ISSN 2063-5346



# INVESTIGATION OF IMPELLER BLADES DESIGN FOR AXIAL FLOW PUMP USING COMPUTATIONAL FLUID DYNAMICS

Tamanampudi Narisi Reddy

Research Scholar, Dept of Industrial Engineering & Management, JSS Academy of Technical Education, Bengaluru, Karnataka State, India nxlinstruments@gmail.com

Vijay Kumar M

Associate Professor, Dept of Industrial Engineering & Management, JSS Academy of Technical Education, Bengaluru, Karnataka State, India <u>mvkjss@gmail.com</u>

Article History: Received: 03.04.2023         Revised: 25.04.2023         Accepted: 15.05.20
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#### Abstract

The investigation of the impeller blade design for axial flow pumps used in power transformers is discussed in this paper. Due to the symmetry of the blade type, axial pumps typically have efficiency levels that are significantly lower than those of classical pumps. In this study, the geometry of the blades is modified to achieve a high efficiency in an axial pump by concentrating on a pump impeller. CF Turbo is the proprietary program used in this simulation. The computational Fluid Dynamics results for hydraulic efficiency, head torque, and pump torque were compared. To identify the key effects and their ideal design factors, Orthogonal Array, Analysis of variance and optimization techniques are used. The best ideal solution for satisfying the pump torque and head limitations is discovered when an effective design variable in impeller blades is taken into account.

Keywords—Axial Flow Pump, Impeller Blade, Computational Fluid Dynamics, ANOVA.

#### 1. Introduction

Oil cooling circulation in power distribution transformers transfer the transformer insulating oil by axial flow pump. inline glandless oil submerged motor on common shaft with impeller. design and performance Mechanical standards are based on the complicated interaction of fluid dynamics variables and the empirical equation. The relationship between the experimental loss and pump design was well explained by Stepan off and Neumann's study findings. The crucial component of pump design is striking a balance between maximum efficiency and the head of fluid flow caused by rotating the impeller.

The airfoil, chord, and angle of the intake and outlet, as well as the directional axial flow pump. Guide vane construction is more constrained than directed axial flow pump for reducing resistance loss. From this perspective, the goal of this study is to optimize the geometry of the impeller blades increase hydrodynamic to performance head of torque, and efficiency.

Computational Fluid dynamics simulations for directional axial flow pumps require a substantial amount of calculation time due to the complex geometrical and physical constraints. In order to save time and do correct calculations, use the surrogate model with approximate design optimization. Orthogonal polynomials are used in the construction of the surrogate model, together with sensitivity data derived from the analysis of variance. Through the design optimization process, the better impeller blades form that balances torque, head, and efficiency is shown with respect to the initial impeller blades shape. Moreover, explain why the better inner fluid flow and effectiveness compared to the original models.

#### 2. Directional Axial Flow Pump

i.Principle of Pump operation and composition

A directional axial flow pump shown in Fig .1 comprises of three components. the component of the motor positioned within the pump. At the tip of the suction shaft is an impeller. Moreover, the inside of the pump has 5 vanes fitted.

The internal electric motor rotates the impeller in the direction of feeding the transformer oil to recirculation for cooling in the power transformer. This is the fundamental operating concept. Five guide vanes have been fitted within the pump pipe to fix the motor and stabilize the internal flow as it changes the flow from cylindrical to axial.



Fig 1. Schematic Diagram of Axial Transformer oil pump

The impeller in the pump is the cause of the guiding vanes' interior installation. The fluid inlet angle is altered by the impeller, which also causes oil to rotate. Due to this, the hydrodynamic performance efficiency has decreased the total Pressure and Magnitude of Velocity.



Fig 2. Impeller Grid and Mesh Formation

The Computational Fluid Dynamics model uses a fluid a transformer oil, on the surfaces and cutting plane with a specific density of 0.856 kg/m3. The design goals considered are Pump head is 2.5 meters, rated flow is 1800 LPM and Impeller rotation speed is 1500 rpm.

#### ii.Flow Analysis

ANSYS CFX v.13 is used to examine the fluid performance of the directional axial flow pump. Shear Stress Transport-SST based on k- was employed as the turbulence model and high resolution based on an upwind biased approach was used for the convection scheme. Hexahedron, tetrahedron, and prism grid types were merged to create a grid system using ANSYS ICEM CFD.

Using ANSYS ICEM CFD, a mixed tetragonal, hexagonal, and prism mesh grid system was constructed. Tetragonal mesh was used to cover the space around the impeller and the pump, while hexagonal and prism mesh was used to cover another area.



Fig 3. Flow chart of Optimization process

These are the boundary conditions: To assess the performance fluctuation of the

pump due to variation in volume flow rate, no-slip conditions are applied around the walls, zero pressure applied at the intake region, and 0.03 m3/s volume, 108 m3/h flow rate is applied at the outlet area.The outcome of the CFD analysis for the base model is shown in Fig. 2. The overall pressure deviation from the rated flow was 2.5 Meters. 402 Nm of torque, and 67.4% efficiency were measured. Design optimization is necessary for higher efficiency and to satisfy the design criteria.

#### 3. Optimization Of Design ImpellerBlades

The process of impeller blade design optimization of bidirectional axial flow pumps is shown in Fig. 3. The fundamental approach for design optimization consists of two steps: creating a surrogate model using an ANOVA and optimizing it using that model. The hydrodynamic performance and efficiency of the optimization model can be estimated as a computationally intensive function interacting with a straightforward analytical or computational model.

Table1: -DesignVariable and theirLevels								
Designvariables	Unit	L1	L2	L3	L4			
<i>x</i> 1	mm	6	6.5	7	7.5			
x2	mm	55.9	65.9	75.9	85.9			
<i>x</i> 3	mm	44.6	58.6	72.6	86.6			
<i>x</i> 4	Deg.	52.4	53.4	54.4	55.5			
$X_5$	Deg.	65.1	67.6	70.1	72.6			





Fig 4. Impeller Blades design Variables

Design factors are shown in Fig. 4 and are as follows: Maximum blade thickness, blade length at hub, chord, and chord angle are all expressed as x1, x2, x3, and x5, respectively (angle at hub). The design variables function independently, serving as the foundation for the original model. The design variables and range of variables are displayed in Table 1.

# 4. Formulation of Mathematical Model

The goal of this study is to build an axial flow pump, and in order to achieve maximum efficiency, the impeller blades' shape must be determined within the restrictions of head and torque.

Find x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>, x<sub>4</sub>, x<sub>5</sub>.

(1)

To Maximise y<sub>efficiency</sub> (x<sub>i</sub>)

Subjet to  $y_{torque}(x_i) \leq [N.m]$ 

 $y_{head}(x_i) \ge 2.5 [m] i=1,2,3,4,5$ 

where y(xi)= model of each response. When the maximum torque is 600Nm, the first and second constraints were conducted with a minimum head of 2.5 m. Using the ANSYS v. 11 optimization module and the feasible direction approach, obtain the precise solution to the approximation model.

# 5. ANOVA Orthogonal Model

The optimization method of impeller design consists of designing tests using the response surface approach. The orthogonal polynomial series is used compute the specific design solutions to the response surface model.

The impacts of any design variable on the effects of other design factors will not be affected. Consequently, it is appropriate to analyze response range using an experiment design with orthogonal design variables. Also, because each term in an orthogonal polynomial is independent, the coefficient is obtained sequentially from lower to higher terms. Using the orthogonal polynomial p (x) n that has n degrees of design variable, a response surface model for the geometry of impeller blades is presented. If they can be represented as a second-order polynomial and are equivalent to Eq (2).

$$y = b_0 + b_1 (x - \underline{x})^2 + b_2 \left[ (x - \underline{x})^2 - \frac{a^2 - 1}{12} h^2 \right] + b_3 \left[ (x - \underline{x})^3 - \frac{3a^2 - 7}{20} (x - \underline{x})h^2 \right] + b_n P_n(x) + \cdots$$
(2)

where h =space between design variable levels,

x= average of the design variable levels.

The degree n must be lower than a.

Then bo and bi can be written as regression coefficients by the quation 3 and 4.

$$b_{0} = T/h_{n} = \underline{y}....(3)$$
  

$$b_{i} = \sum_{i=1}^{a} p_{n}(x_{i})y_{i}/\sum_{i=1}^{a} p_{n}^{2}(x_{i}),$$
  

$$i=1,2,...a ....(4)$$

Table2: -Computational Fluid Dynamics for 1.25 Orthogonal Arrays								
Exp.	$x_1$	<i>x</i> <sub>2</sub>	<i>x</i> <sub>3</sub>	<i>X</i> 4	<i>x</i> <sub>5</sub>	Torque(N·m)	Head(m)	η
								(%)
1	1	1	1	1	1	223.4	11.5	69.1
2	2	2	2	2	2	313.5	13.7	69.4
3	3	3	3	3	3	401.6	16.7	66.3
4	4	4	4	4	4	483.3	22.3	63.1
5	5	5	5	5	5	558.1	24.1	58.3
6	1	2	3	4	5	513.2	21.1	61.1
7	2	3	4	5	1	398.3	18.2	68.2
8	3	4	5	1	2	255.7	11.3	67.1
9	4	5	1	2	3	401.1	19.4	68.2
10	5	1	2	3	4	412.3	18.1	67.7
11	1	3	5	2	4	397.6	19.4	68.1
12	2	4	1	3	5	518.6	22.7	61.3
13	3	5	2	4	1	387.5	19.5	68.4
14	4	1	3	5	2	407.3	18.2	68.6
15	5	2	4	1	3	276.1	14.1	69.2
16	1	4	2	5	3	509.8	23.2	63.1
17	2	5	3	1	4	408.1	19.6	67.4
18	3	1	4	2	5	392.1	18.6	66.3
19	4	2	5	3	1	269.3	13.7	68.1
20	5	3	1	4	2	400.7	17.7	66.3
21	1	5	4	3	2	388.7	17.5	68.3
22	2	1	5	4	3	396.3	17.2	67.7
23	3	2	1	5	4	501.3	20.1	62.1
24	4	3	2	1	5	416.1	17.7	64.1
25	5	4	3	2	1	266.3	12.1	71.1

Table 3: - ANOVA forTorque								
Design variable		Variation	DF	Variance	F-Value	EffectiveRatio (%)		
<i>x</i> <sub>1</sub>	Linear	1734	1	17	2.16	1.10		
<i>X</i> 2	Linear	12327	1	23	3.11	1.58		
<i>X</i> 3	Linear	3968	1	211	27.35	13.1		
<i>X</i> 4	Linear	80263	1	6	0.61	1.34		
<i>X</i> 5	Linear	91742	1	145	18.46	11.10		
	Quadratic	22	1	95	11.7	7.59		
$x_1x_2$	Interchange	67	1	0	0.01	0.01		
<i>x</i> 1 <i>x</i> 3	Interchange	97	1	1	0.06	0.03		
<i>X</i> 1 <i>X</i> 4	Interchange	39	1	9	1.03	0.51		
<i>x</i> <sub>1</sub> <i>x</i> <sub>5</sub>	Interchange	64	1	131	16.98	8.33		
<i>x</i> <sub>2</sub> <i>x</i> <sub>3</sub>	Interchange	74	1	133	16.81	8.24		
<i>x</i> <sub>2</sub> <i>x</i> <sub>4</sub>	Interchange	48	1	24	3.31	1.63		
<i>x</i> <sub>2</sub> <i>x</i> <sub>5</sub>	Interchange	407	1	0	0.01	0.01		
<i>X</i> 3 <i>X</i> 4	Interchange	309	1	371	48.75	24.82		
<i>x</i> <sub>3</sub> <i>x</i> <sub>5</sub>	Interchange	46	1	14	1.88	1.31		
<i>x</i> 4 <i>x</i> 5	Interchange	257	1	256	33.1	16.69		
Error		61	8	60				
Total			24		185.33	100		

Table4: -ANOVA for Head								
Design variable		Variation	DF	Variance	F-Value	Effective		
						Ratio (%)		
<i>x</i> <sub>1</sub>	Linear	2.751	1	0.045	1.03	0.38		
<i>x</i> <sub>2</sub>	Linear	20.712	1	0	0	0		
<i>x</i> <sub>3</sub>	Linear	5.138	1	1.180	26.83	10.10		
<i>X</i> 4	Linear	123.77	1	0.058	1.32	0.51		
	Linear	108.83	1	1.321	30.06	11.32		
<i>X</i> 5	Quadratic	1.098	1	1.113	25.31	9.53		
<i>x</i> <sub>1</sub> <i>x</i> <sub>2</sub>	Interchange	1.72	1	0.004	0.1	0.04		
<i>x</i> <sub>1</sub> <i>x</i> <sub>3</sub>	Interchange	4.648	1	0.001	0.02	0.01		
<i>x</i> 1 <i>x</i> 4	Interchange	0.121	1	0.378	8.59	3.24		
<i>x</i> <sub>1</sub> <i>x</i> <sub>5</sub>	Interchange	0.714	1	0.828	18.82	7.09		

<i>x</i> <sub>2</sub> <i>x</i> <sub>3</sub>	Interchange	1.18	1	0.635	14.44	5.44
<i>X</i> 2 <i>X</i> 4	Interchange	0.099	1	0.154	3.5	1.32
<i>x</i> <sub>2</sub> <i>x</i> <sub>5</sub>	Interchange	0.29	1	0.64	14.54	5.48
<i>X</i> 3 <i>X</i> 4	Interchange	2.402	1	2.097	66.07	24.89
<i>x</i> <sub>3</sub> <i>x</i> <sub>5</sub>	Interchange	0.576	1	0.08	1.81	0.68
<i>X</i> 4 <i>X</i> 5	Interchange	2.329	1	2.329	52.92	19.94
Error		0.352	8	0.352		
Total			24		263.41	100

Table5: -ANOVA for Efficiency							
Design variable		Variation	DF	Variance	F-Value	Effective	
						ratio(%)	
	Linear	2.01	1	0.723	4.53	5.80	
$x_1$	Quadratic	1.43	1	0.203	2.55	2.90	
	Linear	1.35	1	0.565	3.32	4.31	
<i>x</i> <sub>2</sub>	Quadratic	1.71	1	0.533	3.03	5.98	
	Cubic	1.53	1	0.543	3.13	4.13	
	Linear	4.33	1	0.663	4.01	5.21	
<i>x</i> <sub>3</sub>	Quadratic	8.47	1	1.427	11.85	12.39	
	Linear	34.23	1	0.081	1.62	1.77	
<i>X</i> 4	Quadratic	6.03	1	0.663	4.05	5.26	
	Linear	133.23	1	0.125	1.95	2.18	
<i>X</i> 5	Quadratic	1.27	1	0.124	1.95	2.18	
	Cubic	1.11	1	0.131	1	2.26	
<i>x</i> <sub>1</sub> <i>x</i> <sub>2</sub>	Interchange	0.76	1	0.202	1.54	1.90	
<i>x</i> <sub>1</sub> <i>x</i> <sub>3</sub>	Interchange	16.14	1	3.178	24.17	29.80	
<i>x</i> <sub>2</sub> <i>x</i> <sub>5</sub>	Interchange	0.44	1	0.083	1.63	0.78	
<i>x</i> <sub>3</sub> <i>x</i> <sub>4</sub>	Interchange	0.78	1	0.446	3.39	4.19	
<i>x</i> <sub>3</sub> <i>x</i> <sub>5</sub>	Interchange	1.36	1	0.608	4.65	5.71	
<i>x</i> <sub>4</sub> <i>x</i> <sub>5</sub>	Interchange	0.33	1	0.338	2.57	3.25	
Error		0.789	6	0.789			
Total			24		80.94	100	

Table6:-Results for optimal objective Function and DesignVariables									
Iteration	$\begin{array}{c} x_1 \\ (mm) \end{array}$	<i>x</i> <sub>2</sub> (mm)	<i>x</i> <sub>3</sub> (mm)	(Degree)	x <sub>5</sub> (Degree)	Torque (N·m)	Head (m)	Efficienc y (%)	
1	6.6	151.0	191.7	115.1	25.4	381.7	19.3	71.4	
2	6.1	151.0	205.5	13.3	27.0	358.3	17.4	70.5	
3	8.1	116.3	161.3	11.1	27.0	217.1	10.3	72.6	
4	6.7	151.7	191.3	14.7	22.9	321.5	16.1	74.7	
5	6.7	151.7	191.1	14.7	23.0	323.1	16.1	74.5	
6	6.7	151.0	191.2	15.1	23.4	337.3	17.3	74.1	
7	6.8	151.0	191.1	14.7	23.8	335.3	16.7	73.5	
0pt.	6.7	151.7	191.4	14.7	24.7	315.1	15.7	75.1	
Reanalysis		311.3	15.3	71.1					

If y is the average data at each level and pn(xi) denotes each level of x. In the ANOVA, using an orthogonal polynomial is useful. Each effect in an orthogonal polynomial ANOVA is estimated separately.

Table 2 displays the computational fluid Dynamics design matrix for the L25 orthogonal arrays. ANOVA takes into account the order choice and each design variable of the surrogate model. These also employ the coefficient of the Chebyshev orthogonal polynomial. Lower order terms in Chebyshev orthogonal polynomials are independent of one another's rank. So, even if there were undiscovered higher rank coefficients or big coefficient gaps, the coefficient is calculated sequentially from lower to higher rank.

The results of the torque, head, and efficiency ANOVA are displayed in Tables 3 to 5. Each design variable's impact on the responsiveness is assessed independently using polynomial components in an ANOVA. Design variable interactions can be quantitatively verified. For instance, the torque effect of design variables x3 (shroud length) and x5 (hub angle) is 31.3%. The overall interchange effectiveness is also quite high at 63.4%, and the torque effect of the interaction x3x4 is 23.8%. having a similar

pattern to torque, the head. In comparison, efficiency has a 15.1%, a 19.4% impact on x2 (hub length), and x3 (shroud length). The Table 6 gives the summary of optimal solution from surrogate model.

R2 adj is a method for using orthogonal polynomials to increase the surrogate model's correctness. By applying the feasible direction approach, torque, head, and efficiency exhibit good approximation quality as 99.9%, 99.6%, and 98.6%, respectively. Using a surrogate model, find the best answer; reanalysis is required confirm accuracy. The optimal to solution's re-analysis revealed a maximum efficiency of 71.1%, a torque of 313.3 Nm, and a head of 15.3 m. Using a surrogate model, the best answers are  $x_{1}=7.8$  mm, x2=152.9 mm, x3=192.5 mm, x4=15.9°, and x5=23.7°. Efficiency errors are 1.7%, 2.5% for head and torque, and 0.7% for efficiency. As a result, this model exhibits excellent accuracy.

The initial and ideal model shapes are shown in Fig. 5. There have been significant adjustments to the impeller blade angles at the inlet and outflow when compared to the original model. Also, it shortens the chord and decreases a significant amount of torque that happens throughout the entire impeller when the airfoil impeller blade angle is taken into account.

According to this theory, torque decreased as total pump head decreased due to shape design optimization eliminating surplus design. Below is the pump performance curve as a result of ideal design: torque decreased by 22% to 313.3 Nm. Also Maximum head increased by 15.3 m (17.5%), efficiency increased by 71%, and it is 5% better than the original model. Overall pump head was 19.5metres when it was designed. Yet, the ideal model increases efficiency while satisfying the 15.5m of head. While the ideal model exhibits maximum efficiency, the rate flow of the first model shows decreased efficiency.

The fluid's vector distribution on the horizontal plane is depicted in Fig. 6. The fluid collapses on the leading edge of the initial model, and the change in velocity is substantial. Where the model is ideal, the cylindrical direction vector is steady. Moreover, leading edge collapse and unstable fluid flow have diminished or vanished. The vector is depicted horizontally in Fig. 7. Fluid flow became unstable from the hub area. However, the fluid flow becomes less unstable or the cvlindrical direction vector becomes stable.

### 6. Conclusion

The outcomes of shape optimization of pump impeller are presented in this research. An approximation model is designed for the effective optimal solution using an orthogonal polynomial in the Design of experiments. The key findings are as follows:

- (1) The hub region in the early impeller model had an unstable flow, while the cylindrical direction vector in the ideal model was steady.
- (2) The optimal impeller blade's maximum torque decreased by 21% at 313.7 Nm, head increased by17% at

15.5 m, and efficiency increased by 5.5% to 71.5% .

(3) Using the approximate model, the best values for the impeller blade are: x1=7.8 mm, x2=152.9 mm, x3=192.5 mm,  $x4=15.9^{\circ}$ , and  $x5=23.7^{\circ}$ . If the results of the CFD reanalysis are compared to the ideal solution, the margin of error for efficiency, maximum head, and torque, respectively, were roughly 1.7%, 2.5%, and 0.7%, respectively, and these values very accurately reflect actual situations.

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