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GEODETIC RESOLVING NUMBER OF SOME CYCLE RELATED GRAPHS

[1]T. Jebaraj and [2] Sajitha D

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Abstract

A set $L = \{l_1, l_2, \dots, l_k\} \subseteq V(G)$ is a geodetic resolving set of G if L is a geodetic set and for every $l_i, l_j \in V(G)$, the representations are distinct, that is, $R(l_i|L = \{l_1, l_2, \dots, l_k\}) \neq R(l_j|L = \{l_1, l_2, \dots, l_k\})$ for all $l_i, l_j \in V(G)$. The minimum cardinality of geodetic resolving set is known as a geodetic resolving number, it is denoted by $g_{res}(G)$. Here, we construct the extended mesh and enhanced mesh of ladder graph and step ladder graphs. Also we found the geodetic resolving number of honey comb regular triangulene mesh $HRrTM(n)$ and honey comb derived regular triangulene mesh $HDRrTM(n)$.

Keywords: *Geodetic set, Resolving set, Extended Mesh, Enhanced Mesh, Geodetic Resolving set.*

[1]Assistant Professor, Department of Mathematics, Malankara Catholic College, Mariagiri, Kanyakumari District, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamilnadu, India, Email ID: jebaraj.math@gmail.com
[2]Research Scholar, Reg. no: 20213082092001, Department of Mathematics, Malankara Catholic College, Mariagiri, Kanyakumari District, Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamilnadu, India, Email ID: sajithajomrv@gmail.com

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INTRODUCTION

In 1945, the concept " Resolving set" defined by Slater and in 2000, Chartrand et al. defined the concept of resolving number [see 1,2,7]. Extended mesh $EX(2,n)$ is a $2 \times n$ mesh in which every 4-cycle is turned into a complete graph. There are mn vertices in an extended mesh and we denote each vertex as (k,l) for $k = 1,2,\dots\dots,n$ and $l = 1,2$. Enhanced mesh $EN(2,n)$ is obtained by replacing each 4 cycle of $M(2,n)$ by a wheel, the hub of the wheel being a new vertex. Let $h_{kl}, 1 \le k \le n-1, l = 1$ be the hub vertices.

1. Geodetic resolving number of cycle related graphs

Definition 1.1

A set $T = \{t_1, t_2, \dots, t_k\} \subseteq V(G)$ is a geodetic resolving set of G if T is a geodetic set and for every $t_i, t_j \in V(G), R(t_i|T = \{t_1, t_2, \dots, t_k\}) \neq R(t_j|T = \{t_1, t_2, \dots, t_k\})$, that is, the representations are distinct for all $t_i, t_j \in V(G)$. The minimum cardinality of geodetic resolving set is known as a geodetic resolving number, it is denoted by $g_{res}(G)$.

Example 1.2

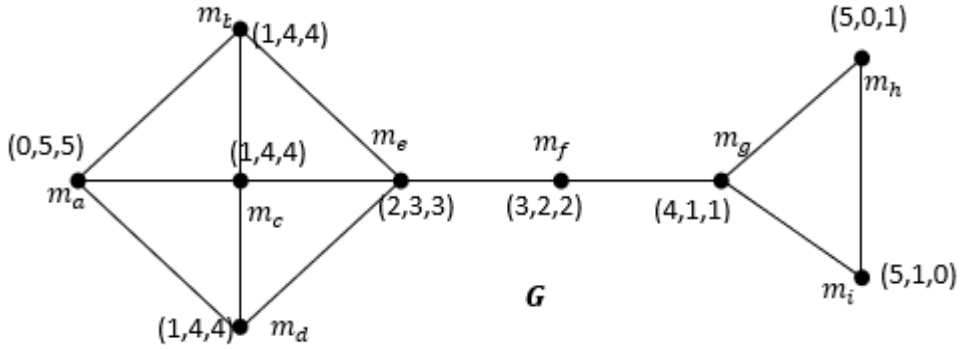

Figure 1.1: $R(m|M = \{m_a, m_h, m_i\})$; $m \in V(G)$

In Figure 1.1, $M = \{m_a, m_h, m_i\}$ is a geodetic set and $R(m_b|M = \{m_a, m_h, m_i\}) = R(m_c|M = \{m_a, m_h, m_i\}) = R(m_d|M = \{m_a, m_h, m_i\})$. Thus M does't form a geodetic resolving set in G. Thereafter, append the vertex m_b to M, then we observed from Table (1) , $M' = \{m_a, m_b, m_h, m_i\}$ make a geodetic resolving set in G so as $g_{res}(G) = 4$.

Table 1: $R(m|M')$

$m \in V(G)$	Representations
$R(m_a\|M')$	(0,1,5,5)
$R(m_b\|M')$	(1,0,4,4)
$R(m_c\|M')$	(1,1,4,4)
$R(m_d\|M')$	(1,2,4,4)
$R(m_e\|M')$	(2,1,3,3)
$R(m_f\|M')$	(3,2,2,2)
$R(m_g\|M')$	(4,3,1,1)
$R(m_h\|M')$	(5,4,0,1)
$R(m_i\|M')$	(5,4,1,0)

Remark: 1.3

From Figure 1.1, $g(G) = 3, res\,(G) = 2$ and $g_{res}(G) = 4$. Thus the geodetic resolving number and resolving number can be different.

Theorem: 1.4

For the Gear graph G_n , $g_{res}(G_n) = \begin{cases} n & \text{if} \quad n = 3 \\ n-1 & \text{if} \quad n = 4, 5 \\ n-2 & \text{if} \quad n \geq 6 \end{cases}$

Proof

Let $V(G_n) = \{l_1, l_2, \dots\dots, l_n\} \cup \{t_1, t_2, \dots\dots, t_n\} \cup v$, where $\deg(t_j) = 3, \deg(l_i) = 2;\ i, j \in [1, n]$ and the central vertex is v.

Case (i) If $n = 3$.

Suppose $N_1 = \{l, t\}$ forms a geodetic resolving set of G_3. Since $d(l, t) = \text{diam}\,(G_3)$ and every vertex of G_3 lies in $I[N_1]$. Also $R(l_i|N_1 = \{l, t\}) = R(t_j|N_1 = \{l, t\})$ for some l_i , $t_j \in V(G_3)$,which is conflict to the description of geodetic resolving set. Obviously $N_1' = \{l_1, l_2, l_3\}$ is a geodetic resolving set of G_3. Thus $g_{res}(G_3) = 3 = n$.

Table (2): Finding the representation of geodetic resolving set for G_n (n is even).

$v \in V(G_n)$	$R(v\|M_1' = \{l_1, l_2, \dots\dots, l_n\} - \{l_1, l_i = \frac{n+2}{2}\})$	$v \in V(G_n)$	$R(v\|M_1' = \{l_1, l_2, \dots\dots, l_n\} - \{l_1, l_i = \frac{n+2}{2}\})$
l_1	$2, \underbrace{4, 4, \dots\dots\dots, 4}_{n-4}, 2$	t_1	$\underbrace{3,3, \dots\dots, 3}_{n-3}, 1$
l_2	$0, 2, \underbrace{4, 4, \dots\dots\dots, 4}_{n-4}$	t_2	$1, \underbrace{3,3, \dots\dots, 3}_{n-3}, 1$
l_3	$2, 0, 2, \underbrace{4, 4, \dots\dots\dots\dots, 4}_{n-5}$	t_3	$1, 1, \underbrace{3,3, \dots\dots, 3}_{n-4}$
l_4	$4, 2, 0, 2, 4, 4, \dots\dots, 4$	t_4	$3, 1, 1, \underbrace{3,3, \dots\dots, 3}_{n-5}$
.	.	.	.
.	.	.	.
$l_{\frac{n}{2}}$	$\underbrace{4,4, \dots, 4}_{n-(i+2)}, 2, 0, \underbrace{4,4, \dots\dots, 4}_{n-i}$	$t_{\frac{n}{2}}$	$\underbrace{3,3, \dots\dots, 3}_{n-(i+2)}, 1, 1, \underbrace{3,3, \dots, 3}_{(n-i)}$
$l_{\frac{n+2}{2}}$	$\underbrace{4,4, \dots\dots, 4}_{n-(i+1)}, 2, 2, \underbrace{4,4, \dots\dots, 4}_{n-(i+1)}$	$t_{\frac{n+2}{2}}$	$\underbrace{3,3, \dots, 3}_{n-(i+1)}, 1, \underbrace{3,3, \dots, 3}_{n-i}$
.	$\underbrace{4,4, \dots\dots, 4}_{(n-i)}, 0, 2, \underbrace{4,4, \dots 4}_{n-(i+2)}$	$t_{\frac{n+4}{2}}$	$\underbrace{3,3, \dots, 3}_{n-i}, 1, \underbrace{3,3 \dots 3}_{n-(i+1)}$
.		$t_{\frac{n+6}{2}}$	$\underbrace{3,3, \dots, 3}_{n-i}, 1, 1, \underbrace{3,3, \dots, 3}_{n-(i+2)}$
.	.	.	.
.	.	.	.
l_n	$\underbrace{4,4, \dots 4}_{n-4}, 2, 0$	t_n	$\underbrace{3,3, \dots 3}_{n-4}, 1, 1$

Case (ii) If $n = 4, 5$

Let $M_1 = \{l_1, l_2, \dots\dots, l_{n-1}\}$ be the geodetic resolving set of G_n. Since $g(G_4) = 2 = \{t_t, l_j\}$ where $d(l_t, l_j) = diam\,(G_n)$and $g(G_5) = 3 = \{l, l_t, l_j\}$; $d(l, l_t) = d(l, l_j) = diam\,(G_n)$. Also $R(l|M_1 = \{l_1, l_2, \dots\dots, l_{n-1}\}) \neq R(t|M_1 = \{l_1, l_2, \dots\dots, l_{n-1}\})$ for every $l, t \in V(G_n)$. Hence $g_{res}(G_n) = n - 1, n \geq 4, 5$.

Case (iii) If $n \geq 6$

Choose $M_1' = \{l_1, l_2, \dots\dots, l_n\} - \{l_i, l_j\}$, where l_i, l_j doesn't have common neighbors. Based on Table (2), all the representations are different. Also M_1' forms a geodetic set, hence $I[M_1'] = V(G_n)$. Clearly M_1' make a geodetic resolving set of G_n and $|M_1'| = n - 2$. Suppose $g_{res}(G_n) < n - 2$. Assume $g_{res}(G_n) = n - 3$. Let $N_1' = \{l_1, l_2, \dots\dots, l_{n-3}\}$ be the geodetic resolving set of G_n. Since all the vertices of G_n $belongs$ to $I[N_1']$ and it satisfies the geodetic condition. But $R(l|N_1' = \{l_1, l_2, \dots\dots, l_{n-3}\}) = R(t|N_1' = \{l_1, l_2, \dots\dots, l_{n-3}\})$ for some l, t in G_n, which is conflict to the definition of resolving set. Thus $N_1' = \{l_1, l_2, \dots\dots, l_{n-2}\}$ is the geodetic resolving set of G_n. Hence $g_{res}(G_n) = n - 2$.

Theorem: 1.5

For the connected graph G with cardinality n, $1 \leq res\ (G) \leq g(G) \leq g_{res}(G) \leq n$.

Proof:

A resolving set needs atleast one vertex and so $res(G) \geq 1$. Also each geodetic set contains minimum two vertices and size of the resolving set is doesn't exceeds the geodetic set, so as $res\ (G) \leq g(G)$. As well as, every geodetic resolving set act as a geodetic set, so as $g(G) \leq g_{res}(G)$. Further more, each geodetic set needs atmost n vertices so that $g_{res}(G) \leq n$.

Remark: 1.6

From table (3), the bounds are clear **.**

Table (3): [Bounds of G]

G	$g(G)$; $g_{res}(G)$; $res\ (G)$
Path graph $P_n, n \geq 3$	$res\ (P_n) = 1$ $g(P_n) = g_{res}(P_n)\ and\ g_{res}(P_n) < n.$
Cycle graph $C_{2n}, n \geq 2$	$res\ (C_{2n}) > 1$ $res\ (C_{2n}) = g(C_{2n})$ $g(C_{2n}) < g_{res}(C_{2n})$
Complete graph $K_n, n \geq 3$	$res(K_n) < g(K_n)$ $g_{res}(K_n) = n$

Theorem: 1.7

If $G = C_{12+8n}H_{8+4n}$ is an n- acene$[b, h]$biphenylene, then $g_{res}(G) = 3$

Proof:

Consider $G = C_{12+8n=t}H_{8+4n=m}$

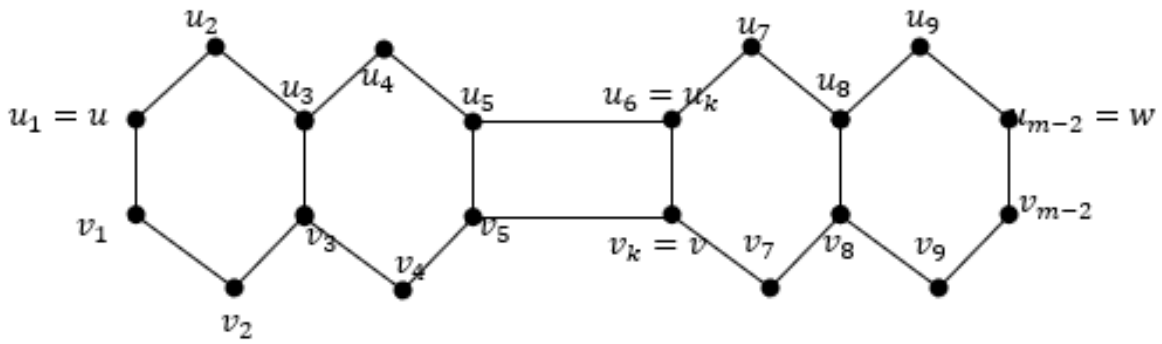

Figure1.2 (a): Dibenzo [b, h] biphenylene $G = C_{20}H_{12}$

Suppose $M = \{u, v_{m-2}\}$ forms a geodetic resolving set of G. Thereafter $d(u, v_{m-2}) = diam(G)$. From Figure 1.2 (a) u, v_{m-2} are mutually eccentric vertices and all the vertices of G lies in $I[u, v_{m-2}]$ but the representations $R(u_i|M)$ and $R(v_j|M)$ are equal for some u_i, v_j in G.

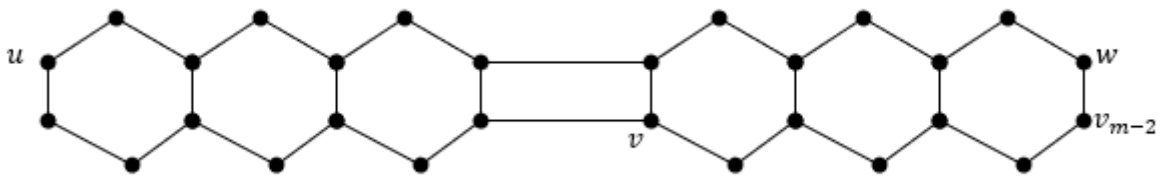

Figure 1.2 (b): Dinaphtho[b,h]biphenylene $\boldsymbol{G} = C_{28}H_{16}$

It is conflict to the description of geodetic resolving set. Let $M' = \{u, v, w\}$ be the geodetic resolving set of G. From Figure 1.2 (b), $d(u, w) = diam(G) - 1$; u, w exist on the first and last cycle of G, and $d(u, v) = rad(G), d(v, w) = rad(G) - 1$. Since all the vertices of G lies in $I[M']$ and from table (4), the representations $R(u|M')$ and $R(v|M')$ are different for all $u, v \in V(G)$. Thus $g_{res}(G) = 3$

Table (4): Finding the representation of geodetic resolving set for G(Figure 1.2 b)

$u \in V(G)$	$R(u\|M')$	$v \in V(G)$	$R(v\|M')$
u_1	$(0, \quad r, d-1)$	v_1	$(1 \quad , r-1 \ , d)$
u_2	$(1, r-1, d-2)$	v_2	$(2, r-2, d-1)$
u_3	$(2, r-2, d-3)$	v_3	$(3, r-3, d-2)$
.	$(. \quad 3 \quad .)$	.	$(. \quad 2 \quad .)$
.	$(. \quad 2 \quad .)$	.	$(. \quad 1 \quad .)$
u_k	$(r-1, 1 , d-r)$	v_k	$(r, 0, d-(r-1))$
.	$(. \quad 2 \quad .)$	.	$(. \quad 1 \quad .)$
.	$(. \quad 3 \quad .)$	.	$(. \quad 2 \quad .)$
u_{m-3}	$(d-2, r-2, 1)$	v_{m-3}	$(d-1, r-3, 2)$
u_{m-2}	$(d-1, r-1, 0)$	v_{m-2}	$(d , r-2, \quad 1)$

Theorem: 1.8

If $G = C_{4n+2}H_{2n+4}$ is an [n] phenacene then $g_{res}(G) = \begin{cases} 3 & if \quad n = 2 \\ \text{n} & if \quad n \geq 3 \end{cases}$

Proof

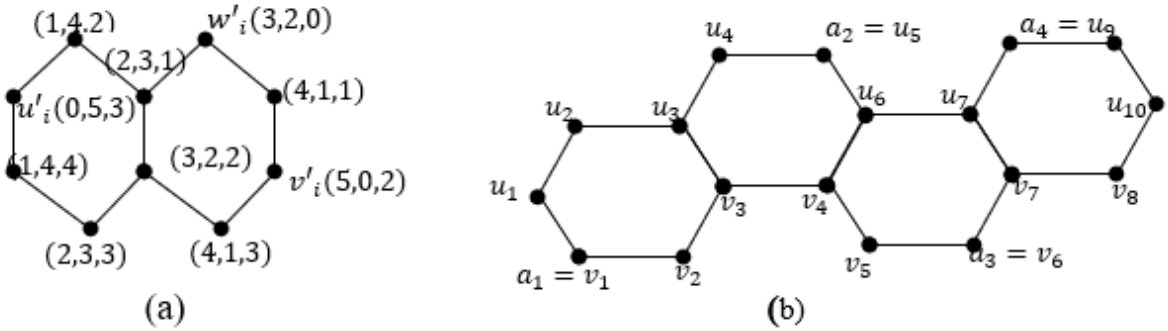

Figure 1.3 Representation of geodetic resolving set of (a) $C_{10}H_8$ (b) $C_{18}H_{12}$

Case (i) $\boldsymbol{If\ n = 2}$

Clearly $G = C_{10}H_8$ and one edge is common for two even cycles. The geodetic number of C_{2n} is 2. Let $S = \{u'_i, v'_i\}$ be the geodetic set of G. Further u'_i and v'_i are mutually eccentric vertices and $d(u'_i, v'_i) = diam(G)$. Obviously every vertex of G lies in $I[S]$ and the representations $R(u_k|S)$ and $R(v_k|S)$ are equal for some u_k, v_k in G. Hence $S' = \{u'_i, w'_i, v'_i\}$ act as a geodetic resolving set in G. Thus $g_{res}(G) = 3$.

Case (ii) If $\boldsymbol{n \geq 3}$

Choose $S = \{a_1, a_2, \dots, a_n\}$ is a geodetic resolving set of G. Since none of the vertex in S belongs to the same cycle of G. Therefore each vertex in S belongs to distinct cycles of G and it satisfies the condition of geodetic resolving set. Obviously S is geodetic and $R(u_i|S)$ and $R(v_j|S)$ are different for every u_i, v_j in G. Suppose $g_{res}(G) < n$. Consider $S' = \{a_1, a_2, \dots, a_{n-1}\}$ be the geodetic resolving set in G. Certainly it is a geodetic set and some representations are same with respect to S', which is a contradiction. Thus $S = \{a_1, a_2, \dots, a_n\}$ act as a geodetic resolving set in G so as $g_{res}(G) = n$.

Table (5): The representation of geodetic resolving number for [4] phenacene [Figure 1.3 (b)]

$v \in V(G)$	$r(v\|S = \{a_1, a_2, a_3, a_4\})$	$v \in V(G)$	$r(v\|S = \{a_1, a_2, a_3, a_4\})$
u_1	(1,4,6,8)	v_1	(0,5,5,7)
u_2	(2,3,5,7)	v_2	(1,4,4,6)
u_3	(3,2,4,6)	v_3	(2,3,3,5)
u_4	(4,1,5,5)	v_4	(3,2,2,4)
u_5	(5,0,4,4)	v_5	(4,3,1,5)
u_6	(4,1,3,3)	v_6	(5,4,0,4)
u_7	(5,2,2,2)	v_7	(6,3,1,3)
u_8	(6,3,3,1)	v_8	(7,4,2,2,)
u_9	(7,4,4,0)		
u_{10}	(8,5,3,1)		

2. Construction of the extended mesh and enhanced mesh from ladder graph

Theorem: 2.1

If a step ladder graph $S(L_n)$, $g_{res}(G) = \begin{cases} 3 & if \quad G = S(L_n) \\ 3 & if \quad G = EN[S(L_n)] \\ n+2 & if \quad G = EX[S(L_n)] \end{cases}$

Proof:

Case (i) : If $G = S(L_n), n \geq 2$

$Let\ V[S(L_n)] = \{x_{11}, x_{12}, \ldots\ldots, x_{1n}\} \cup \{x_{21}, x_{22}, \ldots\ldots x_{2n}\} \cup \{x_{31}, x_{32}, \ldots\ldots x_{3n-1}\} \cup \{x_{41}, x_{42}, \ldots\ldots x_{4n-2}\} \cup \ldots\ldots \cup \{x_{n1}, x_{n2}\}$ and let $H = \{x_{1n}, x_{n1}\}$ be the geodetic set of G. Also $d(x_{1n}, x_{n1}) = diam\ (G)$ and all the vertices of G lies in $I[x_{1n}, x_{n1}]$. Hence $g(G) = 2$. Further more $R(x_{ij} \mid H = \{x_{1n}, x_{n1}\}) = R(x_{ji} \mid H = \{x_{1n}, x_{n1}\})$ for some x_{ij} and $x_{ji}, 1 \leq i \leq j \leq n$. Obviously $H' = \{x_{11}, x_{1n}, x_{n2}\}$ forms a geodetic resolving set of G. Since $g(G) = 2$ and $R(x_{ij} | H' = \{x_{11}, x_{1n}, x_{n2}\}) \neq R\left(x_{ji} \middle| H' = \{x_{11}, x_{1n}, x_{n2}\}\right)$ for every $x_{ij}, x_{ji} \in V\ [S\ (L_n)]$. Thus $g_{res}(G) = 3$.

Case (ii) If $G = EN\ [S(L_n)]$

Consider $V(G) = V[S\ (L_n)] \cup \{h_{11}, h_{12}, \ldots\ldots, h_{1(n-1)}\} \cup \{h_{21}, h_{22}, \ldots\ldots, h_{2(n-2)}\} \cup \ldots\ldots \cup h_{(n-1)1}$,where $h_{ij}, 1 \leq i, j \leq n-1$ is the hub of each wheel. Suppose $H = \{x_{1n}, x_{n1}\}$ is the geodetic resolving set of G. Further $d\ (x_{1n}, x_{n1}) = e\ (x_{1n})$ and x_{1n}, x_{n1} are mutually eccentric vertices of G. But $R(x_{ij} | H = \{x_{1n}, x_{n1}\}) = R\ (x_{ji} | H = \{x_{1n}, x_{n1}\})$ for some x_{ij} and x_{ji} , which is conflict to the description of geodetic resolving set. Clearly from Figure 1.4 (a), $H' = \{x_{11}, x_{1n}, x_{n2}\}$ forms a geodetic resolving set of G. Further it act as a geodetic set as well as it is a resolving set. Thus $g_{res}(G) = 3$.

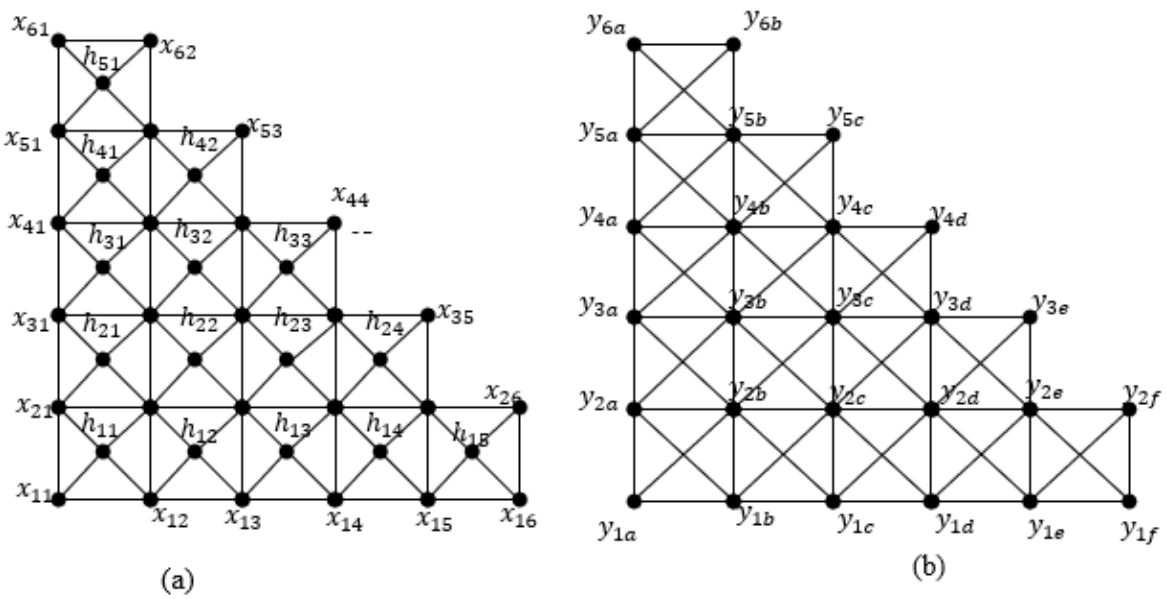

Figure 1.4: (a) EN[S(L_6)] (b): EX[S($L_{n=6}$)]

Case (iii) If $G = EX[S(L_n]$

$EX\ [S(L_n)]$ contains $3n$ complete graphs. Each one is in the form of K_4 and the degree of each vertex in K_4 is 3. Clearly $g(K_4) = 4$. Suppose $H = \{y_{11}, y_{1n}, y_{n1}, y_{n2}\}$ forms a geodetic resolving set in $G = EX[S(L_n]$ .Theorem 1.1[see 11], extreme vertices

$y_{2n}, y_{3(n-1)}, \dots, y_{(n-1)3}$ lies in H, which is a contradiction. Therefore we include each extreme vertices of G to H. Now $H' = \{y_{11}, y_{1n}, y_{2n}, y_{3n-1}, y_{4n-2}, \dots\dots\dots, y_{n2}, y_{n1}\}$ is a geodetic set and the representations are different for all $y_{1i}, y_{in} \in G$. Thus $H' = \{y_{11}, y_{n1}, y_{1n}, y_{2n}, \dots\dots\dots, y_{n2}\}$ act as a geodetic resolving set in G. So as $g_{res}(G) = n+2$

Corollary 2.2

If G is an $2 \times$ n; $(n = gk)$ mesh then $g_{res}(G) = \begin{cases} 3 & if \quad G = L_{gk} \\ 3 & if \quad G = EN(L_{gk}) \\ gk+2 & if \quad G = EX(L_{gk}) \end{cases}$

Poof

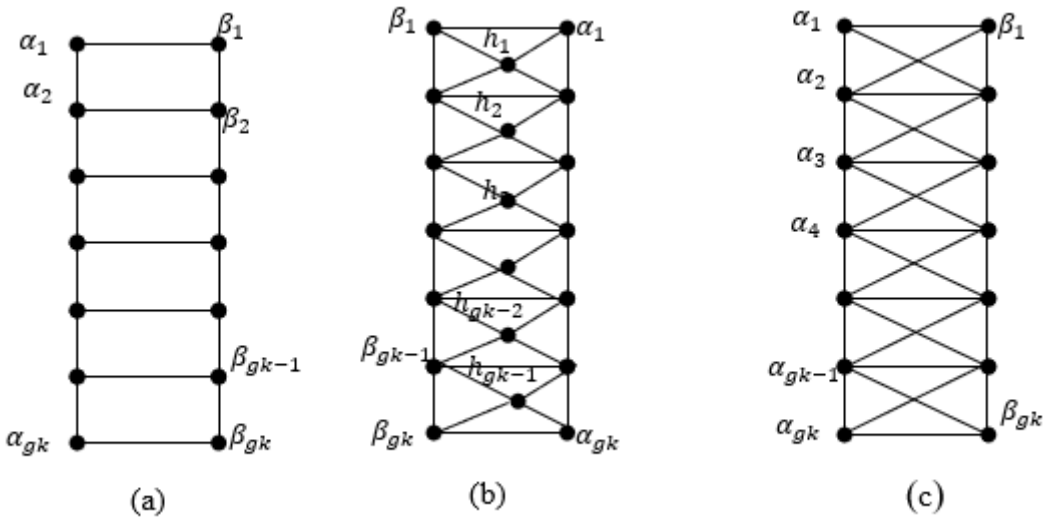

Figure 1.5: (a) L_{gk} (b): $EN(L_{gk})$ (c): $EX(L_{gk})$

[**Table 6**]: The Representation of geodetic resolving set for Figure 1.5 (a) ,(c)

$\boldsymbol{v} \in \boldsymbol{L}_{gk}$	$\boldsymbol{R(v\|H} = \{\boldsymbol{\alpha_1}, \boldsymbol{\alpha}_{gk}, \boldsymbol{\beta}_{gk-1}\}$	$\boldsymbol{R(v\|H} = \{\boldsymbol{\beta_1}, \boldsymbol{\alpha_1}, \boldsymbol{\alpha_2}, \dots\dots \boldsymbol{\alpha}_{gk}, \boldsymbol{\beta}_{gk}\})\boldsymbol{v} \in \boldsymbol{EX(L}_{gk}\boldsymbol{):}gk \geq \mathbf{3}$
α_1	$(0, gk-1, gk-1)$	$(1, 0, 1, 2, 3, \dots\dots (gk-2), (gk-1), (gk-1))$
α_2	$(1, gk-2, gk-2)$	$(1, 1, 0, 1, 2 \dots, gk-3, gk-2, gk-2)$
α_3	$(2, gk-3, gk-3)$	$(2, 2, 1, 0, 1, \dots\dots gk-4, gk-3, gk-3)$
α_4	..	$(3, 3, 2, 1, 0, 1, \dots\dots gk-4, gk-4)$
α_{gk-1}	$(gk-2, 1, 1)$	$\dots\dots$
α_{gk}	$(gk-1, 0, 2)$	$(gk-1, gk-1, gk-2 \dots\dots 1, 0, 1)$
β_1	$(1, gk, gk-2)$	$(0, 1, 1, 2, 3, \dots\dots gk-3, gk-2, gk-1, gk-1)$
β_2	$(2, gk-1, gk-3)$	$(1, 1, 1, 1, gk-(gk-2), \dots\dots\dots gk-3, gk-2, gk-2)$

β_3	$(3, gk-2, gk-4)$	$(2, 2, 1, 1, 1, gk-(gk-2) \ldots\ldots gk-4, gk-3, gk-3)$
β_4	….	$(3, 3, 2, 1, 1, 1, gk-(gk-2), \ldots, gk-5, gk-4, gk-4)$
…	…..	……
β_{gk-1}	$(gk-1, 2, 0)$	$(gk-2, gk-2, gk-3, gk-4, \ldots, gk-(gk-2), 1, 1, 1, 1)$
β_{gk}	$(gk, 1, 1)$	$(gk-1, gk-1, gk-2, gk-3, \ldots\ldots gk-(gk-2), 1, 1\ 0)$

Table (7): The Representation of geodetic resolving set for Figure 1.5 (b)

$\boldsymbol{v} \in \boldsymbol{EN}(\boldsymbol{L}_{gk}$	$\boldsymbol{R}(\boldsymbol{v}/\boldsymbol{H} = \{\alpha_1, \alpha_{gk}, \beta_{gk-1}\})$	$\boldsymbol{v}$	$\boldsymbol{R}(\boldsymbol{v}/\boldsymbol{H} = \{\alpha_1, \alpha_{gk}, \beta_{gk-1}\}$	$\boldsymbol{v}$	$\boldsymbol{R}(\boldsymbol{v}/\boldsymbol{H} = \{\alpha_1, \alpha_{gk}, \beta_{gk-1}\}$
α_1	$(0, gk-1, gk-1)$	h_1	$(1, gk-1, gk-2)$	β_1	$(1, gk, gk-2)$
α_2	$(1, gk-2, gk-2)$	h_2	$(2, gk-2, gk-3)$	β_2	$(2, gk-1, gk-3)$
α_3	$(2, gk-3, gk-3)$	h_3	$(3, gk-3, gk-4)$	.	…..
.	….	.	…..	.	….
.	….	.	…..	.	….
α_{gk-1}	$(gk-2, 1, 1)$	.	…..	β_{gk-1}	$(gk-1, 2, 0)$
α_{gk}	$(gk-1, 0, 2)$	h_{gk-1}	$(gk-1, 1, 1)$	β_{gk}	$(gk, 1, 1)$

Honey Comb Regular Triangulene Mesh $\boldsymbol{HRrTM}(\boldsymbol{n})$

Theorem: 2.3

If $G = HRrTM\ (n)$ is a honey comb regular triangulene Mesh then

$$g_{res}(G) = \begin{cases} \frac{n+7}{3} & if \quad n = 2m+k, \quad m \geq 1, \quad k = m-1 \\ \frac{n+6}{3} & if \quad n = 3m, \quad m \geq 1 \\ \frac{n+5}{3} & if \quad n = 4m-k, \quad m \geq 1, \quad k = m-1 \end{cases}$$

Proof:

Honey comb regular Triangulene Mesh $HRrTM\ (n), n \geq 2$ is a group of hexagons arranged in the form of pyramid. It contain n hexagonal layers and in each layer one cycle is decreased from the previous.

Case (i) $\boldsymbol{n = 2, 3\ or\ 4}$

Let $H = \{l_1, l_2, l_3\}$ be the geodetic resolving set of G. Since $d\ (l_1, l_2) = d(l_2, l_3) = d(l_3, l_1) = diam\ (G) - 1$ and all the vertices of G lies in I[S]. Also representations $R(l|H), R(l'|H)$ are different for every l, l' in G. Thus g_{res} (G) = 3.

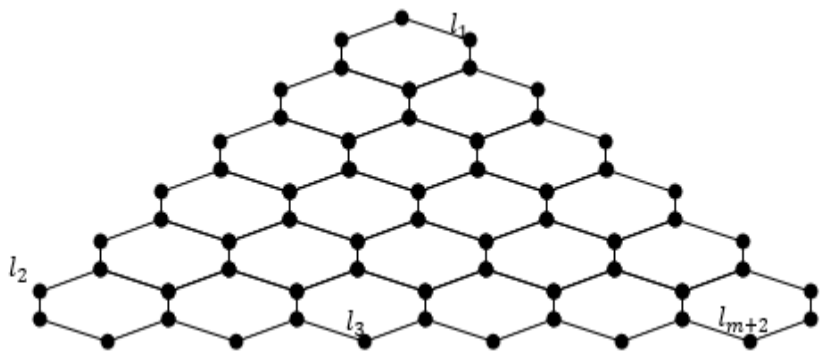

Figure 1.6**:** Representing the geodetic resolving set of HRrTM (6)

Case (ii) : If $n = \begin{cases} 2m + k & m \geq 2 \quad k = m - 1 \\ 3m & m \geq 2 \\ 4m - k & m \geq 2 \quad k = m - 1 \end{cases}$

Let us indicate the vertex l_1 in the final layer and $l_2, l_3, \ldots\ldots l_{m+2}$; $2 \leq m \leq n$ in the initial layer. First we find the lower end of geodetic resolving number is $m + 2$, that is $g_{res}(G) \geq m + 2$. Suppose $g(G) < m + 2$. Accordingly $H = \{l_1, l_2, l_{m+2}\}$. Since each vertex in H forms two distinct geodesic paths between themselves. So it cover only the three side layers of G, it is conflict to the description of geodetic resolving set. Hence we require atleast $m + 2$ vertices to attain the condition of geodetic resolving set. Moreover, we will verify an upper end of geodetic resolving set of G is m+2, that is $g_{res}(G) \leq m + 2$. Let $L = \{l_1, l_2, \ldots, l_{m+2}\}$ be the geodetic resolving set of G. Since $d(l_1, l_2)$; $d(l_1, l_3)$; $\ldots\ldots d(l_1, l_{m+2})$ are same as $\text{diam}(G) - 1$ and $d(l_2, l_{m+2}) = \text{diam}(G) - 1$. Further more all the vertices of G lies in $I[L]$ and $R(l|L)$, $R(l'|L)$ are different for every l, l' in G. Thus $|L| = m + 2$ and $g_{res}(G) \leq m + 2$. For distinct values of m, we get different forms of geodetic resolving number which is same as $m + 2$.

Definition: 2.4

A graph G is called a perfect geodetic resolving if $g(G) = res(G) = g_{res}(G)$.

Theorem: 2.5 If $G = HDRrTM(n)$ is a honey comb Derived Regular Triangulene Mesh then G is a perfect geodetic resolving .

Proof

Honey Comb Derived Regular Triangulene Mesh of dimension 1 is obtained by taking the union of honey comb Regular Triangulene Mesh and its stellation. Each hexagons of HRrTM(n) is turned into a wheel graph. To prove G is a perfect geodetic resolving ie, $g(G) = res(G) = g_{res}(G)$. First we find $g(G) = g_{res}(G)$. Suppose $g(G) \neq g_{res}(G)$ that is only the possibility is $g(G) < g_{res}(G)$. Let $H = \{u'', v'', w''\}$ be the geodetic resolving set of G.

(5,1,6) (2,4,4) (5,6,1)

(6,0,6) (4,3,3)

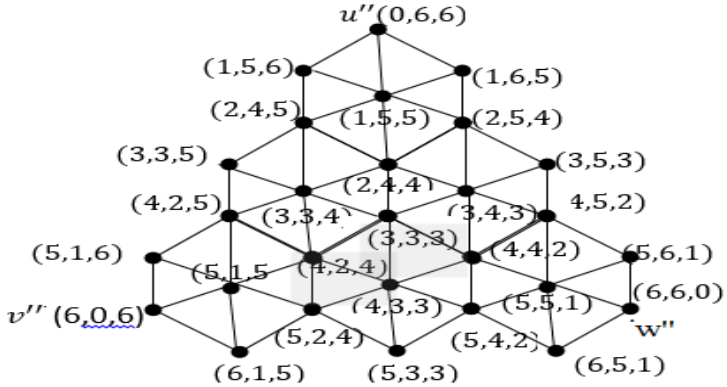

FIGURE 1.7: Representing the geodetic resolving set of $HDRrTM(3)$

Since $d(u'',v'') = d(u'',w'') = d(v'',w'') = \mathrm{diam}(G)$ and all the vertices of G lies in I[H], representations $R(u_i|H)$ and $R(v_i|H)$ are different for every $u_i, v_i \in V(G)$. Consider $H' = \{u'',v''\} \subset H$, then the vertex $w'' \notin I[H']$. Obviously $H' = \{u'',v''\}$ is not a geodetic set, which is conflict to our assumption. Thus $g(G) = g_{res}(G)$. Next we confirm $res(G) = g_{res}(G)$. Assume $res(G) \neq g_{res}(G)$, that is only the possibility is $res(G) < g_{res}(G)$. Let $H = \{u'',v'',w''\}$ be the geodetic resolving set of G. Since $d(u'',v'') = d(u'',w'') = d(v'',w'') = \mathrm{diam}(G)$ then all the vertices of G lies in I[H] and the representations $R(u_i|H)$ and $R(v_i|H)$ are different for every $u_i, v_i \in V(G)$. Consider $H'' = \{v'',w''\} \subset H$, then the representations $R(u_i|H'')$ and $R(v_i|H'')$ are equal for some $u_i, v_i \in V(G)$, which is conflict to our assumption. Thus $res(G) = g_{res}(G)$, and we attain $g(G) = res(G) = g_{res}(G)$. Hence G is a perfect geodetic resolving ■

REFERENCES

[1] Chartrand G, Eroh L, Jhonson M A, Oellermann O R 2000 Resolvability in graphs and the metric dimension of a graph , Discrete Applied Mathematics. 105 (1-3): 99-113.

[2] Brigham R C, Chartrand G, Dutton R D and Zhang P 2003 Resolving domination number of a graph Mathematica Bohemica 128(1): 25-36.

[3] Diestel R 2005 Graph Theory, p.8.3rd Edition Springer- Verlag.

[4] R. Muntean and P. Zhang, On geodomination in graphs, Congr. Numer.143 (2000) 161–174.

[5] J. Xu, Topological Structure and Analysis of Interconnection Networks, Springer, 2001.

[6] G. Chartrand, C. Poisson, P. Zhang: Resolvability and the upper dimension of graphs. International J. Comput. Math. Appl. 39 (200), 19-28.

[7] G.Chartrand, F.Harary and P.Zhang, On the geodetic number of a graph, Networks 39 (2002) 1-6.

[8] Slater, PJ 1975, 'Leaves of trees', Congr. Numer., vol. 14, pp. 549-559.