 - e^{-(\lambda_0+\lambda_2+\lambda_3)t}\right]g_e(t)dt$$

$dQ_{81}(t) = pg_a(t)dt$

$dQ_{8,10}(t) = qg_a(t)dt$

$dQ_{10,1}(t) = g_e(t)dt$

The non-zero elements p_{ij} are as follows:

$p_{01} = \frac{\lambda_0+\lambda_1}{\lambda_0+\lambda_1+\lambda_2+\lambda_3}$, $p_{02} = \frac{\lambda_2}{\lambda_0+\lambda_1+\lambda_2+\lambda_3}$, $p_{03} = \frac{\lambda_3}{\lambda_0+\lambda_1+\lambda_2+\lambda_3}$,
$p_{12} = \frac{\lambda_2}{\lambda_0+\lambda_2+2\lambda_3}$, $p_{13} = \frac{\lambda_3}{\lambda_0+\lambda_2+2\lambda_3}$, $p_{14} = \frac{\lambda_3}{\lambda_0+\lambda_2+2\lambda_3}$, $p_{15} = \frac{\lambda_0}{\lambda_0+\lambda_2+2\lambda_3}$, $p_{18}^{(5)} = \frac{\lambda_0}{\lambda_0+\lambda_2+2\lambda_3}$,
$p_{20} = p_{30} = 1$, $p_{40} = pg_a^*(\lambda_0+\lambda_2+\lambda_3)$, $p_{42} = \frac{\lambda_2}{\lambda_0+\lambda_2+\lambda_3}[1-g_a^*(\lambda_0+\lambda_2+\lambda_3)]$,
$p_{43} = \frac{\lambda_3}{\lambda_0+\lambda_2+\lambda_3}[1-g_a^*(\lambda_0+\lambda_2+\lambda_3)]$, $p_{46} = qg_a^*(\lambda_0+\lambda_2+\lambda_3)$,
$p_{47} = \frac{\lambda_0}{\lambda_0+\lambda_2+\lambda_3}[1-g_a^*(\lambda_0+\lambda_2+\lambda_3)]$, $p_{41}^{(7)} = \frac{p\lambda_0}{\lambda_0+\lambda_2+\lambda_3}[1-g_a^*(\lambda_0+\lambda_2+\lambda_3)]$,
$p_{4,10}^{(7)} = \frac{q\lambda_0}{\lambda_0+\lambda_2+\lambda_3}[1-g_a^*(\lambda_0+\lambda_2+\lambda_3)]$,
$p_{60} = g_e^*(\lambda_0+\lambda_2+\lambda_3)$, $p_{62} = \frac{\lambda_2}{\lambda_0+\lambda_2+\lambda_3}[1-g_e^*(\lambda_0+\lambda_2+\lambda_3)]$,
$p_{63} = \frac{\lambda_3}{\lambda_0+\lambda_2+\lambda_3}[1-g_e^*(\lambda_0+\lambda_2+\lambda_3)]$, $p_{69} = \frac{\lambda_0}{\lambda_0+\lambda_2+\lambda_3}[1-g_e^*(\lambda_0+\lambda_2+\lambda_3)]$,
$p_{61}^{(9)} = \frac{\lambda_0}{\lambda_0+\lambda_2+\lambda_3}[1-g_e^*(\lambda_0+\lambda_2+\lambda_3)]$, $p_{81} = p$, $p_{8,10} = q$, $p_{10,1} = 1$

From the transition probabilities, it can be verified that

$$p_{01}+p_{02}+p_{03} = 1$$
$$p_{12}+p_{13}+p_{14}+p_{15} = p_{12}+p_{13}+p_{14}+p_{18}^{(5)} = 1$$
$$p_{20} = p_{30} = 1$$
$$p_{40}+p_{42}+p_{43}+p_{46}+p_{47} = p_{40}+p_{42}+p_{43}+p_{46}+p_{41}^{(7)}+p_{4,10}^{(7)} = 1$$
$$p_{60}+p_{62}+p_{63}+p_{69} = p_{60}+p_{62}+p_{63}+p_{61}^{(9)} = 1$$
$$p_{81}+p_{8,10} = p+q = 1\ ,\quad p_{10,1} = 1$$

5. Mean Sojourn Time

If T denotes mean sojourn time in state 0, then

$\mu_0 = \int P(T>t)dt = \frac{1}{\lambda_0+\lambda_1+\lambda_2+\lambda_3}$, $\mu_1 = \frac{1}{\lambda_0+\lambda_2+2\lambda_3}$, $\mu_2 = \int_0^\infty \overline{G_2(t)}dt$, $\mu_3 = \int_0^\infty \overline{G_3(t)}dt$,
$\mu_4 = \frac{1-g_a^*(\lambda_0+\lambda_2+\lambda_3)}{\lambda_0+\lambda_2+\lambda_3}$, $\mu_6 = \frac{1-g_e^*(\lambda_0+\lambda_2+\lambda_3)}{\lambda_0+\lambda_2+\lambda_3}$,

$$\mu_8 = \int_0^\infty \overline{G_a(t)}dt,\ \mu_{10} = \int_0^\infty \overline{G_e(t)}dt$$

The unconditional mean time taken by the system to transit to any regenerative state i when it is counted from the epoch of entrance into that state is, mathematically, stated as

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -\frac{d}{ds}q_{ij}^*|_{s=0}$$
$$\therefore m_{01}+m_{02}+m_{03} = \mu_0$$
$$m_{12}+m_{13}+m_{14}+m_{15} = \mu_1$$
$$m_{40}+m_{42}+m_{43}+m_{46}+m_{47} = m_{40}+m_{42}+m_{43}+m_{46}+m_{41}^{(7)}+m_{4,10}^{(7)} = \mu_4$$
$$m_{60}+m_{62}+m_{63}+m_{69} = m_{60}+m_{62}+m_{63}+m_{61}^{(9)} = \mu_6$$
$$m_{12}+m_{13}+m_{14}+m_{18}^{(5)} = k_1$$

6. Mean time to system failure

To determine the MTSF of the system, we regard the failed states of the system as absorbing. By probabilistic arguments, we have

$$\phi_0(t) = Q_{02}(t)+Q_{03}(t)+Q_{01}(t)Ⓢ\phi_1(t)$$
$$\phi_1(t) = Q_{12}(t)+Q_{13}(t)+Q_{15}(t)+Q_{14}(t)Ⓢ\phi_4(t)$$
$$\phi_4(t) = Q_{42}(t)+Q_{43}(t)+Q_{47}(t)+Q_{40}(t)Ⓢ\phi_0(t)+Q_{46}(t)Ⓢ\phi_6(t)$$
$$\phi_6(t) = Q_{62}(t)+Q_{63}(t)+Q_{69}(t)+Q_{60}(t)Ⓢ\phi_0(t)$$

Now the MTSF, given that the system started at the beginning of state 0 is

$$T_0 = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{\mu_{0+}P_{01}\{\mu_1+P_{14}[\mu_4+P_{46}\mu_6]\}}{1-p_{01}p_{14}(p_{40}+p_{46}p_{60})}$$

7. Availability Analysis

$M_i(t)$ denotes the probability that the system starting in up regenerative state is up at time t without passing through any regenerative state. Thus, we have

$M_0(t) = e^{-(\lambda_0+\lambda_1+\lambda_2+\lambda_3)t}$, $M_1(t) = e^{-(\lambda_0+\lambda_2+2\lambda_3)t}$, $M_4(t) = e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_a(t)}$
$M_6(t) = e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_e(t)}$

Taking Laplace transform of the above equations and solving them for $s\to 0$, we get

$M_0^*(0) = \mu_0$, $M_1^*(0) = \mu_1$, $M_4^*(0) = \mu_4$, $M_6^*(0) = \mu_6$

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy

$$A_0(t) = M_0(t)+q_{01}(t)©A_1(t)+q_{02}(t)©A_2(t)+q_{03}(t)©A_3(t)$$
$$A_1(t) = M_1(t)+q_{12}(t)©A_2(t)+q_{13}(t)©A_3(t)+q_{14}(t)©A_4(t)+q_{18}^{(5)}(t)©A_8(t)$$
$$A_2(t) = q_{20}(t)©A_0(t)$$
$$A_3(t) = q_{30}(t)©A_0(t)$$

$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{42}(t)©A_2(t) + q_{43}(t)©A_3(t) + q_{46}(t)©A_6(t) + q_{41}^{(7)}(t)©A_1(t) + q_{4,10}^{(7)}(t)©A_{10}(t)$

$A_6(t) = M_6(t) + q_{60}(t)©A_0(t) + q_{62}(t)©A_2(t) + q_{63}(t)©A_3(t) + q_{61}^{(9)}(t)©A_1(t)$

$A_8(t) = q_{81}(t)©A_1(t) + q_{8,10}(t)©A_{10}(t)$

$A_{10}(t) = q_{10,1}(t)©A_1(t)$

The steady state availability of the system is given by $A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1}$

Where $N_1 = \mu_0\left[p_{14}p_{46}p_{81} - p_{14}p_{46}p_{61}^{(9)} - p_{14}p_{4,10}^{(7)} + p_{18}^{(5)}p_{8,10}\right] + p_{01}[\mu_1 + p_{14}\{\mu_4 + p_{46}\mu_6\}]$

and $D_1 = \mu_0\left[1 - p_{18}^{(5)} - p_{14}\left\{p_{41}^{(7)} + p_{4,10}^{(7)} + p_{46}p_{61}^{(9)}\right\}\right] + p_{01}\xi_1 + p_{01}[p_{12}\mu_2 + p_{13}\mu_3 + p_{14}\{p_{42}\mu_2 + p_{43}\mu_3 + p_{46}(p_{62}\mu_2 + p_{63}\mu_3)\}] + p_{02}\mu_2 + p_{43}\mu_3 + p_{46}(p_{62}\mu_2 + p_{63}\mu_3) + (p_{02}\mu_2 + p_{03}\mu_3)\left[1 - p_{18}^{(5)} - p_{14}\left\{p_{41}^{(7)} + p_{4,10}^{(7)} + p_{46}p_{61}^{(9)}\right\}\right] + p_{01}\left[p_{14}(\xi_2 + p_{46}\xi_3) + p_{18}^{(5)}\mu_8 + \left(p_{14}p_{4,10}^{(7)} + p_{18}^{(5)}p_{8,10}\right)\mu_{10}\right]$

8. Busy period analysis

Busy period analysis of the assistant repairman:

Using probabilistic arguments, we have

$B_0^a(t) = q_{01}(t)©B_1^a(t) + q_{02}(t)©B_2^a(t) + q_{03}(t)©B_3^a(t)$

$B_1^a(t) = q_{12}(t)©B_2^a(t) + q_{13}(t)©B_3^a(t) + q_{14}(t)©B_4^a(t) + q_{18}^{(5)}(t)©B_8^a(t)$

$B_2^a(t) = q_{20}(t)©B_0^a(t)$

$B_3^a(t) = q_{30}(t)©B_0^a(t)$

$B_4^a(t) = W_4(t) + q_{40}(t)©B_0^a(t) + q_{41}^{(7)}(t)©B_1^a(t) + q_{42}(t)©B_2^a(t) + q_{43}(t)©B_3^a(t) + q_{46}(t)©B_6^a(t) + q_{4,10}^{(7)}(t)©B_{10}^a(t)$

$B_6^a(t) = q_{60}(t)©B_0^a(t) + q_{61}^{(9)}(t)©B_1^a(t) + q_{62}(t)©B_2^a(t) + q_{63}(t)©B_3^a(t)$

$B_8^a(t) = W_8(t) + q_{81}(t)©B_1^a(t) + q_{8,10}(t)©B_{10}^a(t)$

$B_{10}^a(t) = q_{10,1}(t)©B_1^a(t)$

Where $W_4(t) = \frac{1}{\lambda_0+\lambda_2+\lambda_3}[\lambda_0 + (\lambda_2 + \lambda_3)e^{-(\lambda_0+\lambda_2+\lambda_3)t}]\overline{G_a(t)}$ and $W_8(t) = \overline{G_a(t)}$

Taking Laplace transform of the above equations and taking $s \to 0$, we will get

$W_4^*(0) = \xi_2$, $W_8^*(0) = \mu_8$

In steady-state solution, the total fraction of time under which system is under repair of assistant repairman is given by $B_0^a = \lim_{s\to 0} s\,B_0^{a^*} = \frac{N_2}{D_1}$

Where $N_2 = p_{01}\left(p_{14}\xi_2 + p_{18}^{(5)}\mu_8\right)$ and D_1 is same as above.

Busy period analysis of the expert repairman:

Using probabilistic arguments, we have

$B_0^e(t) = q_{01}(t)©B_1^e(t) + q_{02}(t)©B_2^e(t) + q_{03}(t)©B_3^e(t)$

$B_1^e(t) = W_1(t) + q_{12}(t)©B_2^e(t) + q_{13}(t)©B_3^e(t) + q_{14}(t)©B_4^e(t) + q_{18}^{(5)}(t)©B_8^e(t)$

$B_2^e(t) = W_2(t) + q_{20}(t)©B_0^e(t)$

$B_3^e(t) = W_3(t) + q_{30}(t)©B_0^e(t)$

$B_4^e(t) = q_{40}(t)©B_0^e(t) + q_{41}^{(7)}(t)©B_1^e(t) + q_{42}(t)©B_2^e(t) + q_{43}(t)©B_3^e(t) + q_{46}(t)©B_6^e(t) + q_{4,10}^{(7)}(t)©B_{10}^e(t)$

$B_6^e(t) = W_6(t) + q_{60}(t)©B_0^e(t) + q_{61}^{(9)}(t)©B_1^e(t) + q_{62}(t)©B_2^e(t) + q_{63}(t)©B_3^e(t)$

$B_8^e(t) = q_{81}(t)©B_1^e(t) + q_{8,10}(t)©B_{10}^e(t)$

$B_{10}^e(t) = W_{10}(t) + q_{10,1}(t)©B_1^e(t)$

Where $W_1(t) = e^{-(\lambda_0+\lambda_2+2\lambda_3)t} + [\lambda_3 e^{-(\lambda_0+\lambda_2+2\lambda_3)t}©1]$, $W_2(t) = \overline{G_2(t)}$, $W_3(t) = \overline{G_3(t)}$,

$W_6(t) = e^{-(\lambda_0+\lambda_2+\lambda_3)t}\overline{G_e(t)} + [\lambda_0 e^{-(\lambda_0+\lambda_2+\lambda_3)t}©1]\overline{G_e(t)}$, $W_{10}(t) = \overline{G_e(t)}$

Taking Laplace transform of the above equations and taking $s \to 0$, we will get

$W_1^*(0) = \xi_1$, $W_2^*(0) = \mu_2$, $W_3^*(0) = \mu_3$, $W_6^*(0) = \xi_3$, $W_{10}^*(0) = \mu_{10}$

In steady-state solution, the total fraction of time for which expert repairman is busy is given by $B_0^e = \lim_{s\to 0} s\,B_0^{e^*} = \frac{N_3}{D_1}$

Where $N_3 = p_{01}[\xi_1 + p_{12}\mu_2 + p_{13}\mu_3 + p_{14}\{p_{42}\mu_2 + p_{43}\mu_3 + p_{46}(p_{62}\mu_2 + p_{63}\mu_3)\}] + (p_{02}\mu_2 + p_{03}\mu_3)\left[1 - p_{14}p_{41}^{(7)} - p_{14}p_{4,10}^{(7)} - p_{18}^{(5)} - p_{14}p_{46}p_{61}^{(9)}\right] + p_{01}p_{14}p_{46}\xi_3 + p_{01}\mu_{10}\left(p_{14}p_{4,10}^{(7)} + p_{18}^{(5)}p_{8,10}\right)$

And D_1 is same as above.

9. Expected number of visits by expert repairman

Using probabilistic arguments, we have the following relations for expert visits i.e. $V_i(t)$:

$$V_0(t) = Q_{01}(t)ⓢ[1 + V_1(t)] + Q_{02}(t)ⓢ[1 + V_2(t)] + Q_{03}(t)ⓢ[1 + V_3(t)]$$

$$V_1(t) = Q_{12}(t)ⓢV_2(t) + Q_{13}(t)ⓢV_3(t) + Q_{14}(t)ⓢV_4(t) + Q_{18}^{(5)}(t)ⓢV_8(t)$$

$$V_2(t) = Q_{20}(t)ⓢV_0(t)$$

$$V_3(t) = Q_{30}(t)ⓢV_0(t)$$

$$V_4(t) = Q_{40}(t)ⓢV_0(t) + Q_{41}^{(7)}(t)ⓢ[1 + V_1(t)] + Q_{42}(t)ⓢ[1 + V_2(t)] + Q_{43}(t)ⓢ[1 + V_3(t)] + Q_{46}(t)ⓢ[1 + V_6(t)] + Q_{4,10}^{(7)}(t)ⓢ[1 + V_{10}(t)]$$

$$V_6(t) = Q_{60}(t)ⓢ[1 + V_0(t)] + Q_{61}^{(9)}(t)ⓢV_1(t) + Q_{62}(t)ⓢV_2(t) + Q_{63}(t)ⓢV_3(t)$$

$$V_8(t) = Q_{81}(t)ⓢ[1 + V_1(t)] + Q_{8,10}(t)ⓢ[1 + V_{10}(t)]$$

$$V_{10}(t) = Q_{10,1}(t)ⓢV_1(t)$$

In steady-state, the number of visits per unit time is given by taking $s \to 0$ and $t \to \infty$

$$V_0 = \lim_{t\to\infty} \frac{V_0(t)}{t} = \lim_{s\to\infty}[sV_0^{**}(s)] = \frac{N_4}{D_1}$$

Where $N_4 = 1 - p_{14}\left(p_{41}^{(7)} + p_{4,10}^{(7)} + p_{46}p_{61}^{(9)}\right) - p_{18}^{(5)} - p_{01}p_{14}p_{40} + p_{01}\left[p_{18}^{(5)} + p_{14} + p_{14}p_{46}p_{60}\right]$

And D_1 is same as above.

10. Cost-Benefit analysis

The expected total profit in steady state is given by

$P_2 = C_0A_0 - C_1B_0^a - C_2B_0^e - C_3V_0$

Where C_0 is total revenue per unit time of the system.

C_1 is cost per unit time for which the assistant repairman is busy.

C_2 is cost per unit time for which the expert repairman is busy.

C_3 is cost per visit of the expert repairman.

11. Graphical Analysis

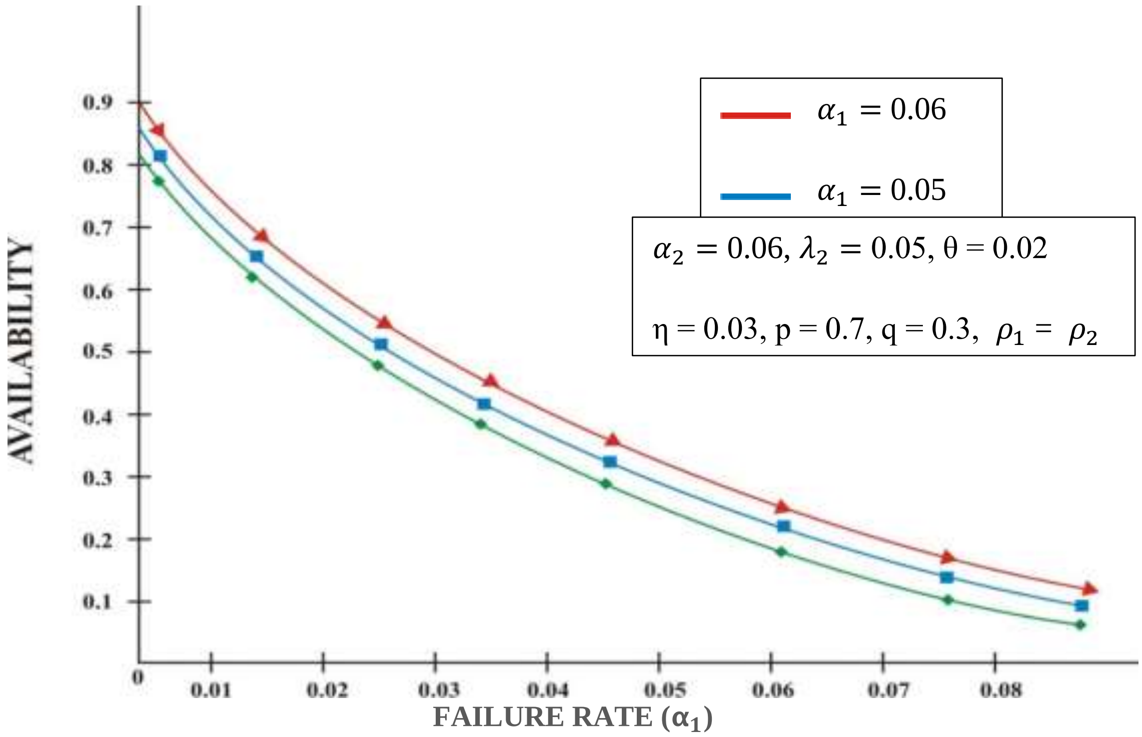

Fig. 2: AVAILABILITY VS. FAILURE RATE (λ_1) FOR DIFFERENT VALUES OF REPAIR RATE (α_1)

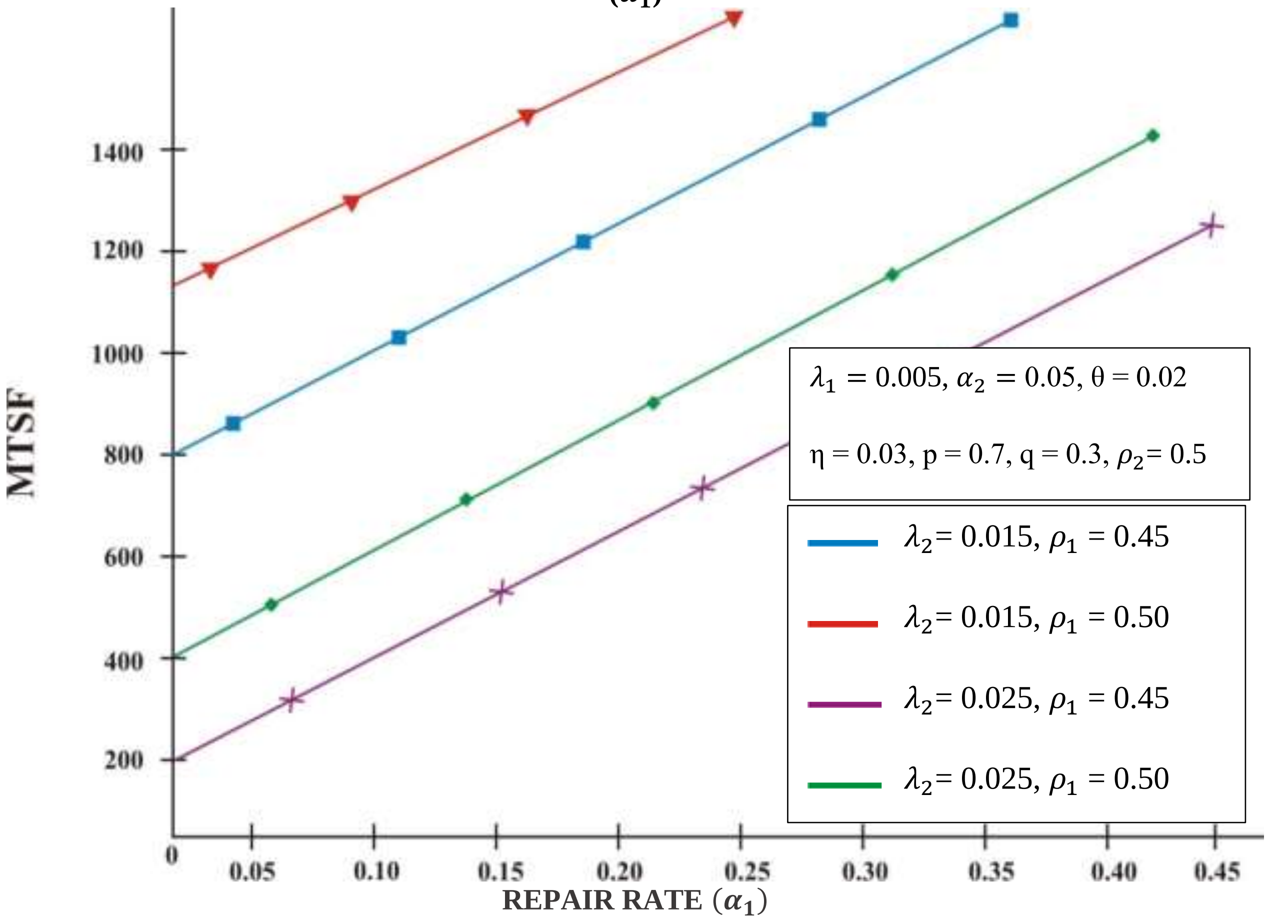

Fig. 3: MTSF Vs. REPAIR RATE (α_1) FOR DIFFERENT VALUES OF FAILURE RATE (λ_2) AND CORRELATION CO-EFFICIENT (ρ_1)

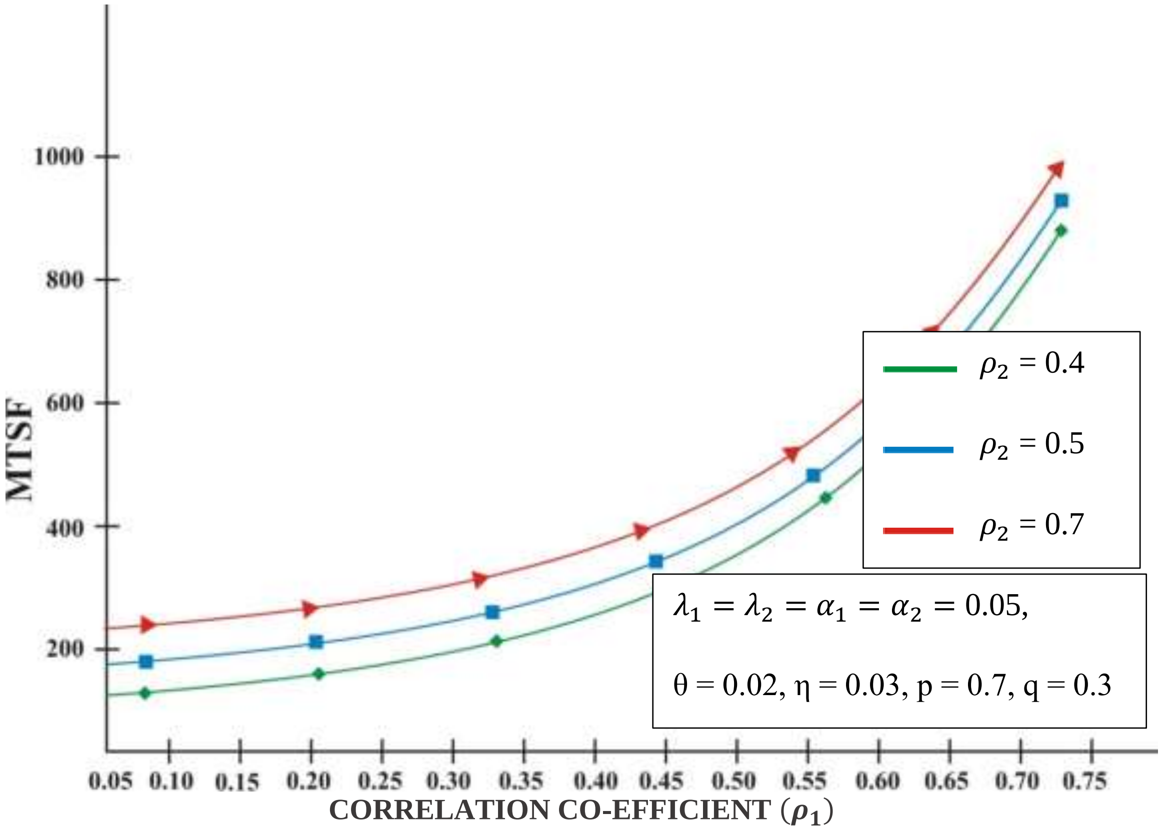

Fig. 4: MTSF Vs. CORRELATION CO-EFFICIENT (ρ_1) FOR DIFFERENT VALUES OF ρ_2

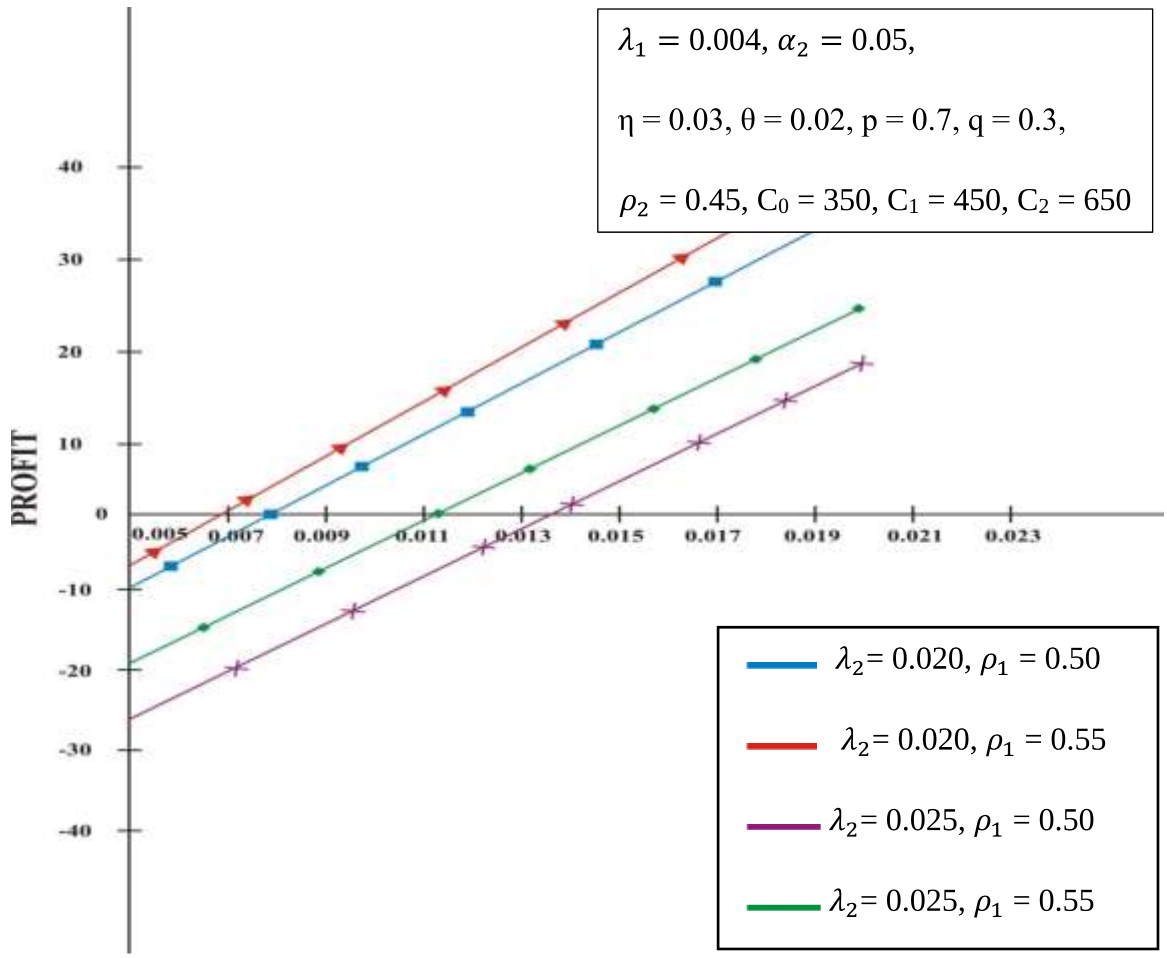

Fig. 5: PROFIT Vs. REPAIR RATE (α_1) FOR DIFFERENT VALUES OF FAILURE RATE (λ_2) AND CORRELATION CO-EFFICIENT (ρ_1)

12. Conclusion

Fig. 2 shows that the behavior of availability with respect to failure rate (λ_1). It gets decrease with increase in the values of failure rate.

Fig. 3 shows the behavior of MTSF with respect to repair rate (α_1) for different values of failure rate (λ_2) and correlation coefficient (ρ_1). It gets increases with increase in the values of repair rate (α_1). It is lower for higher values of failure rate (λ_2) but it is higher for higher values of correlation coefficient (ρ_1).

Fig. 4 reveals the pattern of MTSF with respect to correlation coefficient (ρ_1) for different values of (ρ_2). It is observed that the MTSF gets increase with increase in the values of correlation coefficient (ρ_1) and it is higher for higher values of (ρ_2)

Fig. 5 reveals the pattern of the profit with respect to repair rate (α_1) for different values of failure rate (λ_2) and correlation coefficient (ρ_1). It is clear from the graph that the profit increases with increase in the value of repair rate (α_1) and it is lower for higher value of repair rate (α_1) but higher for the higher values of correlation coefficient (ρ_1).

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