Maximization of profit in a multi-server queue with encouraged arrival customers

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Abstract

To increase profit by enticing more users through the implementation of a multi-server encouraged arrival queue. Users may experience a slow service process as a result of server difficulties, resulting in an increase in waiting costs and waiting time. To increase the transactions thereby to reduce the waiting cost and waiting time, we introduce the encouraged arrival. We provide numerical examples to show the effectiveness of the reduction of the estimated total cost and the reduction of waiting cost. Little's law is verified.

Keywords: Encouraged arrival, system size, Multi-server, total cost, little's law

1. Introduction

The study of waiting lines or queues is known as queuing theory. An investigation of queues is often known as "Queuing theory." In this work, we maximize the system size to gain a significant profit thereby minimizing the predicted waiting cost and expected total cost by using M/M/K queueing model. Souvik pal, G.Suseendran et al minimized the waiting time and server utilization by using M/M/C model in [1]. Using M/M/C queueing model, the minimum total cost was estimated in [2]. Encouraged arrivals M/M/C/N Feedback queuing model with retention of impatient customers found in [3]. One-server limited capacity Markovian queuing system with encouraged arrival is studied in [4]. Using limited capacity solo server queueing model encouraged arrivals and reverse reneging is found in [5]. A stochastic model is built in [6]. M/M/1/N feedback queue with retention of reneged customers is found in [7]. The system's cost-profit analysis is carried out by creating a cost model in [8]. A single server queueing model of finite size with various vacations and encouraged client arrival was examined in [9]. Under a fuzzy environment, a single service provider finite space queue with encouraged or discouraged arrivals and a modified reneging policy was examined in [10]. A novel and practical method for calculating balking loss as a waiting cost was studied in [12].

This paper is organized as follows; we derived the mathematical model formulation in section 4. The performance measures in section were given in 5. We provide the numerical illustration and simulation results to compare the introduced the encouraged arrival queue with poisson queue for the system size (L_s) and queue (L_q) in section 6. We determined the values for waiting cost, operating cost and total estimated cost for the introduced encouraged arrival M/M/C model in 7. Encouraged arrival and poisson arrival comparision table for operating cost, waiting cost and the estimated total cost shown in 8.

2. Notations

In this section, we provide notations that we use:

$\lambda(1+\zeta)$=The encouraged arrival

μ= service rate

K=no. of servers

P_0=Probability of zero number of customer in the system

P_n = Probability of "n" number of customers in the system

Lq = The expected number of customers in the queue

Ls = The expected number of customers in the system

Wq= The expected waiting time of customers in the queue

Ws= The expected waiting time of customers in the system

ETC=Estimated total cost

OP cost=Operating cost

3. Preliminaries

In this section, we provide the preliminaries that we require:

Definition 3.1 Encouraged arrival

The phrase "encouraged arrivals" arose from the circumstance that a system encounter following the announcement of offers and discounts by enterprises. Encouraged arrivals are a novel element to queuing theory's current consumer behavior.

Definition:3.2 Implementing encouraged arrival queueing model in Digital Payment platforms:

Instead of paying workers or providing services, they provide rewards and discounts. Offering discounts using encouraged arrival queueing model to encourage customers to utilize digital payments to attract more customers and gain extra **profit**. Instead of offering customer assistance and paying customer service representatives, they provide discounts.

4. MATHEMATICAL MODEL FORMULATION

In this section, we derive the governing differential-difference equations:

The governing Differential-difference equations:

$P_0'(t) = -\lambda\,(1 + \zeta)\,P_0(t) + K\mu P_1(t)$ … (1)

$P_n'(t) = \lambda\,(1 + \zeta)P_{n-1}(t) - \{\lambda\,(1 + \zeta) + K\mu\}\,P_n(t) + K\mu P_{n+1}(t)$… (2)

In the steady state, $as\ t \to \infty$, $P_n(t) = P_n$ and therefore, $P_n'(t) = 0\ as\ t \to \infty$ then, the equations are

$0 = -\lambda\,(1 + \zeta)\,P_0 + K\mu P_1$ … (3)

$0 = \lambda\,(1 + \zeta)P_{n-1} - \{\lambda\,(1 + \zeta) + K\mu\}\,P_n + K\mu P_{n+1}$… (4)

Steady state solutions

$$P_n = \begin{cases} \frac{\lambda(1+\zeta)^n}{n!\mu^n} P_0 & where\ \ 0 \le n \le K \\ \frac{\lambda(1+\zeta)}{K^{n-K}k!\mu^n}^{n} P_0 & where\ \ n \ge K \end{cases}$$

$$P_0 = \left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K}{\left(K!\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)\right)} + \sum_{n=0}^{k-1}\left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^n}{n!}\right) \right)^{-1} - - - - - - - - (5)$$

5. Performance measures

The expected number of jobs in the queue

$$L_q = \left(\left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K \left(\frac{\lambda(1+\zeta)}{K\mu}\right)}{K!\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)^2} \right) P_0 \right) - - - - - - - - (6)$$

Expected number of jobs in the system

$$L_s = \left(\left(\left(\frac{\lambda(1+\zeta)}{\mu}\right) + \left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K \left(\frac{\lambda(1+\zeta)}{K\mu}\right)}{K!\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)^2}\right)\right) P_0\right) ------ (7)$$

Expected waiting time of jobs in the queue

$$W_q = \left(\left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K}{K!\,(K\mu)\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)^2}\right) P_0\right) -------- (8)$$

Expected waiting time of jobs in the system

$$W_s = \frac{1}{\mu} + \left(\left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K}{K!\,(K\mu)\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)^2}\right) P_0\right) -------- (9)$$

The average waiting-time

Wait time$= \left(\left(\lambda(1+\zeta)\right)\left(\frac{\left(\frac{\lambda(1+\zeta)}{\mu}\right)^K}{K!(K\mu)\left(1-\left(\frac{\lambda(1+\zeta)}{K\mu}\right)\right)^2}\right) P_0\right) ------- (10)$

Waiting cost

Waiting cost= (s) *(Wait time)$-------------- (11)$

Operating cost (Op cost)

Op cost= (K)*(server charge per time duration) $------ (12)$

Estimated Total cost (ETC)

ETC = Operating cost + Waiting cost $------ (13)$

6. Numerical illustrations:

In this section, we provide the numerical examples to show the effectiveness of the introduced encouraged arrival M/MK model:

Table 6.1. In the following table, we determine the L_q and W_q for $\zeta = 0.05$ and K=2

Poisson Arrival	Encouraged arrival rate 5% discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 5% discount (L_q)	Encouraged Arrival with 5% discount (W_q)	Little's law for system $Lq = \lambda(1+\boldsymbol{\zeta})Wq$
20	21	40	0.033	0.038853003	0.001850143	0.03885
60	63	70	0.1929	0.2285266	0.0036274	0.22852
120	126	122	0.3138	0.3755502	0.0029806	0.37555

Table 6.2. In the following table, we determine the $L_S \; and \; W_S$ for $\zeta = 0.0.5$ and K=2

Poisson Arrival	Encouraged arrival rate 5% discount	Service rate (μ)	Poisson (L_S)	Encouraged arrival with 5% discount (L_S)	Encouraged Arrival with 5% discount (W_S)	Little's law for system $Ls = \lambda(1+\boldsymbol{\zeta})Ws$
20	21	40	0.533	0.563852996	0.026850143	0.56385
60	63	70	1.050	1.1285266	0.017913	1.12852
120	126	122	1.2974	1.4083371	0.0111773	1.40833

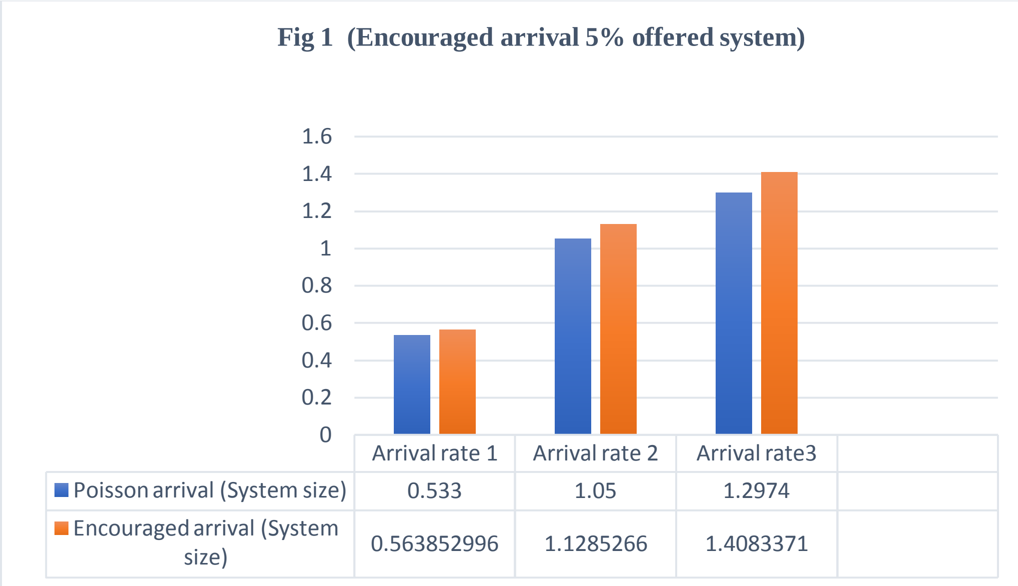

Fig 01: In the following figure, we show the system size, for $\zeta = 0.05$ and K=2

Table 6.3. In the following table, we determine the L_q and W_q for $\zeta = 0.1$ and K=2

Poisson Arrival	Encouraged arrival rate 10% discount	Service rate (µ)	Poisson (L_q)	Encouraged arrival with 10% discount (L_q)	Encouraged Arrival with 10% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	22	40	0.033	0.0449966	0.0040821	0.04499
60	66	70	0.1929	0.2285266	0.0020453	0.22852
120	132	122	0.3138	0.3755502	0.0034919	0.37555

Table 6.4. In the following table, we determine the L_s and W_s for $\zeta = 0.1$ and K=2

Poisson Arrival	Encouraged arrival rate 10 % discount	Service rate (µ)	Poisson (L_S)	Encouraged arrival with 10 % discount (L_S)	Encouraged Arrival with 10 % discount (W_S)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	22	40	0.533	0.5949966	0.0313256	0.59499
60	66	70	1.050	1.2122802	0.0183678	1.21228
120	132	122	1.2974	1.5428989	0.0116886	1.54289

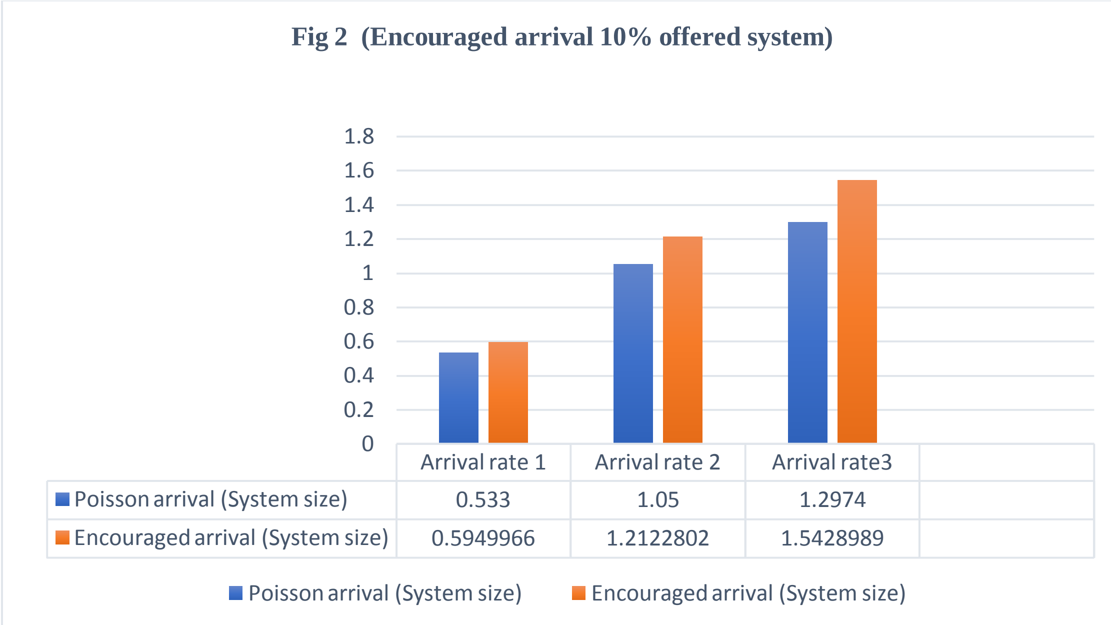

Fig 02: In the following figure, we show the system size, for $\zeta = 0.1$ and K=2

Table 6.5. In the following table, we determine the L_q and W_q for $\zeta = 0.15$ and K=2

Poisson Arrival	Encouraged arrival rate 15 % discount	Service rate (µ)	Poisson (L_q)	Encouraged arrival with 15 % discount (L_q)	Encouraged Arrival with 15% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	23	40	0.033	0.05180975	0.00225298	0.05180
60	69	70	0.1929	0.3162602	0.00458350	0.31626
120	138	122	0.3138	0.5319959	0.00038550	0.53199

Table 6.6. In the following table, we determine the L_s and W_s for $\zeta = 0.15$ and K=2

Poisson Arrival	Encouraged arrival rate 15% discount	Service rate (µ)	Poisson (L_s)	Encouraged arrival with 15% discount (L_s)	Encouraged Arrival with 15% discount (W_s)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	23	40	0.533	0.62680975	0.02725259	0.626809
60	69	70	1.050	1.3019745	0.0188692	1.301974
120	138	122	1.2974	1.6631434	0.0120517	1.663143

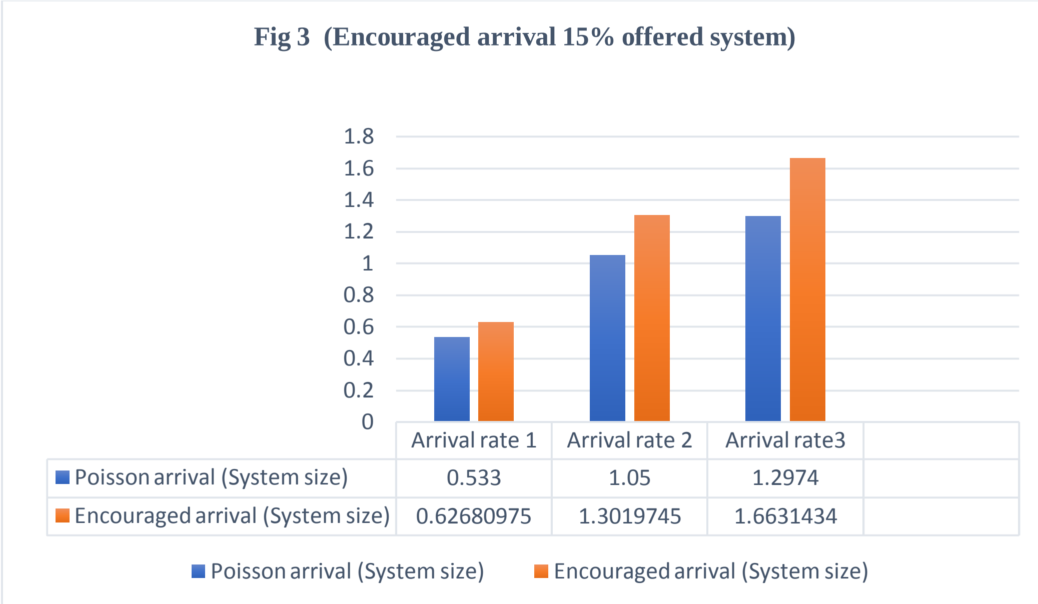

Fig 03: In the following figure, we show the system size, for $\zeta = 0.15$ and K=2

Table 6.7. In the following table, we determine the L_q and W_q for $\zeta = 0.20$ and K=2 then the queue length and waiting time

Poisson Arrival	Encouraged arrival rate 20 % discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 20 % discount (L_q)	Encouraged Arrival with 20% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	24	40	0.033	0.059340659	0.0024725	0.05934
60	72	70	0.1929	0.3698686	0.3698686	0.36986
120	144	122	0.3138	0.6308060	0.0043806	0.63080

Table 6.8. In the following table, we determine the L_q and W_q for $\zeta = 0.20$ and K=2

Poisson Arrival	Encouraged arrival rate 20% discount	Service rate (μ)	Poisson (L_s)	Encouraged arrival with 20% discount (L_s)	Encouraged Arrival with 20% discount (W_s)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	24	40	0.533	0.659340480	0.027472520	0.65934
60	72	70	1.050	1.3984461	0.0194229	1.39844
120	144	122	1.2974	1.8111339	0.0125773	1.81113

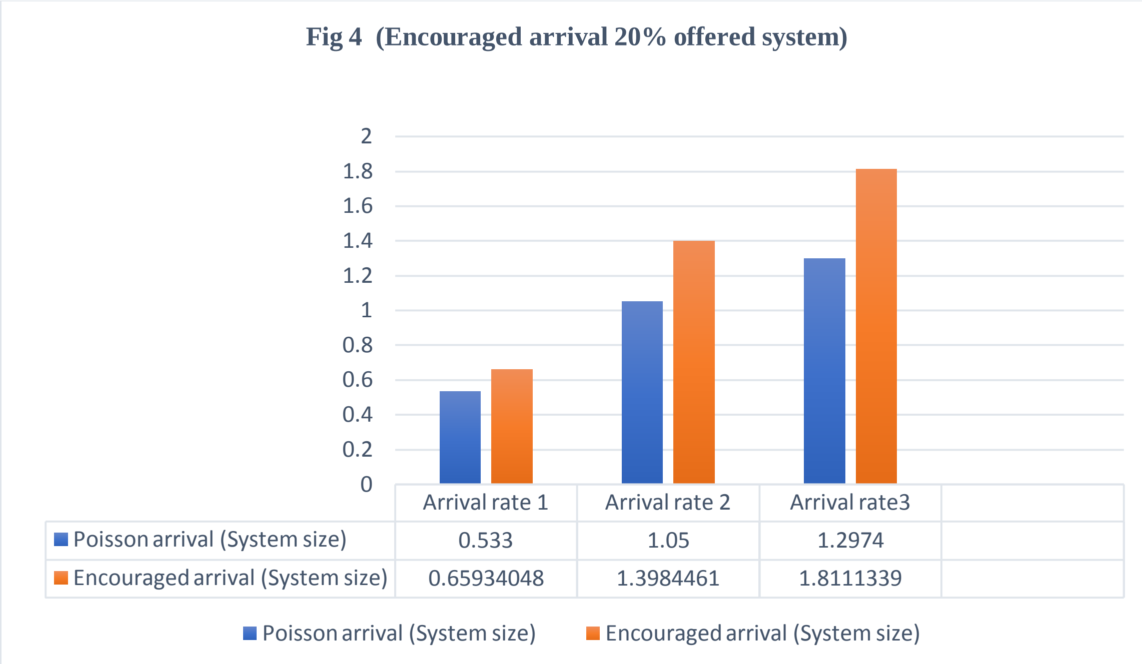

Fig 6: In the following figure, we show the system size, for $\zeta = 0.20$ and K=2

Table 6.9. In the following table, we determine the L_q and W_q for $\zeta = 0.25$ and K=2

Poisson Arrival	Encouraged arrival rate 25 % discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 25 % discount (L_q)	Encouraged Arrival with 25% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	25	40	0.033	0.067640693	0.00270562	0.06764
60	75	70	0.1929	0.4312548	0.0057501	0.43125
120	150	122	0.3138	0.7469472	0.0049796	0.74694

Table 6.10. The following table, we determine the L_s and W_s for $\zeta = 0.25$ and K=2

Poisson Arrival	Encouraged arrival rate 25% discount	Service rate (μ)	Poisson (L_s)	Encouraged arrival with 25% discount (L_s)	Encouraged Arrival with 25% discount (W_s)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	25	40	0.533	0.6926407	0.027705628	0.69264
60	75	70	1.050	1.5026834	0.0057501	1.50268
120	150	122	1.2974	1.9764554	0.0131764	1.97645

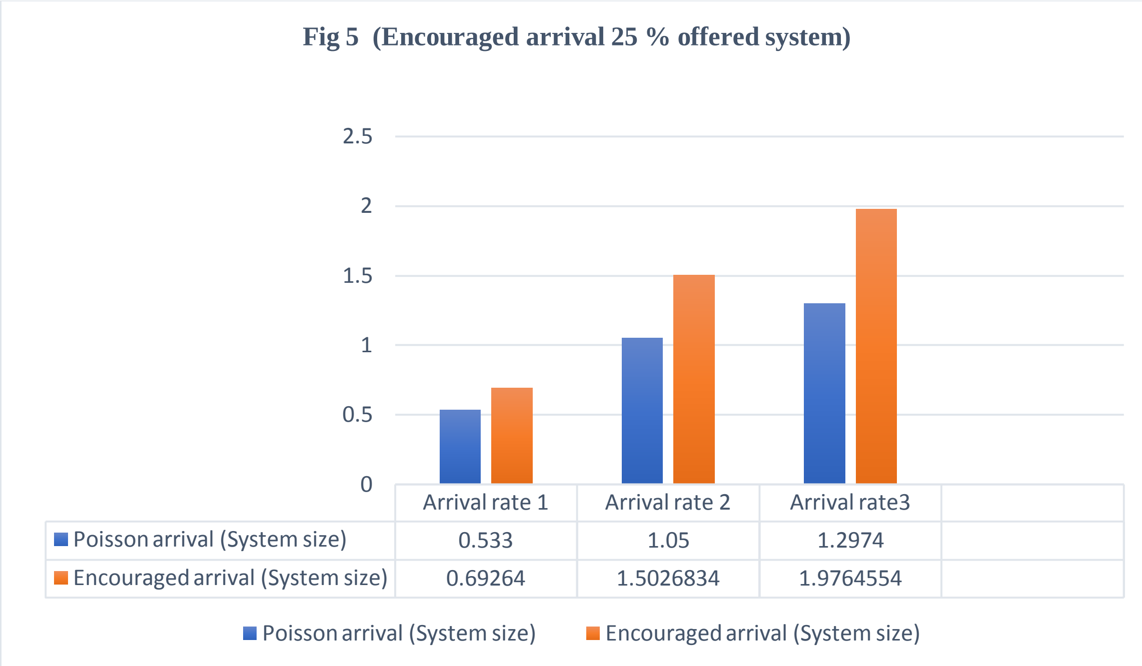

Fig 05: In the following figure, we show the system size, for $\zeta = 0.25$ and K=2

Table 6.11. In the following table, we determine the L_q and W_q for $\zeta = 0.30$ and K=2

Poisson Arrival	Encouraged arrival rate 30 % discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 30 % discount (L_q)	Encouraged Arrival with 30% discount (W_q)	Little's law for system $Lq = \lambda(1+\xi)Wq$
20	26	40	0.033	0.076764500	0.002952481	0.07676
60	78	70	0.1929	0.5015701	0.0064305	0.50157
120	156	122	0.3138	0.8840390	0.0056669	0.88403

Table 6.12. In the following table, we determine the L_s and W_s for $\zeta = 0.30$ and K=2

Poisson Arrival	Encouraged arrival rate 30% discount	Service rate (μ)	Poisson (L_S)	Encouraged arrival with 30% discount (L_S)	Encouraged Arrival with 30 % discount (W_S)	Little's law for system $Ls = \lambda(1+\xi)Ws$
20	26	40	0.533	0.726764500	0.027952481	0.72676
60	78	70	1.050	1.6158628	0.0207162	1.61586
120	156	122	1.2974	2.1627275	0.0138636	2.16272

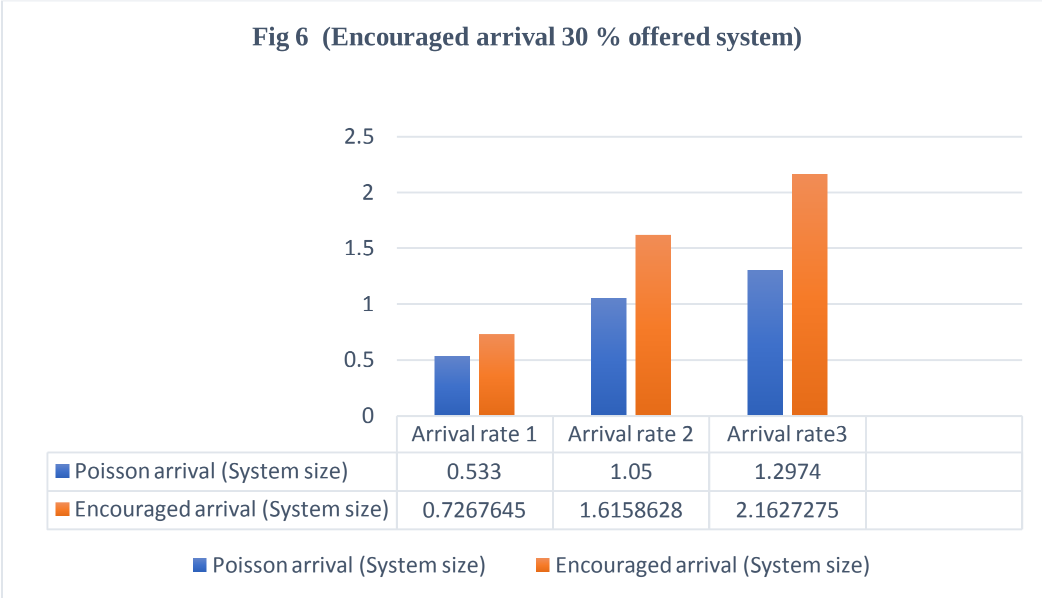

Fig 6: In the following figure, we show the system size, for $\zeta = 0.30$ and K=2

Table 6.13. In the following table, we determine the L_q and W_q for $\zeta = 0.05$ and K=3

Poisson Arrival	Encouraged arrival rate 5% discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 5% discount (L_q)	Encouraged Arrival with 5% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	21	40	0.003	0.003664753	0.0017451207	0.003664
60	63	70	0.0248	0.0300123507	0.000476386	0.030012
120	126	122	0.0426	0.0346196	0.0002748	0.034619

Table 6.14. In the following table, we determine the L_s and W_s for $\zeta = 0.05$ and K=3

Poisson Arrival	Encouraged arrival rate 5% discount	Service rate (μ)	Poisson (L_s)	Encouraged arrival with 5% discount (L_s)	Encouraged Arrival with 5% discount (W_s)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	21	40	0.503	0.528664753	0.025174512	0.546354
60	63	70	0.8819	0.9300123507	0.0147621008	0.930012
120	126	122	1.0262	1.0674065	0.0084715	1.067406

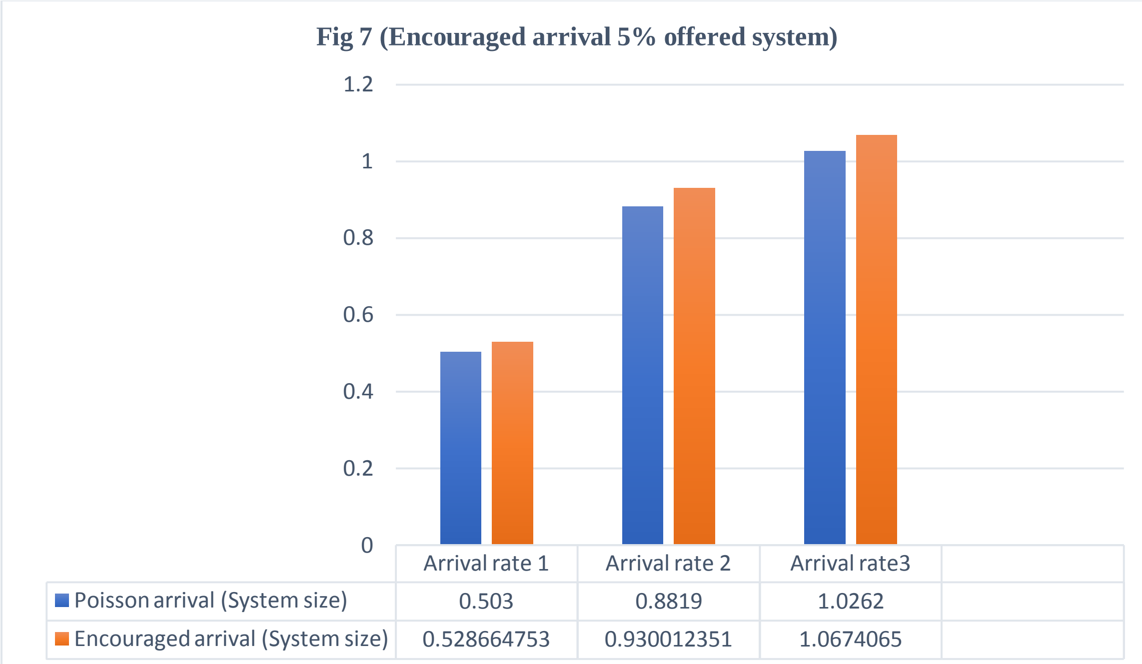

Fig 07: In the following figure, we show the system size, for $\zeta = 0.05$ and K=3

Table 6.15. In the following table, we determine the L_q and W_q for $\zeta = 0.10$ and K=3

Poisson Arrival	Encouraged arrival rate 10% discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 10% discount (L_q)	Encouraged Arrival with 10% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	22	40	0.003	0.004812225	0.000218738	0.0048122
60	66	70	0.0248	0.0360367	0.0005460	0.0360367
120	132	122	0.0426	0.0621381	0.0004707	0.0621381

Table 6.16. In the following table, we determine the L_s and W_s for $\zeta = 0.10$ and K=3

Poisson Arrival	Encouraged arrival rate 10 % discount	Service rate (μ)	Poisson (L_S)	Encouraged arrival with 10 % discount (L_S)	Encouraged Arrival with 10 % discount (W_S)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	22	40	0.503	0.554812225	0.025218738	0.55481
60	66	70	0.8819	0.9788938	0.0148317	0.97889
120	132	122	1.0262	1.1441053	0.0086675	1.14410

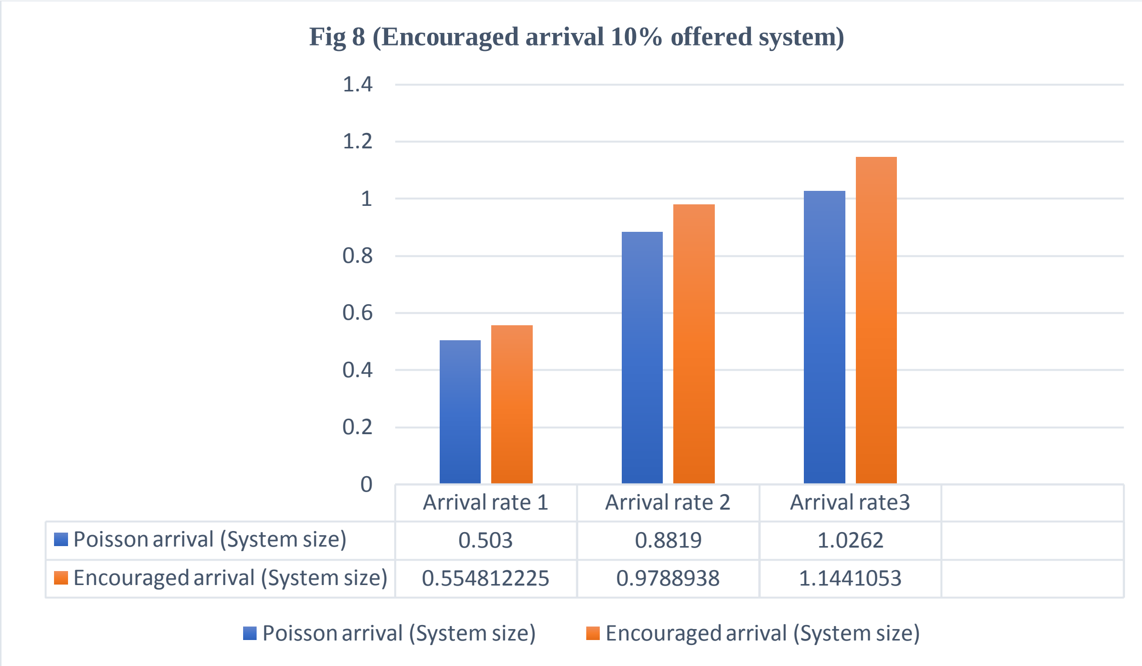

Fig 8: In the following figure, we show the system size, for $\zeta = 0.10$ and K=3

Table 6.17. In the following table, we determine the L_q and W_q for $\zeta = 0.15$ and K=3

Poisson Arrival	Encouraged arrival rate 15 % discount	Service rate (µ)	Poisson (L_q)	Encouraged arrival with 15 % discount (L_q)	Encouraged Arrival with 15% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	23	40	0.003	0.005757861	0.000250342	0.00575
60	69	70	0.0248	0.0429421	0.0006224	0.04294
120	138	122	0.0426	0.0742309	0.0005378	0.07423

Table 6.18. In the following table, we determine the L_s and W_s for $\zeta = 0.15$ and K=3

Poisson Arrival	Encouraged arrival rate 15% discount	Service rate (µ)	Poisson (L_S)	Encouraged arrival with 15% discount (L_S)	Encouraged Arrival with 15% discount (W_S)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	23	40	0.503	0.580757861	0.025250342	0.58075
60	69	70	0.8819	1.0286564	0.0149081	1.02865
120	138	122	1.0262	1.2053784	0.0087345	1.20537

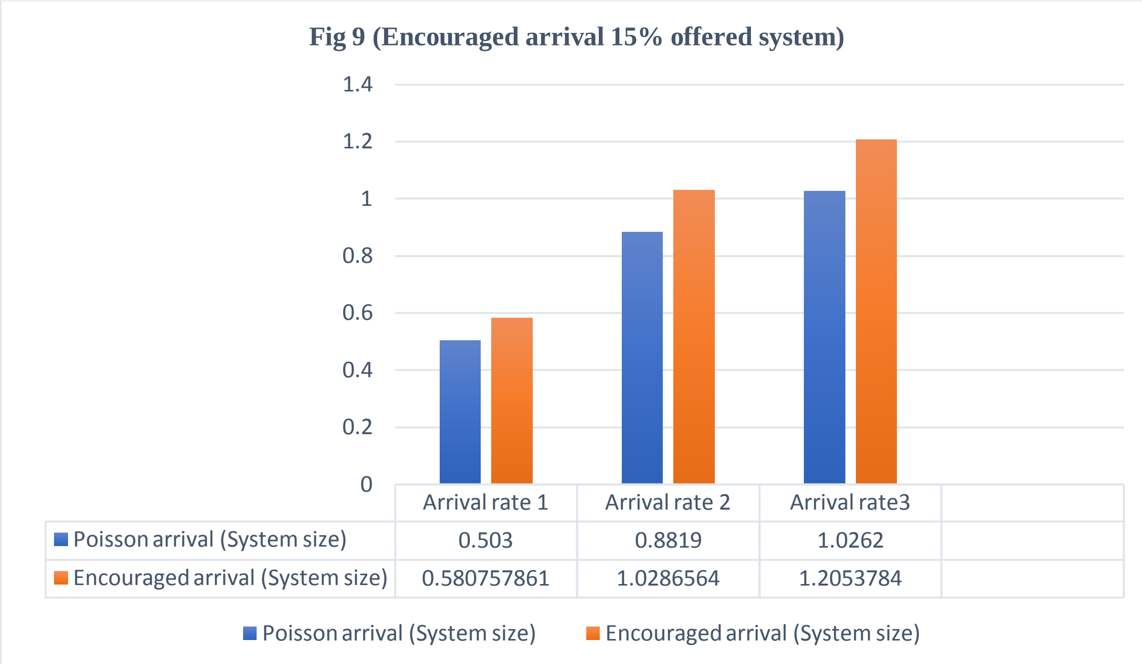

Fig 9: In the following figure, we show the system size, for $\zeta = 0.15$ and K=3

Table 6.19. In the following table, we determine the L_q and W_q for $\zeta = 0.20$ and K=3

Poisson Arrival	Encouraged arrival rate 20 % discount	Service rate (µ)	Poisson (L_q)	Encouraged arrival with 20 % discount (L_q)	Encouraged Arrival with 20% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	24	40	0.003	0.00616438356	0.0002568493	0.006164
60	72	70	0.0248	0.0508178	0.0007058	0.050817
120	144	122	0.0426	0.0880466	0.0006114	0.088046

Table 6.20. In the following table, we determine the L_s and W_s for $\zeta = 0.20$ and K=3

Poisson Arrival	Encouraged arrival rate 20% discount	Service rate (µ)	Poisson (L_S)	Encouraged arrival with 20% discount (L_S)	Encouraged Arrival with 20% discount (W_S)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	24	40	0.503	0.606164384	0.0252568493	0.60616
60	72	70	0.8819	1.0793892	0.0149915	1.07938
120	144	122	1.0262	1.2683744	0.0088082	1.26837

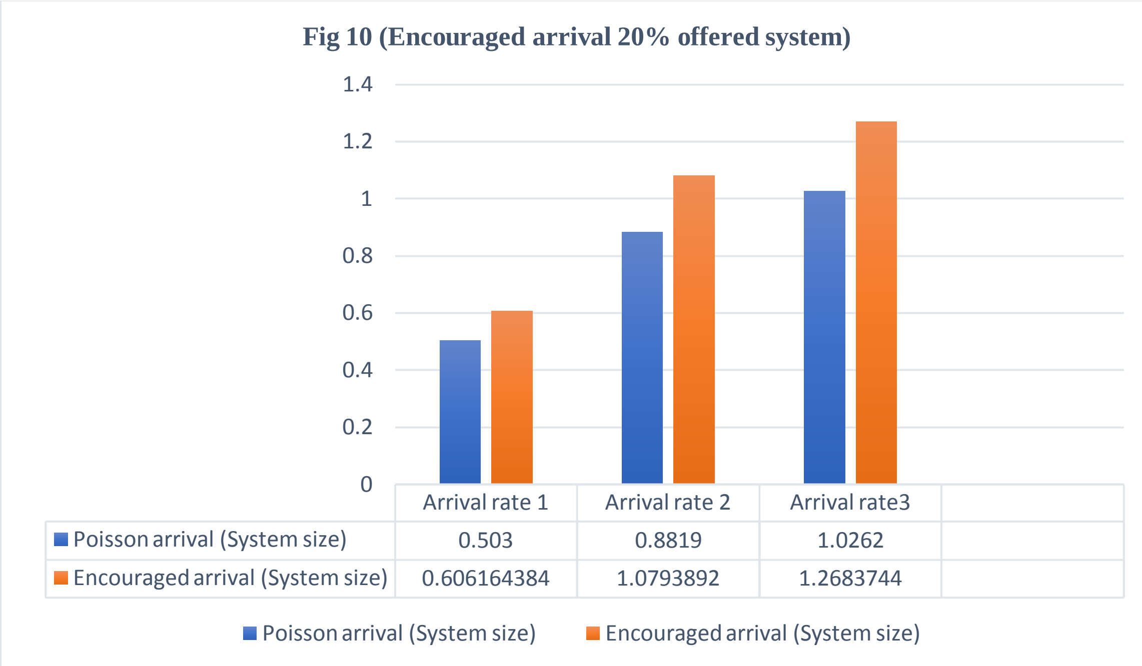

Fig 10: In the following figure, we shown the system size, for $\zeta = 0.20$ and K=3

Table 6.21. In the following table, we determine the L_q and W_q for $\zeta = 0.25$ and K=3

Poisson Arrival	Encouraged arrival rate 25 % discount	Service rate (μ)	Poisson (L_q)	Encouraged arrival with 25 % discount (L_q)	Encouraged Arrival with 25% discount (W_q)	Little's law for system $Lq = \lambda(1+\zeta)Wq$
20	24	40	0.003	0.00722643602	0.000289057441	0.00722
60	72	70	0.0248	0.0597629	0.0007968	0.05976
120	144	122	0.0426	0.1038414	0.0006923	0.10384

Table 6.22 In the following table, we determine the L_s and W_s for $\zeta = 0.25$ and K=3

Poisson Arrival	Encouraged arrival rate 25% discount	Service rate (μ)	Poisson (L_S)	Encouraged arrival with 25% discount (L_S)	Encouraged Arrival with 25% discount (W_S)	Little's law for system $Ls = \lambda(1+\zeta)Ws$
20	24	40	0.503	0.632226436	0.0252890574	0.63222
60	72	70	0.8819	1.1311915	0.0150826	1.13119
120	144	122	1.0262	1.333496	0.0088890	1.33349

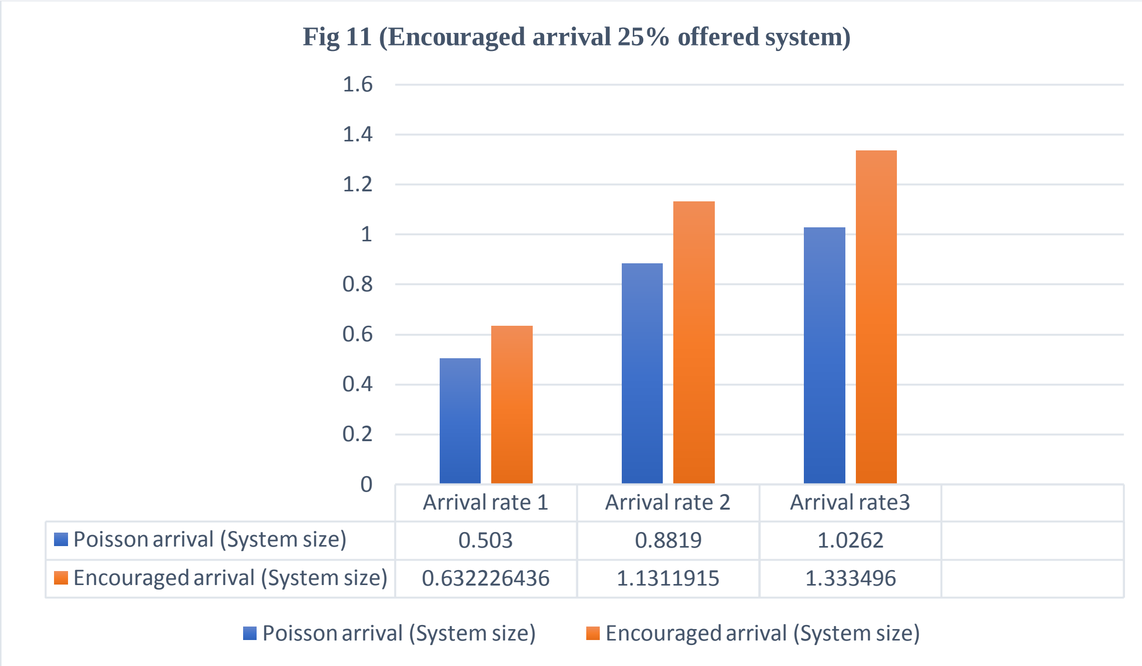

Fig 11: In the following figure, we show the system size, for $\zeta = 0.25$ and K=3

Table 6.23. In the following table, we determine the L_q and W_q for $\zeta = 0.30$ and K=3

Poisson Arrival	Encourage d arrival rate 30 % discount	Servic e rate (µ)	Poisson (L_q)	Encouraged arrival with 30 % discount (L_q)	Encouraged Arrival with 30% discount (W_q)	Little's law for system $Lq = \lambda(1+\boldsymbol{\zeta})Wq$
20	26	40	0.003	0.520919921	0.00032380 63253	0.52091
60	78	70	0.0248	0.0698828	0.0008959	0.06988
120	156	122	0.0426	0.1152707	0.0007389	0.11527

Table 6.24. In the following table, we determine the L_s and W_s for $\zeta = 0.30$ and K=3

Poisson Arrival	Encourage d arrival rate 30% discount	Servic e rate (µ)	Poisson (L_S)	Encouraged arrival with 30% discount (L_S)	Encouraged Arrival with 30 % discount (W_S)	Little's law for system $Ls = \lambda(1+\boldsymbol{\zeta})Ws$
20	26	40	0.503	0.658418961	0.02532388	0.65841
60	78	70	0.8819	1.1841685	0.0151816	1.18416
120	156	122	1.0262	1.3939592	0.0089356	1.39395

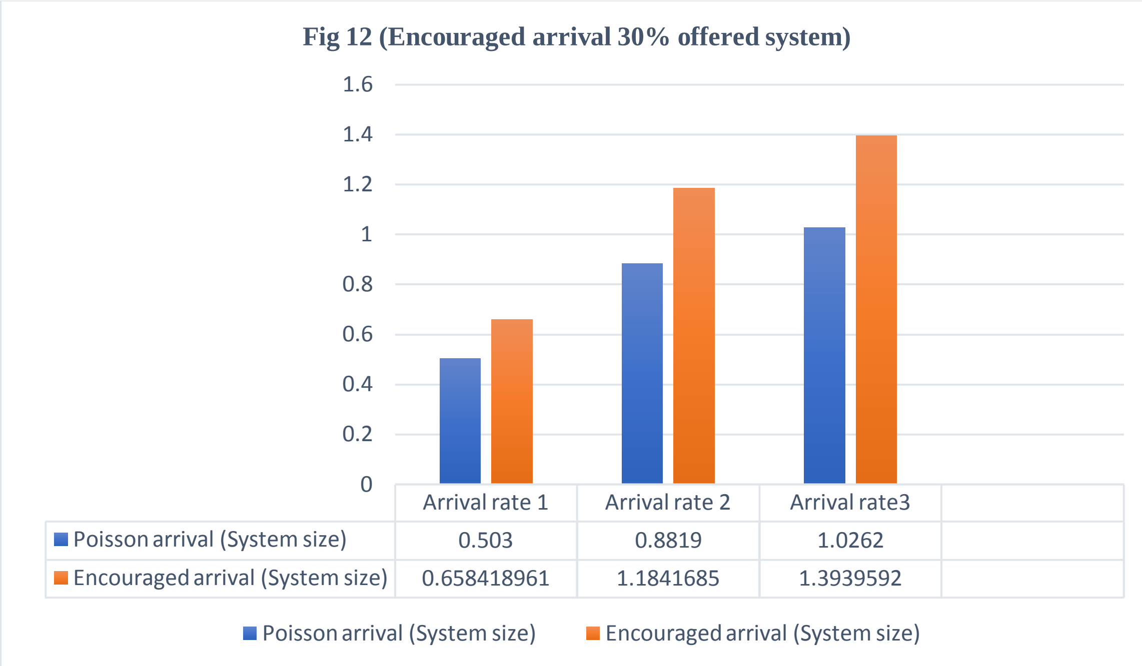

Fig 12: In the following figure, we show the system size, for $\zeta = 0.30$ and K=3

7. Cost table for multiple servers:

We consider the system with encouraged arrivals, where $\lambda = 20, 60, 120$ and $\boldsymbol{\zeta} = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3.$ customers arrive on average per respective time duration for a certain service and the operator service rate is 40, 70 and 120 customers per time duration. When there is sluggish processing, the consumer will withdraw from the payment. In this instance, the company will be deprived of amount "s" per payment for the relevant time period. Let the one server cost to operate Rs. 50/- to operate. We evaluate the total expected cost (**ETC**) of the system and find the number of servers will be opened to reduce the estimated total cost (**ETC**) also we calculate the waiting cost and operating cost (**OP**) for the introduced encouraged arrival M/M/K queueing system.

Table 7.1. shows the encouraged arrival with the rate of $\lambda(1+\boldsymbol{\zeta})$ where $\lambda = 20$, $\boldsymbol{\zeta} = 0.05$ to 0.30 *and* $s = 10$ then, the waiting time, waiting cost, operating cost and total cost for up to 4 servers are,

Encouraged arrival Rate $\boldsymbol{\lambda(1+\zeta)}$	Waiting time	Waiting cost	Operating cost (2 servers)	Total cost
21	0.0389	0.3885	100.00	100.3885
22	0.0450	0.4500	100.00	100.4500
23	0.0518	0.5182	100.00	100.5182
24	0.0593	0.5934	100.00	100.5934
25	0.0676	0.6764	100.00	100.6764
26	0.0769	0.7676	100.00	100.7676

Encouraged arrival Rate $\boldsymbol{\lambda}(\mathbf{1}+\boldsymbol{\zeta})$	**Waiting time**	**Waiting cost**	**Operating cost (3 servers)**	**Total cost**
21	0.0037	***0.0366***	***150.00***	***150.0366***
22	0.0048	0.0481	150.00	150.0481
23	0.0058	0.0576	150.00	150.0576
24	0.0062	0.0616	150.00	150.0616
25	0.0072	0.0723	150.00	150.0723
26	0.0084	0.0842	150.00	150.0842
Encouraged arrival Rate $\boldsymbol{\lambda}(\mathbf{1}+\boldsymbol{\zeta})$	**Waiting time**	**Waiting cost**	**Operating cost (4 servers)**	**Total cost**
21	0.0003	0.0033	200.00	200.0033
22	0.0004	0.0041	200.00	200.0041
23	0.0005	0.0050	200.00	200.0050
24	0.0006	0.0062	200.00	200.0062
25	0.0007	0.0075	200.00	200.0075
26	0.0009	0.0090	200.00	200.0090

Table 7.2. shows the encouraged arrival with the rate of $\lambda(1+\boldsymbol{\zeta})$ where $\lambda = 60$ *and* $\boldsymbol{\zeta}$ =0.05 to 0.30 and s=10 then, the waiting time, waiting cost, operating cost and total cost for up to 4 servers are,

Encouraged arrival Rate $\boldsymbol{\lambda}(\mathbf{1}+\boldsymbol{\zeta})$	**Waiting time**	**Waiting cost**	**Operating cost (2 servers)**	**Total cost**
63	0.2285	2.2853	100.00	102.2853
66	0.2694	2.6942	100.00	102.6942
69	0.3391	3.3906	100.00	103.3906
72	0.3699	3.6988	100.00	103.6988
75	0.4313	4.3126	100.00	104.3126
78	0.5016	5.0158	100.00	105.0158
Encouraged arrival Rate $\boldsymbol{\lambda}(\mathbf{1}+\boldsymbol{\zeta})$	**Waiting time**	**Waiting cost**	**Operating cost (3 servers)**	**Total cost**
63	0.0300	0.3001	150.00	150.3000
66	0.0360	0.3604	150.00	150.3604
69	0.0429	0.4295	150.00	150.4295
72	0.0508	0.5082	150.00	150.5082
75	0.0598	0.5976	150.00	150.5976
78	0.0699	0.6988	150.00	150.6988
Encouraged arrival Rate $\boldsymbol{\lambda}(\mathbf{1}+\boldsymbol{\zeta})$	**Waiting time**	**Waiting cost**	**Operating cost (4 servers)**	**Total cost**
63	0.0042	0.0461	200.00	200.0461
66	0.0054	0.0541	200.00	200.0541

69	0.0064	0.0636	200.00	200.0636
72	0.0078	0.0776	200.00	200.0776
75	0.0094	0.0938	200.00	200.0938
78	0.0113	0.1126	200.00	200.1126

Table 7.3 Shows the encouraged arrival with the rate of $\lambda(1+\zeta)$ where $\lambda = 120$ *and* ζ =0.05 to 0.30 and s=10 then, the waiting cost, operating cost and total costs for up to 4 servers are,

Encouraged arrival Rate $\lambda(1+\zeta)$	**Waiting time**	**Waiting cost**	**Operating cost (2 servers)**	**Total cost**
126	0.3756	3.7556	100.00	103.7556
132	0.4609	4.6093	100.00	104.6093
138	0.5320	5.3199	100.00	105.3199
144	0.6308	6.3081	100.00	106.3081
150	0.7469	7.4694	100.00	107.4694
156	0.8840	8.8404	100.00	108.8404
Encouraged arrival Rate $\lambda(1+\zeta)$	**Waiting time**	**Waiting cost**	**Operating cost (3 servers)**	**Total cost**
126	**0.0346**	***0.3462***	***150.00***	***150. 3462***
132	0.0621	0. 6213	150.00	150.6213
138	0.0742	0. 7422	150.00	150.7422
144	0.8804	0.8804	150.00	150.8804
150	0.1038	1.0385	150.00	151.0385
156	0.1152	1.1527	150.00	151.1527
Encouraged arrival Rate $\lambda(1+\zeta)$	**Waiting time**	**Waiting cost**	**Operating cost (4 servers)**	**Total cost**
126	0.0079	0.0791	200.00	200.0791
132	0.0098	0.0982	200.00	200.0982
138	0.0121	0.1207	200.00	200.1207
144	0.0147	0.1471	200.00	200.1471
150	0.0178	0.1777	200.00	200.1777
156	0.0213	0.2132	200.00	200.2132

8. Cost comparison:

In this section we determine the waiting cost, OP cost and ETC for the introduced M/M/K queue with encouraged arrival model.

Table 8.1 we determine the results of the introduced M/M/K queue with encouraged arrival with that of waiting cost and the minimum (ETC) of *poisson* and *encouraged arrivals*.

Poisson Arrival Rate	Encouraged arrival Rate $\lambda(1+\zeta)$	servers	Waiting time (poisson)	Waiting Time $\lambda(1+\zeta)$	Waiting Cost (poisson)	Waiting Cost $\lambda(1+\zeta)$	Op cost	Total Cost (poisson)	ETC (Encouraged) $\lambda(1+\zeta)$
20	21	3	0.0040	0.0037	0.0400	0.0366	150.00	150.0400	150.0366
120	126	3	0.0480	0.0346	0.4800	0.3462	150.00	150.4800	150.3462

From the above table 8.1, It is evident that for the introduced M/M/C queue with encouraged arrival, the waiting cost and ETC are less when it is compared with the Poisson queue refer [2] model with rates of ζ =0.05, λ=20, K=3 and ζ =0.05, λ=120, K=3 have lower costs than the Poisson arrival cost model. Three servers are sufficient for handling expenses and users, according to our cost estimations.

8.1 Remarks:

Remark 8.1.1: From the figures Fig. 1 to Fig 12, we find that the system size is effectively expanded. As a result of this increased user base, we identify that the M/M/K queue with encouraged arrival queueing model gives more system size and thereby increasing the profit when it is compared with the poisson arrival M/M/K model [1].

Remark 8.1.2: In Table 7.1 to 7.3 we have calculated the cost analysis of encouraged arrival for the introduced M/M/K queue with encouraged arrival model and is used to the cost effectiveness of the M/M/K queue with encouraged arrival.

Remark 8.1.3: From the table Table 8.1, It is evident that for the introduced M/M/K queue with encouraged arrival, the waiting cost and ETC are less when it is compared with the poisson queue cost model refer [2]. Our model with rates of ζ =0.05, λ=20, K=3 and ζ =0.05, λ=120, K=3 have *lower costs* than the poisson arrival cost model. Three servers are sufficient for handling expenses and users, according to our cost estimations.

9. Conclusion

The simulation results shows that the system size was efficiently increased. Gaining more users will produce more profit. We have determined that the introduced M/M/K queue with encouraged arrival queueing model is more efficient and profit than the multi-server poisson arrival queueing model. From the cost comparison table, we have identified that the introduced M/M/K queue with encouraged arrival queueing model has minimum waiting cost and minimum estimated total cost than that of the multi-server poisson arrival queueing model. As a result, M/M/K queue with encouraged arrival queueing model will be effective for cost management.

According to cost estimations determined, it is wisely found that three servers are adequate to handle expenses and users**.**

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