STAGNATION- POINT FLOW AND HEAT TRANSFER OF NANOFLUID OVER A SHRINKING SHEET

Vijayalakshmi A.R.

Department of Mathematics, Maharani's Science College for women,

Palace road, INDIA.Bengaluru – 560 001.E-mail: drarv@rediffmail.com

Abstract

The present paper is regarding the combined effects ofBrownian motion, thermophoresis and thermal radiation on stagnation–point flow and heat transfer due to nanofluid towards a nonlinearly shrinking sheet. Using a similarity transformation, the governing mathematical equations are transformed into coupled nonlinear ordinary differential equations which are then solved numerically using fourth order Runga–Kutta method with shooting technique.

Key words: Stagnation–point flow, heat transfer, Brownian motion, thermophoresis, thermal radiation, shrinking sheet

2010 Mathematics Subject Classification: 37E35, 74SXX

1 Introduction

In many industries such as power, manufacturing and transportation fluid heating and cooling are important. Effective cooling techniques are needed for cooling any sort of high energy device. To enhance the thermal conductivity of base fluids namely water, engine oil or ethylene glycol, oil and other lubricants and polymer solutions, nanoparticles namely oxide ceramics, nitride ceramics, carbide ceramics, metals, semiconductors are suspended in the fluids. Using a nanofluid as a heat transfer working fluid, has gained much attention due to its potential advantages which include higher thermal conductivity than the pure fluids, excellent stability and little increase in pressure drop. Due to better performance of heat exchange, the nanofluids can be used in several engineering and industrial applications which include power generation in a power plant, production of microelectronics, advanced nuclear system and many others. Buongiorno [5] studied convective transportation in nanofluids by considering seven slip conditions that may produce a relative velocity within the base fluid and nanoparticles. Only Brownian motion and thermophoresis out of these seven slip mechanisms were found to be important mechanisms. By using Buogiorno's model, Kuznetsov and Nield [7] investigated the nanofluid boundary layer uniformly convecting fluid flow.The boundary layer flow behavior towards a linearly or non-linearly stretching sheet plays a significant role for solving engineering problems and has applications in metal spinning, rubber sheet manufacturing, production of glass fibres, wire drawing, extrusion of polymer sheets, petroleum industries, polymer processing. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching.

The study of stagnation point flow and heat transfer over a stretching surface has a large number of applications such as in the colons of electronic devices, paper production, glass blowing and continuous casting, aerodynamic extrusion of plastic sheets, etc. In certain polymeric and metallurgical processes, nonlinearly stretching /shrinking effects are very much important because the final product is strongly influenced by the processes of stretching and the rate of cooling. In the case of flow over a shrinking sheet, the fluid is stretched towards a slot and the flow is quite different from the stretching case. The vorticity generated due to shrinking sheet is not confined within the boundary layer and consequently a situation appears where some other external forces are to be imposed. Several researchers studied the boundary layer flow over a shrinking surface under different physical conditions (Bhattacharyya etal [3], Bhattacharyya [4], Mahapatraetal.[9]). Stagnation point flow and mass transfer with chemical reaction past a permeable stretching/ shrinking sheet in a nanofluid was studied byRosca et al. [16]. Unsteady boundary layer flow and heat transfer of nanofluid over a permeable stretching/ shrinking sheet was investigated by Bachok et al. [2] and they found that dual solutions exist for shrinking sheet. The unsteady flow over a continuously shrinking surface with mass suction in a nanofluid was investigated byRohini et al. [15]. Sarma and Rao [16] examined the viscoelastic fluid flow by considering stretching sheet. Vajravelu [17] studied flow and heat transfer in a viscous fluid over a nonlinear stretching sheet without using the impact of viscous dissipation. Prasad etal [13] examined the mixed convection heat transfer aspects with variable fluid flow properties over a non-linear stretching surface. Using numerical methods, Bachok etal [12] studied the two-dimensional stagnation-point flow over a stretching/shrinking sheet in a nano fluid and the effects of the solid volume fraction on the fluid flow and heat transfer characteristics were examined.Najwa Najib etal [11] investigated numerically steady two-dimensional stagnation point flow and heat transfer in nanofluid over a stretching/shrinking cylinder in the presence of slip effect and studied stability analysis. The effect of slip at the boundary caused to decrease the surface stress but increase the heat transfer rate at the surface. Anuar etal [1] studied the effect of slip on stagnation point flow and heat transfer over an exponentially stretching/shrinking sheet in hybrid nano fluid using numerical methods.Stability analysis showed that the solution is linearly stable.Ismail etal [6] studied numerically the stability analysis for the stagnation-point flow and heat transfer of nanofluid over a shrinking surface in the presence of magnetic field and thermal radiation with slip effect. Dual solutions exist at certain ranges in the study. The stability analysis determine which one is stable between the two solutions.Muhammad Ramzan etal [10] studied the comparison of single walled and multi walled cabon nano tube and water towards a stretching surface influenced by nonlinear thermal radiation and found that single walled cabon nano tube nano particles has greater thermal radiation than the multi walled cabon nano tube nano particles. Magyariand Pantokratoras[8] concluded that the thermal radiation in the linearized Rosseland approximation reduces to a simple rescaling of the Prandtl number by a factor involving the radiation parameter.

In this paper we study the dynamics of the natural convection boundary layer stagnation point flow and heat transfer of a viscous incompressible nanofluid over a nonlinearly shrinking surface in the presence of thermal radiation.

2 Mathematical Formulation

We consider the steady two dimensional stagnation-point flow of a viscous incompressible nanofluid over a nonlinearly stretching/shrinking sheet in the presence of thermal radiation. The coordinates (x, y) are such that x is along the sheet and y is normal to the sheet. It is assumed that the velocity of the sheet is $u_w(x) = c\,x^n$ where c is the stretching/ shrinking rate and n is an arbitrary positive constant known as the stretching index. The velocity outside the boundary layer is $u(x) = a\,x^n$, where $a > 0$ is a constant that denotes the strength of the stagnation flow.

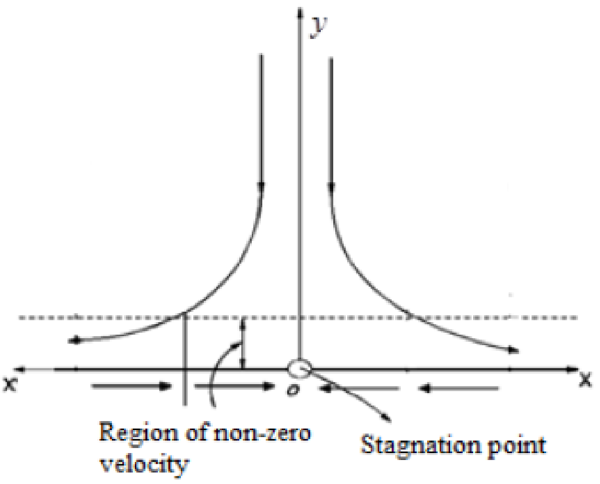

Fig.1 Physical model and coordinate system

We assume that the constant temperature and constant nanoparticle fraction at the surface of the sheet are T_w and C_w, these values in the basic state are denoted by T_∞ and C_∞ . A schematic representation of this problem is shown in Fig.1. Under these assumptions, the basic equations of mass, momentum, thermal energy and nanoparticle fraction can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{\rho_f C_p}\frac{\partial q_r}{\partial y}, \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{2.4}$$

where u and v are the velocity components along x and y axes respectively, υ is the kinematic viscosity, ρ is the density, C_p is the specific heat at constant pressure, α_m is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis coefficient, q_r is the radiative heat flux, τ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid and U(x) is the free stream velocity.

Equation (2.3) shows that heat can be transported in a nanofluid by convection, conduction, nanoparticle diffusion and radiation. Equation (2.4) shows that the nanoparticles can move homogeneously within the fluid but they also possess a slip velocity relative to the fluid due to Brownian diffusion and the thermophoresis.

The boundary conditions for the problem are

$$u=U_w(x)=cx^n, v=0, T=T_w, C=C_w \text{ at } y=0,$$

$$u\to U(x)=ax^n,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty. \tag{2.5}$$

The radiative heat flux is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.6}$$

where σ^* is the Stefan Boltzmann constant and k^* is the Rosseland mean absorption coefficient. Assuming the temperature difference within the flow is such that T^4 can be expanded in a Taylor series about T_∞ and neglecting higher order terms, we get $T^4=4T_\infty^3 T-3T_\infty^4.$

Using this result and equation (2.6), we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (2.7)$$

We look for a similarity solution of equations (2.2) - (2.4) together with the boundary conditions (2.5) of the following form

$$\psi = \sqrt{\frac{2\upsilon a}{n+1}}\, x^{\frac{n+1}{2}} F(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \eta = y\sqrt{\frac{a(n+1)}{2\upsilon}}\, x^{\frac{n-1}{2}} \quad (2.8)$$

where the stream function ψ is defined as

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \quad (2.9)$$

Hence from equation (2.8) we have

$$v = -x^{\frac{n-1}{2}}\sqrt{\frac{a\upsilon(n+1)}{2}}\left(F(\eta) + \frac{n-1}{n+1}\eta F'(\eta)\right), u = ax^n F'(\eta) \quad (2.10)$$

Substituting equations (2.7) and (2.8) into equations (2.2) – (2.4), we have

$$F''' + FF'' + \frac{2n}{n+1}\left(1 - F'^2\right) = 0, \quad (2.11)$$

$$\frac{1}{\Pr_{eff}}\theta'' + F\,\theta' + Nb\,\theta'\varphi' + Nt\,\theta'^2 = 0, \quad (2.12)$$

$$\varphi'' + LeF\varphi' + \frac{Nt}{Nb}\theta'' = 0, \quad (2.13)$$

where the prime denotes the differentiation with respect to η. The dimensionless parameters for this problem are $\Pr_{eff} = \frac{\Pr}{1 + 4R/3}$, the effective Prandtl number, $\Pr = \frac{\upsilon}{\alpha_m}$ is the Prandtl number, $R = \frac{4\sigma^* T_\infty^3}{k^* \alpha_m}$ is the radiation parameter, $Nb = \frac{\tau D_B (C_w - C_\infty)}{\upsilon}$ is the Brownian motion parameter, $Nt = \frac{\tau D_T (T_w - T_\infty)}{\upsilon T_\infty}$ is the thermophoresis parameter and $Le = \frac{\upsilon}{D_B}$ is the Lewis number.

The boundary conditions then become

$$F(0) = 0, F'(0) = \frac{c}{a} = \alpha, \theta(0) = 1, \varphi(0) = 1$$

$$F'(\eta)\to 1, \theta(\eta)\to 0, \varphi(\eta)\to 0 \text{ as } \eta\to\infty. \tag{2.14}$$

The skin friction coefficient C_f, the local Nusselt number Nu_x and the local Sherwood number Sh_x and the local Reynolds number, Re_x are given by

$$C_f = \frac{\tau_w}{\rho U_w^2},\ Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x q_m}{D_B(C_w - C_\infty)},\ \mathrm{Re}_x = \frac{U_w x}{\upsilon} \tag{2.15}$$

where k is the thermal conductivity of the nanofluid, τ_w is the surface shear stress, q_w is the surface heat flux and q_m is the mass flux at the surface and are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_m = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=o} + (q_r)_{y=0}. \tag{2.16}$$

Using (2.8) and (2.16) in equation (2.15), we get

$$\mathrm{Re}_x^{1/2} C_f = F''(0), \mathrm{Re}_x^{-1/2} Sh_x = -\varphi'(0), \mathrm{Re}_x^{-1/2} Nu_x = -\left(1+\frac{4}{3}R\right)\theta'(0) \tag{2.17}$$

3 Results and Discussion

In this paper, using similarity transformation the problem of stagnation-point flow and heat transfer in a nanofluid past a nonlinearly shrinking sheet is studied. The system of nonlinear ordinary differential equations (2.11)–(2.13) with the boundary conditions (2.14) was solved numerically using shooting method. We have found that non–unique solutions exist for certain chosen parameters and the flow and heat transfer are significantly influenced by these parameters.

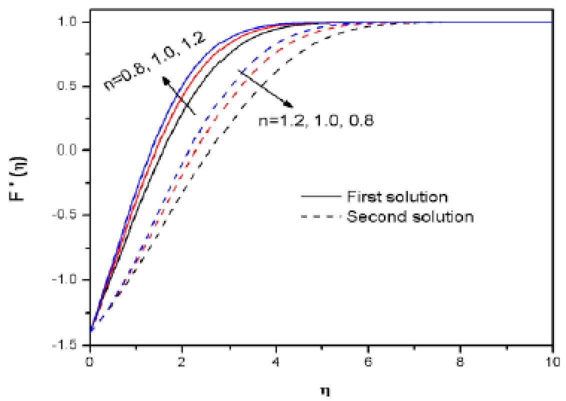

Fig. 2. Variation of velocity profiles $F'(\eta)$ for several values of the power-law index parameter n with $\alpha = -1.40$

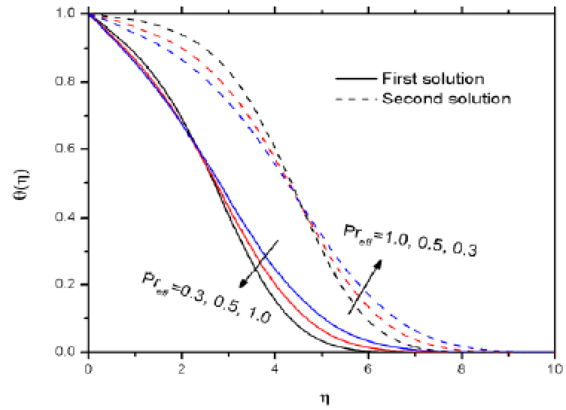

Fig.3. Variation of temperature distribution $\theta(\eta)$ for several values of effective Prandtl number with $\alpha = -1.40, Nb = 0.1, Nt = 0.1, Le = 1, n = 1.2$

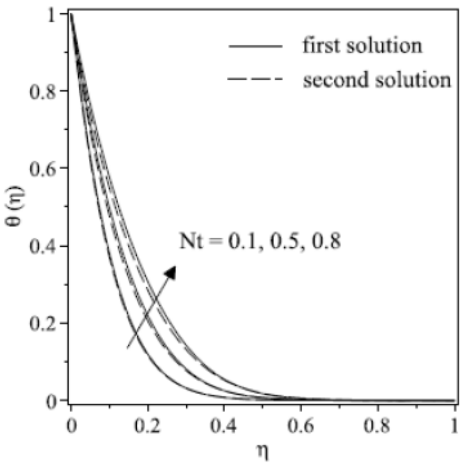

Fig.4. Effect of the thermophoresis parameter Nt **on the temperature distribution** $\theta(\eta)$ **when** $\alpha = -1.40$, $Nb = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

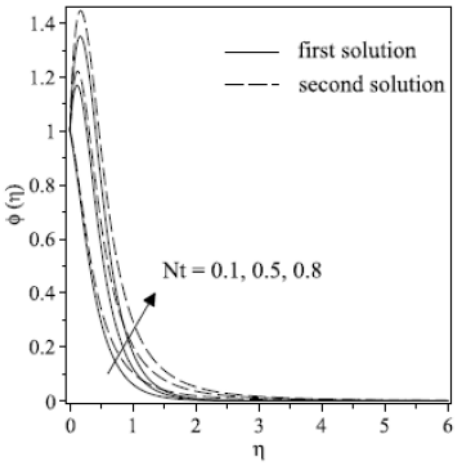

Fig.5. Effect of the thermophoresis parameter Nt **on the concentration distribution** $\phi(\eta)$ **when** $\alpha = -1.40$, $Nb = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

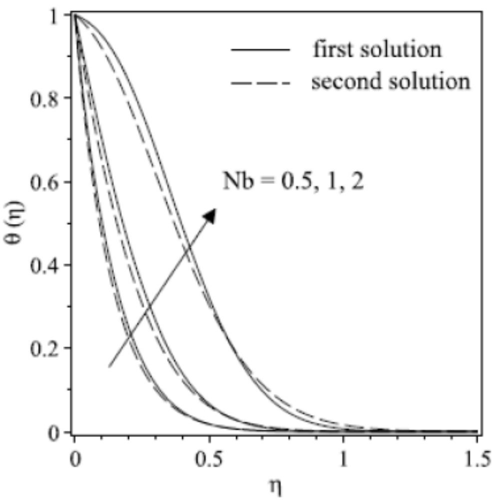

Fig.6. Effect of the Brownian motion parameter Nb **on the temperature distribution** $\theta(\eta)$ **when** $\alpha = -1.40$, $Nt = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

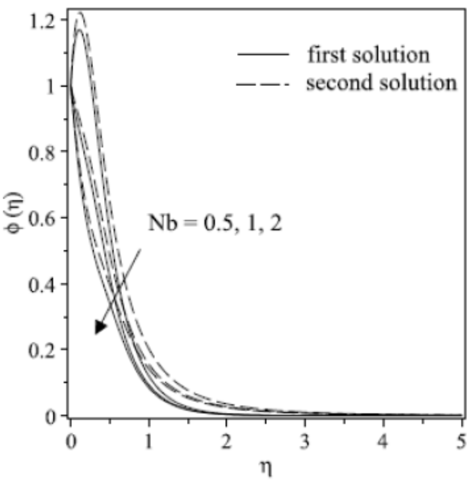

Fig.7. Effect of the Brownian motion parameter Nb **on the concentration distribution** $\phi(\eta)$ **when** $\alpha = -1.40$, $Nt = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

Fig. 2. shows the variations of velocity profiles $F'(\eta)$ for different values of the stretching index parameter n. The value $n = 0$ represents a uniformly moving surface. In this paper we have considered only positive values of n. The value $n = 1$ denotes linear case and $n \neq 1$ denotes nonlinear case.Fig. 3 shows the influence of the effective Prandtl number $\Pr_{eff}$ on the temperature profiles $\theta(\eta)$ for other fixed parameters. As $\Pr_{eff}$ increases, the temperature at a point decreases except in a small region near the sheet (for first solution branch). The

second solution branch is different, for the given parameters as $\Pr_{eff}$ increases temperature at a point also increases.

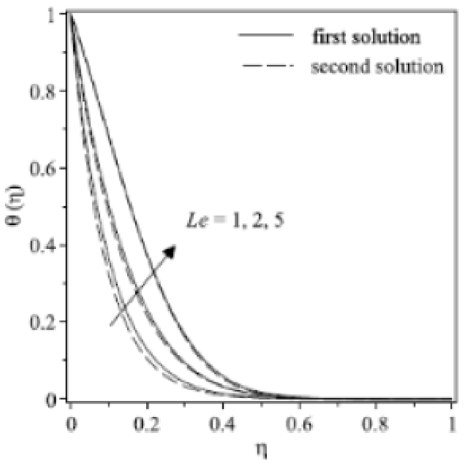

Fig.8. Effect of the Lewis number Le on the temperature distribution $\theta(\eta)$ when $\alpha = -1.40$, $Nb = 0.1$, $Nt = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

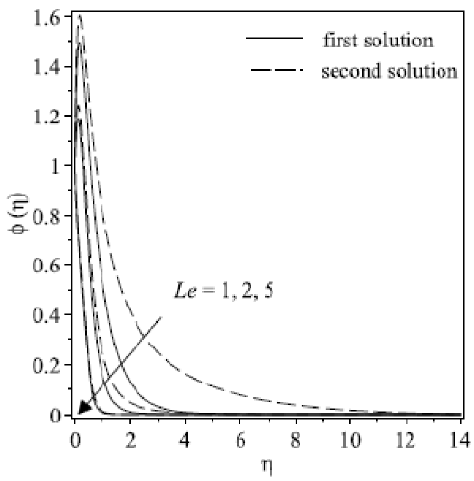

Fig.9. Effect of the Lewis number Le on the concentration distribution $\phi(\eta)$ when $\alpha = -1.40$, $Nb = 0.1$, $Nt = 0.1$, $Le = 1, n = 1.2$, $\Pr_{eff} = 1$

Fig. 4 shows that the temperature profiles as well as the boundary layer thickness of the thermal field increase with increasing Nt. As a result, increasing values of Nt decreases the local Nusselt number. Fig. 5 indicatethat increasing Nt is to increase the concentration for both first and second solutions, and as a result decrease the nanoparticle concentration gradient at the surface resulting in decrease of the local Sherwood number. This is due to the thermophoresis effect, which warm the fluid in the boundary layer. This result is in good agreement with that reported in Fig. 4 in the paper by Rana and Bhargava [14] for the case of flow and heat transfer over a nonlinearly stretching sheet in a nanofluid without suction effect. Fig. 6 isdrawn to observe the effect of the Brownian motion parameter Nb on the temperature profiles which indicate that by increasing the Brownian motion parameter Nb, the temperature and the thermal boundary layer thickness increase. This phenomenon leads to decrease the local Nusselt number. From fig. 7 we observe that nanoparticle concentration is decreasing as Nb increases. The Brownian motion acts to warm the fluid in the boundary layer and at the same time intensify particle deposition away from the fluid regime to the surface which results in a decrease of the nanoparticle concentration boundary layer thickness for both solutions. Hence the concentration gradient at the surface increases and in turn increases the local Sherwood number. This finding is in accordance with the result reported in Fig. 4 by Rana and

Bhargava[14]. Fig. 6 and Fig.7indicates that Brownian motion parameter provides important effect on temperature and concentration. Fig. 8 shows the effect of the Lewis number Le on the temperature profiles. It can be seen that as Le increases the thermal boundary layer thickness also increases which reduces the local Nusselt number. Fig. 9shows the effects of the Lewis number Le on the concentration profiles. It is found that the concentration of both solutions decrease as Le Le increases.

4 Conclusion

It is observed that nanofluid velocity increases with the increase ofthe non-linearity stretching index n for the first solution and the flow velocity decreases with the increase of n.An increase in Prandtl number means a decrease of fluid thermal conductivity which ceases the reduction of the thermal boundary thickness.

The increasing of thermophoresis parameter Nt and the Brownian motion parameter Nb is to increase the temperature in the boundary layer which consequently reduces the heat transfer rate at the surface and as Nb increases and Nt decreases, the nanoparticle concentrationdecreases which results in increase of the local Sherwood number.

The increase of Lewis number Le leads to an increase of the temperature but a decrease in the nanoparticle concentration.

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